Capital Allocation using the RMK Algorithm

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Capital Allocation: The Problem

- How can total capital (and costs) be allocated to sources of risk, so that:
 - Components add up to subtotals and the total
 - Capital is in proportion to risk contributed
 - Diversification is attributed to its sources
 - The user specifies the risk metric
 - Theory behind the method is connected to financial pricing theory

An algorithm

"RMK" has these properties, plus:

- Relatively simple it's weighted averages
- Can be explained fairly easily
- Evaluates risk from the total-company, "top-down" view
 - Vs evaluating each line's stand-alone risk

RMK Algorithm

Central principle

Each component is evaluated to measure its contribution to total-company risk.

RMK Algorithm: Steps

- Simulate possible outcomes by component & total.
- Calculate expected values E[x] of everything
- Select a risk measure on total company outcomes
- Express the risk measure as leverage factors (higher factors for worse outcomes)
- Calculate risk-adjusted expected values E[Rx]
 - These are the weighted averages
- Allocate capital in proportion to risk, by:
- Risk ~ Risk-Adjusted Expected Value Expected Value **Risk ~ E[Rx] – E[x]**

			Total		Actual	Risk-Adjusted
<u>Scenario</u>	<u>Underwriting</u>	<u>Investment</u>	<u>Company</u>	<u>Risk Leverage</u>	<u>Probability</u>	<u>Probability</u>
1	-1,700	700	-1,000	3.50	10%	24%
2	-300	-700	-1,000	3.50	10%	24%
3	-800	1,100	300	1.50	10%	10%
4	1,000	0	1,000	1.10	10%	8%
5	-300	1,800	1,500	0.90	10%	6%
6	200	1,400	1,600	0.90	10%	6%
7	-200	2,100	1,900	0.85	10%	6%
8	-500	2,600	2,100	0.80	10%	6%
9	2,000	800	2,800	0.70	10%	5%
10	1,800	2,200	4,000	0.60	<u>10%</u>	<u>4%</u>
					100%	100%
Expected Income	120	1,200	1,320	1.44		
Risk-Weighted Expected Income	-368	716	348			
Risk Measurement	488	484	972			
Capital Allocation	50%	50%	100%			
Capital	5,020	4,980	10,000			
Return on Risk-Adjusted Capital	2.4%	24.1%	13.2%			
Hurdle Rate for Value Creation	9.7%	9.7%	9.7%			
Value Creation	-368	716	348			

				Total	
<u>Scenario</u>	Underwriting	Property	Casualty	<u>Company</u>	<u>Risk Leverage</u>
1	-1,700	-500	-1,200	-1,000	3.50
2	-300	-700	400	-1,000	3.50
3	-800	-600	-200	300	1.50
4	1,000	100	900	1,000	1.10
5	-300	-100	-200	1,500	0.90
6	200	500	-300	1,600	0.90
7	-200	300	-500	1,900	0.85
8	-500	100	-600	2,100	0.80
9	2,000	800	1,200	2,800	0.70
10	1,800	700	1,100	4,000	0.60
Expected Income	120	60	60	1,320	
Risk-Weighted Expected Income	-368	-231	-137	348	
Risk Measurement	488	291	197	972	
Capital Allocation	50%	30%	20%	100%	
Capital	5,020	2,994	2,026	10,000	
Return on Risk-Adjusted Capital	2.4%	2.0%	3.0%	13.2%	
Hurdle Rate for Value Creation	9.7%	9.7%	9.7%	9.7%	
Value Creation	-368	-231	-137	348	

					Total	
<u>Scenario</u>	<u>Investment</u>	Equities	Fixed Income	Other Invested	<u>Company</u>	<u>Risk Leverage</u>
1	700	1,100	-400	0	-1,000	3.50
2	-700	-400	-100	-200	-1,000	3.50
3	1,100	100	1,300	-300	300	1.50
4	0	-700	800	-100	1,000	1.10
5	1,800	500	1,800	-500	1,500	0.90
6	1,400	400	400	600	1,600	0.90
7	2,100	-100	1,700	500	1,900	0.85
8	2,600	200	1,300	1,100	2,100	0.80
9	800	200	200	400	2,800	0.70
10	2,200	100	1,600	500	4,000	0.60
Expected Income	1,200	140	860	200	1,320	
Risk-Weighted Expected Income	716	203	463	50	348	
Risk Measurement	484	-63	397	150	972	
Capital Allocation	50%	-6%	41%	15%	100%	
Capital	4,980	-650	4,084	1,545	10,000	
Return on Risk-Adjusted Capital	24.1%	-21.6%	21.1%	12.9%	13.2%	
Hurdle Rate for Value Creation	9.7%	9.7%	9.7%	9.7%	9.7%	
Value Creation	716	203	463	50	348	

							Total	
<u>Underwriting</u>	Property	Casualty	<u>Investment</u>	Equities	Fixed Income	Other Invested	<u>Company</u>	<u>Risk Leverage</u>
-1,700	-500	-1,200	700	1,100	-400	0	-1,000	3.50
-300	-700	400	-700	-400	-100	-200	-1,000	3.50
-800	-600	-200	1,100	100	1,300	-300	300	1.50
1,000	100	900	0	-700	800	-100	1,000	1.10
-300	-100	-200	1,800	500	1,800	-500	1,500	0.90
200	500	-300	1,400	400	400	600	1,600	0.90
-200	300	-500	2,100	-100	1,700	500	1,900	0.85
-500	100	-600	2,600	200	1,300	1,100	2,100	0.80
2,000	800	1,200	800	200	200	400	2,800	0.70
1,800	700	1,100	2,200	100	1,600	500	4,000	0.60
120	60	60	1,200	140	860	200	1,320	
-368	-231	-137	716	203	463	50	348	
488	291	197	484	-63	397	150	972	
50%	30%	20%	50%	-6%	41%	15%	100%	
5,020	2,994	2,026	4,980	-650	4,084	1,545	10,000	
2.4%	2.0%	3.0%	24.1%	-21.6%	21.1%	12.9%	13.2%	
9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	9.7%	
-368	-231	-137	716	203	463	50	348	
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Selecting a risk measure

- Many standard risk measures (such as TVaR) can be expressed in the form of weights.
- See Kreps, PCAS 2005 for major examples.
- <u>Example:</u> Net loss outcomes > 1, net gain outcomes = 1.
 - Measures tail of distribution where losses occur.
- In general, risk measure weights are:
 - Non-negative,
 - Higher for worse ("riskier") outcomes, lower for better outcomes.

Summary of useful properties

- General framework for applying additive capital allocation methods
- Flexible choice of risk measure can experiment
- Allocates risk down to detail level (state, tier)
- Consistent with financial theory
 - Can be used to generate risk-neutral prices
- Relatively simple / transparent

Selected References

- Halliwell, "Conjoint Prediction of Paid and Incurred Losses," *CAS Forum*, Summer 1997, volume 1 (thank you Dave Clark for this one)
- Ruhm / Mango, "A Risk Charge Calculation Based on Conditional Probability," Bowles Symposium, Atlanta, April 2003
- Kreps, "Riskiness Leverage Ratios," *Proceedings* of the CAS, 2005
- Clark, "Reinsurance Applications for the RMK Framework," *CAS Forum*, Spring 2005