CP-1: Beyond the Cost Model: Understanding Price Elasticity and Its Applications

CAS 2013 Ratemaking and Product Management

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March 2013

TOWERS WATSON

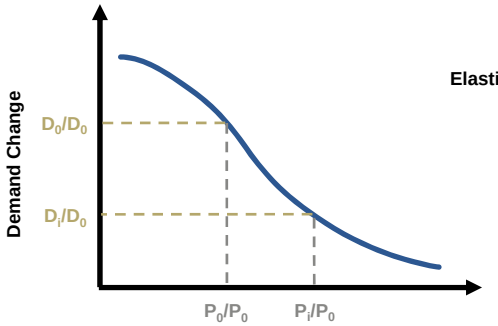
Agenda

- What is elasticity?
- What affects elasticity?
- How to model elasticity?

What is Elasticity?

Elasticity defined

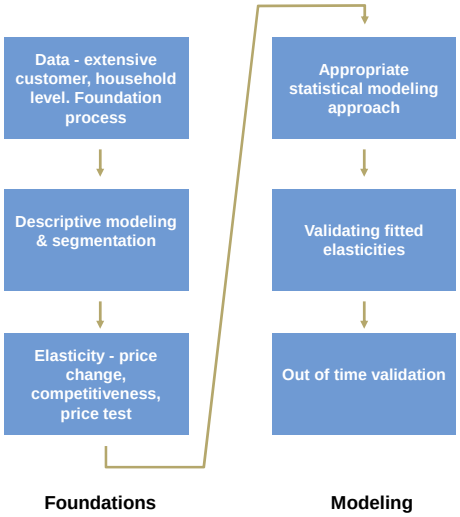
- Rate of response in quantity demanded given a specified change in price



$$\text{Elasticity} = \frac{\text{\% Change in Demand}}{\text{\% Change in Price}}$$

$$= \frac{D_i/D_0 - D_0/D_0}{P_i/P_0 - P_0/P_0}$$

Elasticity modeling - a rigorous approach



How to model elasticity?

Modeling tools

Classification and Regression Trees

- Interpolate missing data
- Identify initial main effects
- Identify key segments for models
- Identify complex interactions

Generalized Linear Models

- Parameterizes model structure
- Complex interaction strategies
- Issues
  - Possible "negative" elasticity

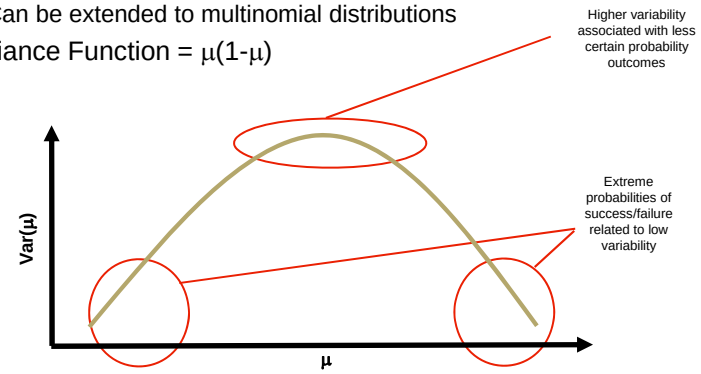
Generalized Non Linear Models

- Interacts price factors with all non price factors
- Non linear element forces positive elasticity
- Issues
  - Tendency to overfit
  - Ignores real "negative" elasticity

- Modeling requires flexibility in choosing the right strategy for the right data set

Distribution Function

- Binomial
  - Basic functional form in decision modeling
  - Belongs to the exponential family of distributions
  - Can be extended to multinomial distributions
- Variance Function = $\mu(1-\mu)$



Link Function

- Recall the following basic model form

Link function ($g=h^{-1}$)
Links random and systematic components

$$\hat{Y} = \mu = h(X\beta)$$

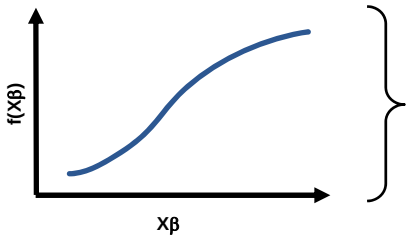
- Standard link functions used in loss cost models
  - Multiplicative: $\exp(X\beta)$
  - Identity: $X\beta$
  - Reciprocal: $1/(X\beta)$

Link Function

- Logit link function used in binomial models:

$$\frac{1}{1+\frac{1}{\exp(X\ \beta)}}$$

- Properties of the logit link function:



- S-shape curve “traps” the predictive value to the probability range

What do we mean by "price elasticity"?

- Most people define elasticity as ….
  - **Percentage change in demand / percentage change in price**
  - "Classical elasticity"
  - Definition found in economics textbooks
- But sometimes …
  - **Absolute change in linear predictor / percentage change in price**
  - "Linear predictor elasticity"
  - Doesn't vary with demand

Price Elasticity Definitions

- Classical

$$Demand_1 = \frac{1}{1 + \frac{1}{\exp(\beta_0 + \alpha_1 \times \frac{P_1}{P_0})}}$$

$$Demand_2 = \frac{1}{1 + \frac{1}{\exp(\beta_0 + \alpha_1 \times \frac{P_2}{P_0})}}$$

$$Classical = \frac{\frac{Demand_2 - Demand_1}{Demand_1}}{\frac{P_2 - P_1}{P_1}}$$

Price Elasticity Definitions

- Linear Predictor

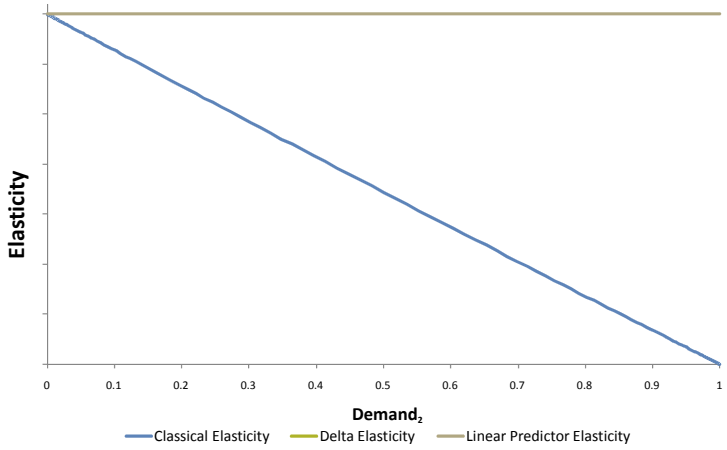
$$Demand_1 = \frac{1}{1 + \frac{1}{\exp(\beta_0 + \alpha_1 \times \frac{P_1}{P_0})}}$$

$$Demand_2 = \frac{1}{1 + \frac{1}{\exp(\beta_0 + \alpha_1 \times \frac{P_2}{P_0})}}$$

$$Linear = \frac{\beta_0 + \alpha_1 \times \frac{P_2}{P_0} - \beta_0 + \alpha_1 \times \frac{P_1}{P_0}}{\frac{P_2 - P_1}{P_1}} = \alpha_1$$

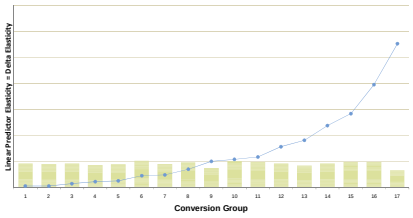
What does logit imply about elasticity?

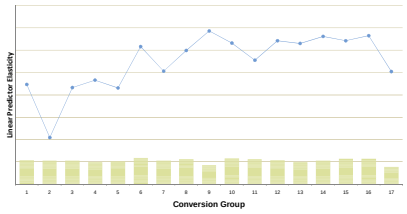
- If there are ***no interactions with price change factors***



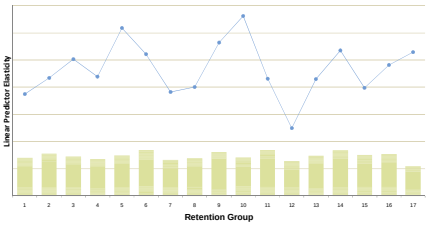
The case for logistic GLMs

Normal





Probit



Logistic

How to model elasticity?

Generalized Non-Linear Models

- Allow models to be fitted where the linear predictor is not a linear combination of factors

Generalized Linear Models

$$y = \frac{1}{1+\exp\left(-X\beta_{segments} + \underbrace{\Delta P \beta_{\Delta P}}\right)} + error$$

ΔP can either be represented by a categorical factor or by a curve

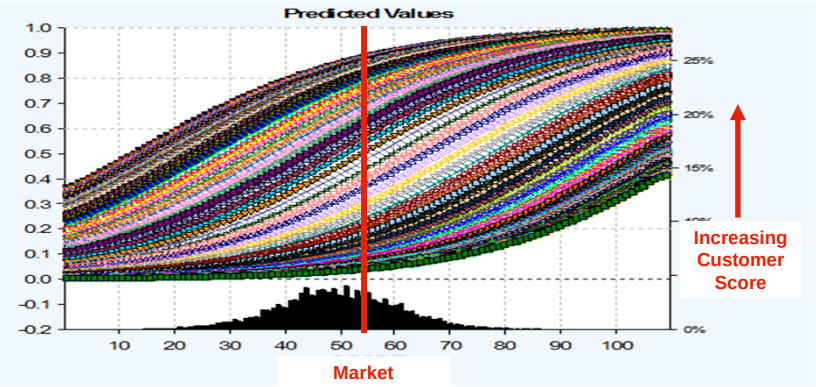
Generalized Non-Linear Models

$$y = \frac{1}{1+\exp(-X\beta + \underbrace{\Delta P e^{Z\chi}})} + error$$

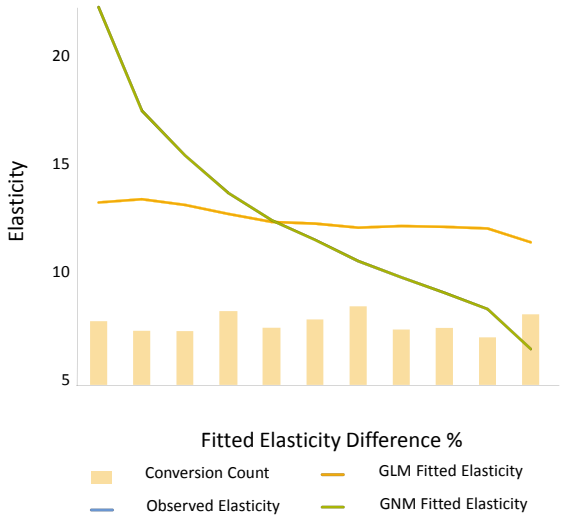
Forces elasticity to be positive

Competitive Demand: US Auto New Business

- Non price parameters compiled into a customer score interacted with competitive ratio



New business – linear vs. non-linear



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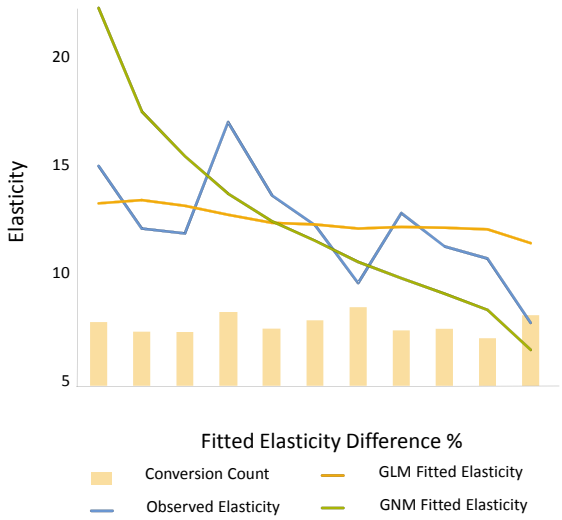
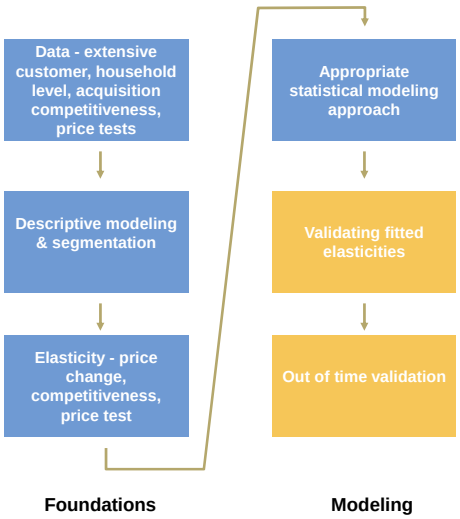
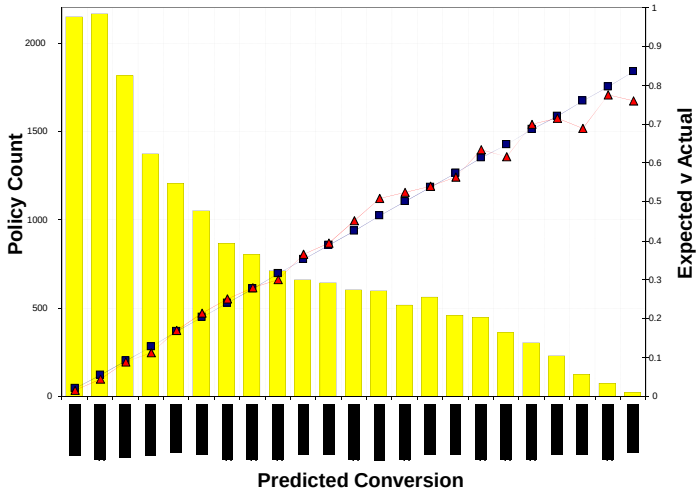


Elasticity modeling - a rigorous approach



New business - out of time validation



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