Extending the Asset Share Model

Recognizing the Value of Options in P&C Insurance Rates

Greg McNulty, FCAS
SCOR Reinsurance

SCOR

CAS Antitrust Notice

- The Casualty Actuarial Society is committed to adhering strictly to the letter and spirit of the antitrust laws. Seminars conducted under the auspices of the CAS are designed solely to provide a forum for the expression of various points of view on topics described in the programs or agendas for such meetings.
- Under no circumstances shall CAS seminars be used as a means for competing companies or firms to reach any understanding – expressed or implied – that restricts competition or in any way impairs the ability of members to exercise independent business judgment regarding matters affecting competition.
- It is the responsibility of all seminar participants to be aware of antitrust regulations, to prevent any written or verbal discussions that appear to violate these laws, and to adhere in every respect to the CAS antitrust compliance policy.

SCOR 2

Extending the Asset Share Model

1	**Background and Motivation**
2	**The Extended Asset Share Model**
3	**Simple Example – Importance of Variables**
4	**Complex Example – "Accident Forgiveness" and Value of Options**
5	**Further Considerations**

SCOR 3

Background and Motivation

- Well known that renewal business has lower loss costs than new business
- Asset share model calculates premium required to produce desired expected profit over the lifetime of a policy incepting today
  - Company may take a loss during the initial terms, then make it up with profitable renewal terms
- Formula:

$$EPV[P] = \frac{F + EPV[L]}{1 - U - V}$$

  - Extra parameters needed: premium and loss trend, discount rate, renewal probability

SCOR 4

Background and Motivation

- What is the shortcoming of the asset share model? It assumes constant inflation of premium and loss, ignoring risk class transition
- Example 1:
  - 2 risk classes, Low and High
  - Low can have an accident and be reclassified as High
- Premium and loss trend include the effect of Low risks being reclassified and having higher premium and loss
  - E.g. 10% chance of transition * 40% higher premium = 4% trend
  - If we change High risk rate, Low risk trend assumptions no longer valid

SCOR 5

Background and Motivation

- Example 2:
  - Experienced vs Inexperienced Drivers
- Inexperienced drivers charged more, asset share model assumes higher premium will keep trending into the future
  - Premium must drop to experienced rate eventually, leading to less renewal term profit than anticipated
- Is there a way to adapt the asset share model to recognize interactions between the risk classes over time and remedy these problems?

SCOR 6

Background and Motivation

- Accident forgiveness, price lock, rate guarantees are common policy features in personal auto
  - Economics depend on transition between risk classifications
- Common theme is the right but not obligation to purchase an asset (insurance policy) in the future at a price set today
  - That's a call option
- Will our new model be able to price these products?

Extending the Asset Share Model

The Extended Asset Share Model

- The solution is to use linear algebra to calculate the asset share price for all risks at the same time and explicitly account for risk class transition
- Instead of separate series of parameters for each class used independently, use series of vectors for premium, loss and expense in a unified analysis
  - Each dimension represents one risk class
- Also need a series of transition matrices A(n) which describe how each risk class renews and moves into the others at the n-th renewal

The Extended Asset Share Model

- The Extended Asset Share Model Formula:

$$\sum_{n=0}^{\infty} P(n) * v_n * A_n(R) = \frac{1}{1-U-V}\left[F + \sum_{n=0}^{\infty} v_n * A_n(L_n)\right]$$

- Simple linear equation of form Ax = b
  - Scalars: P, v, U, V; Vectors: R, F, L; Matrix: A
- LHS is the lifetime present value of premium, RHS is the loaded fixed expense and lifetime present value of loss
  - Transition matrix tracks policies as they renew into different classes or fail to renew

SCOR 10

The Extended Asset Share Model

- We can apply the inverse of the left hand side matrix to get the current premium vector (relativity times base rate) alone:

$$P(0) * R = \frac{1}{1-U-V}\left(\sum_{n=0}^{\infty} \frac{P(n)}{P(0)} * v_n * A_n\right)^{-1}\left(F + \sum_{n=0}^{\infty} v_n * A_n(L_n)\right)$$

- $\frac{P(n)}{P(0)}$ is the rate of overall premium inflation excluding effect of risk class transition, i.e. base rate increases
- Closed form solution for premium and relativities

SCOR 11

Extending the Asset Share Model

1	Background and Motivation
2	The Extended Asset Share Model
3	Simple Example – Importance of Variables
4	Complex Example – "Accident Forgiveness" and Value of Options
5	Further Considerations

SCOR 12

Simple Example – Importance of Variables

- Consider the following simple example:
  - Annual policies with two risk classes: Low and High
  - Losses L_{low} = 50 and L_{high} = 70 for the first term
  - Fixed expense 10 at time 0; 20% variable expense every term
  - Low risk renewal probability 90% every term; High 70%
  - Loss trend 1% Low risk, 3% High risk
  - Premium trend 4% per year
  - 5% discounting; 5% profit
  - Every term 10% chance Low risk reclassified as High; High risk not reclassified
  - Low risk class policies renew with 70% probability when reclassified as High

Standard Asset Share Model Data

Extended Model Additional Data

SCOR

13

Simple Example – Importance of Variables

- Given that the retention rate interacts with risk class transition and premium trend depends on the resulting rates this would be difficult to do with standard methods
- The extended model can calculate the required premiums with just a few lines of programming code:
  - $P_{low} = 56.22$
  - $P_{high} = 90.92$
- More interesting: what is the impact of the High risk class variables on indicated Low risk class premium, e.g. loss trend?

SCOR

14

Simple Example – Importance of Variables

- Look at derivative of P_{low} with respect to input variables:
  - $\partial ln(P_{low})/\partial ln(L_{low}) = 0.99$
  - $\partial ln(P_{low})/\partial ln(r_0) = 3.91$
  - $\partial ln(P_{low})/\partial ln(r_1) = 1.73$
  - r_0 = Low risk loss trend factor
    - *E.g. 1.01*
  - r_1 = High risk loss trend factor
    - *E.g. 1.03*
- Changing the High risk loss trend by +/- 1% has about 2x the impact on P_{low} as changing L_{low} by +/- 1% (multiplicatively), and 0.5x the impact of changing Low risk trend
- Surprising since transition probability is only 10%; result will vary under different assumptions

SCOR

15

Extending the Asset Share Model

SCOR 16

Complex Example – "Accident Forgiveness" and Value of Options

- Consider the following additions to the simple example:
  - Third risk class, Medium
  - L_{med} = 55; loss trend 2%
  - Low risks transition only to Medium risk with 10% probability each renewal
  - Medium risk renewal probability 95%
  - Medium risks transition only to High risk with 25% probability each renewal
  - $P_{med} = P_{low}$, i.e. higher expected loss is "forgiven"
- What should we charge Low/Medium risks? Where's the option and what's it worth?
  - Again, difficult for standard methods to solve but easy using Extended Asset Share model

SCOR 17

Complex Example – "Accident Forgiveness" and Value of Options

- Extended Asset Share model formula gives three equations in three unknowns: P_{low}, P_{med} , P_{high}
- Normally we set each premium equal to lifetime EPV of loss and expense, but with forgiveness we set $P_{med} = P_{low}$ and solve for the other two
- Indicated premiums:
  - P_{low} (no forgiveness) = \$54.98
  - P_{med} (no forgiveness) = \$64.25
  - P_{low} (with forgiveness) = P_{med} (with forgiveness) = \$57.65
  - P_{high} = \$90.92

SCOR 18

Complex Example – "Accident Forgiveness" and Value of Options

- Low risk policy with forgiveness is given ***the right but not obligation to buy*** renewal policy at the Low risk price, even if reclassified as Medium
- Call option on insurance policy costs Low/Medium risks $2.67 per term
  - Single term value is less for Low risk policies; more for Medium risk policies when option is "in the money"
- Extended Model adds amortized value of the lifetime EPV of option costs to the price

SCOR 19

Extending the Asset Share Model

SCOR 20

Further Considerations

- Price elasticity of demand
  - Transition matrices will have demand functions as entries
  - Computationally more complex
- More sophisticated premium formulas
  - IRR, RORAC, etc.
- Regulatory constraints
  - Is higher cost of options a valid reason to charge higher rates for otherwise identical risks?

SCOR 21