Capital Allocation using the RMK Algorithm

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Capital Allocation: The Problem

- How can total capital (and costs) be allocated to sources of risk, so that:
  - Components add up to subtotals and the total
  - Capital is in proportion to risk contributed
  - Diversification is attributed to its sources
  - The user specifies the risk metric
  - Theory behind the method is connected to financial pricing theory

An algorithm

"RMK" has these properties, plus:

- Relatively simple – it's weighted averages
- Can be explained fairly easily
- Evaluates risk from the total-company, "top-down" view
  - Vs evaluating each line's stand-alone risk

RMK Algorithm

Central principle

Each component is evaluated to measure its contribution to total-company risk.

RMK Algorithm: Steps

- Simulate possible outcomes by component & total.
- Calculate expected values E[x] of everything
- Select a risk measure on total company outcomes
- Express the risk measure as leverage factors (higher factors for worse outcomes)
- Calculate risk-adjusted expected values E[Rx]
  - These are the weighted averages
- Allocate capital in proportion to risk, by:

Risk ~ Risk-Adjusted Expected Value – Expected Value

Risk ~ E[Rx] – E[x]

RMK Algorithm: A Capital Allocation Example

Scenario	Underwriting	Investment	Total Company	Risk Leverage	Actual Probability	Risk-Adjusted Probability
1	-1,700	700	-1,000	3.50	10%	24%
2	-300	-700	-1,000	3.50	10%	24%
3	-800	1,100	300	1.50	10%	10%
4	1,000	0	1,000	1.10	10%	8%
5	-300	1,800	1,500	0.90	10%	6%
6	200	1,400	1,600	0.90	10%	6%
7	-200	2,100	1,900	0.85	10%	6%
8	-500	2,600	2,100	0.80	10%	6%
9	2,000	800	2,800	0.70	10%	5%
10	1,800	2,200	4,000	0.60	10%	4%
					100%	100%
Expected Income	**120**	**1,200**	**1,320**	1.44		
Risk-Weighted Expected Income	-368	716	348			
Risk Measurement	488	484	972			
Capital Allocation	**50%**	**50%**	**100%**			
Capital	5,020	4,980	10,000			
Return on Risk-Adjusted Capital	**2.4%**	**24.1%**	**13.2%**			
Hurdle Rate for Value Creation	9.7%	9.7%	9.7%			
Value Creation	**-368**	**716**	**348**			

RMK Algorithm: A Capital Allocation Example

Scenario	Underwriting	Property	Casualty	Total Company	Risk Leverage
1	-1,700	-500	-1,200	-1,000	3.50
2	-300	-700	400	-1,000	3.50
3	-800	-600	-200	300	1.50
4	1,000	100	900	1,000	1.10
5	-300	-100	-200	1,500	0.90
6	200	500	-300	1,600	0.90
7	-200	300	-500	1,900	0.85
8	-500	100	-600	2,100	0.80
9	2,000	800	1,200	2,800	0.70
10	1,800	700	1,100	4,000	0.60
Expected Income	**120**	60	60	**1,320**	
Risk-Weighted Expected Income	**-368**	-231	-137	**348**	
Risk Measurement	**488**	291	197	**972**	
Capital Allocation	**50%**	30%	20%	**100%**	
Capital	**5,020**	2,994	2,026	**10,000**	
Return on Risk-Adjusted Capital	**2.4%**	2.0%	3.0%	**13.2%**	
Hurdle Rate for Value Creation	**9.7%**	9.7%	9.7%	**9.7%**	
Value Creation	**-368**	-231	-137	**348**	

RMK Algorithm: A Capital Allocation Example

Scenario	Investment	Equities	Fixed Income	Other Invested	Total Company	Risk Leverage
1	700	1,100	-400	0	-1,000	3.50
2	-700	-400	-100	-200	-1,000	3.50
3	1,100	100	1,300	-300	300	1.50
4	0	-700	800	-100	1,000	1.10
5	1,800	500	1,800	-500	1,500	0.90
6	1,400	400	400	600	1,600	0.90
7	2,100	-100	1,700	500	1,900	0.85
8	2,600	200	1,300	1,100	2,100	0.80
9	800	200	200	400	2,800	0.70
10	2,200	100	1,600	500	4,000	0.60
Expected Income	**1,200**	**140**	**860**	**200**	**1,320**	
Risk-Weighted Expected Income	716	203	463	50	348	
Risk Measurement	484	-63	397	150	972	
Capital Allocation	**50%**	**-6%**	**41%**	**15%**	**100%**	
Capital	4,980	-650	4,084	1,545	10,000	
Return on Risk-Adjusted Capital	**24.1%**	**-21.6%**	**21.1%**	**12.9%**	**13.2%**	
Hurdle Rate for Value Creation	9.7%	9.7%	9.7%	9.7%	9.7%	
Value Creation	**716**	**203**	**463**	**50**	**348**	

RMK Algorithm: A Capital Allocation Example

Scenario	Underwriting	Property	Casualty	Investment	Equities	Fixed Income	Other Invested	Total Company	Risk Leverage
1	**-1,700**	-500	-1,200	**700**	1,100	-400	0	**-1,000**	3.50
2	**-300**	-700	400	**-700**	-400	-100	-200	**-1,000**	3.50
3	**-800**	-600	-200	**1,100**	100	1,300	-300	**300**	1.50
4	**1,000**	100	900	**0**	-700	800	-100	**1,000**	1.10
5	**-300**	-100	-200	**1,800**	500	1,800	-500	**1,500**	0.90
6	**200**	500	-300	**1,400**	400	400	600	**1,600**	0.90
7	**-200**	300	-500	**2,100**	-100	1,700	500	**1,900**	0.85
8	**-500**	100	-600	**2,600**	200	1,300	1,100	**2,100**	0.80
9	**2,000**	800	1,200	**800**	200	200	400	**2,800**	0.70
10	**1,800**	700	1,100	**2,200**	100	1,600	500	**4,000**	0.60
Expected Income	**120**	60	60	**1,200**	140	860	200	**1,320**	
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Hurdle Rate for Value Creation	**9.7%**	9.7%	9.7%	**9.7%**	9.7%	9.7%	9.7%	**9.7%**	
Value Creation	**-368**	-231	-137	**716**	203	463	50	**348**	

Selecting a risk measure

- Many standard risk measures (such as TVaR) can be expressed in the form of weights.
- See Kreps, PCAS 2005 for major examples.
- Example: Net loss outcomes > 1, net gain outcomes = 1.
  - Measures tail of distribution where losses occur.
- In general, risk measure weights are:
  - Non-negative,
  - Higher for worse ("riskier") outcomes, lower for better outcomes.

Summary of useful properties

- General framework for applying additive capital allocation methods
- Flexible choice of risk measure – can experiment
- Allocates risk down to detail level (state, tier)
- Consistent with financial theory
  - Can be used to generate risk-neutral prices
- Relatively simple / transparent

Selected References

- Halliwell, "Conjoint Prediction of Paid and Incurred Losses," *CAS Forum*, Summer 1997, volume 1 (thank you Dave Clark for this one)
- Ruhm / Mango, "A Risk Charge Calculation Based on Conditional Probability," Bowles Symposium, Atlanta, April 2003
- Kreps, "Riskiness Leverage Ratios," *Proceedings of the CAS*, 2005
- Clark, "Reinsurance Applications for the RMK Framework," *CAS Forum*, Spring 2005