

Loss Cost Modeling VS. Frequency and Severity Modeling

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Description of Frequency-Severity Modeling

- Claim Frequency = Claim Count / Exposure
Claim Severity = Loss / Claim Count
- It is a common actuarial assumption that:
 - Claim Frequency has an over-dispersed Poisson distribution
 - Claim Severity has a Gamma distribution
- Loss Cost = Claim Frequency \times Claim Severity
- Can be much more complex

Description of Frequency-Severity Modeling

- A more sophisticated Frequency/Severity model design
 - Frequency – Over-dispersed Poisson
 - Capped Severity – Gamma
 - Propensity of excess claim – Binomial
 - Excess Severity – Gamma
 - Expected Loss Cost = Frequency x Capped Severity
+ Propensity of excess claim x Excess Severity
 - Fit a model to expected loss cost to produce loss cost indications by rating variable

Description of Loss Cost Modeling

Tweedie Distribution

- It is a common actuarial assumption that:
 - Claim count is Poisson distributed
 - Size-of-Loss is Gamma distributed
- Therefore the loss cost (LC) distribution is Gamma-Poisson **Compound** distribution, called Tweedie distribution
 - $LC = X_1 + X_2 + \dots + X_N$
 - $X_i \sim \text{Gamma}$ for $i \in \{1, 2, \dots, N\}$
 - $N \sim \text{Poisson}$

Description of Loss Cost Modeling

Tweedie Distribution (Cont.)

- Tweedie distribution is belong to exponential family
 - $\text{Var}(LC) = \phi\mu^p$
 - ϕ is a scale parameter
 - μ is the expected value of LC
 - $p \in (1,2)$
 - p is a free parameter – must be supplied by the modeler
 - As $p \rightarrow 1$: LC approaches the Over-Dispersed Poisson
 - As $p \rightarrow 2$: LC approaches the Gamma

Data Description

- Structure – On a vehicle-policy term level
- Total 100,000 vehicle records
- Separated to Training and Testing Subsets:
 - Training Dataset: 70,000 vehicle records
 - Testing Dataset: 30,000 Vehicle Records
- Coverage: Comprehensive

Numerical Example 1

GLM Setup – In Total Dataset

- Frequency Model
 - Target = Frequency = Claim Count / Exposure
 - Link = Log
 - Distribution = Poisson
 - Weight = Exposure
 - Variable =
 - Territory
 - Agegrp
 - Type
 - Vehicle_use
 - Vehage_group
 - Credit_Score
 - AFA
- Severity Model
 - Target = Severity = Loss / Claim Count
 - Link = Log
 - Distribution = Gamma
 - Weight = Claim Count
 - Variable =
 - Territory
 - Agegrp
 - Type
 - Vehicle_use
 - Vehage_group
 - Credit_Score
 - AFA
- Loss Cost Model
 - Target = loss Cost = Loss / Exposure
 - Link = Log
 - Distribution = Tweedie
 - Weight = Exposure
 - P=1.30
 - Variable =
 - Territory
 - Agegrp
 - Type
 - Vehicle_use
 - Vehage_group
 - Credit_Score
 - AFA

Numerical Example 1

How to select “ p ” for the Tweedie model?

- Treat “ p ” as a parameter for estimation
- Test a sequence of “ p ” in the Tweedie model
- The Log-likelihood shows a smooth inverse “U” shape
- Select the “ p ” that corresponding to the “maximum” log-likelihood

Value p Optimization

Log-likelihood	Value p
-12192.25	1.20
-12106.55	1.25
-12103.24	1.30
-12189.34	1.35
-12375.87	1.40
-12679.50	1.45
-13125.05	1.50
-13749.81	1.55
-14611.13	1.60

Numerical Example 1

GLM Output (Models Built in Total Data)

		Frequency Model		Severity Model		Frq * Sev	Loss Cost Model (p=1.3)	
		Estimate	Rating Factor	Estimate	Rating Factor	Rating Factor	Estimate	Rating Factor
Intercept		-3.19	0.04	7.32	1510.35	62.37	4.10	60.43
Territory	T1	0.04	1.04	-0.17	0.84	0.87	-0.13	0.88
Territory	T2	0.01	1.01	-0.11	0.90	0.91	-0.09	0.91
Territory	T3	0.00	1.00	0.00	1.00	1.00	0.00	1.00
.....
agegrp	Yng	0.19	1.21	0.06	1.06	1.28	0.25	1.29
agegrp	Old	0.04	1.04	0.11	1.11	1.16	0.15	1.17
agegrp	Mid	0.00	1.00	0.00	1.00	1.00	0.00	1.00
Type	M	-0.13	0.88	0.05	1.06	0.93	-0.07	0.93
Type	S	0.00	1.00	0.00	1.00	1.00	0.00	1.00
Vehicle_Use	PL	0.05	1.05	-0.09	0.92	0.96	-0.04	0.96
Vehicle_Use	WK	0.00	1.00	0.00	1.00	1.00	0.00	1.00

Numerical Example 1

Findings from the Model Comparison

- The LC modeling approach needs less modeling efforts, the FS modeling approach shows more insights.
 - What is the driver of the LC pattern, Frequency or Severity?
 - Frequency and severity could have different patterns.

Numerical Example 1

Findings from the Model Comparison – Cont.

- The loss cost relativities based on the FS approach could be fairly close to the loss cost relativities based on the LC approach, when
 - Same pre-GLM treatments are applied to incurred losses and exposures for both modeling approaches
 - Loss Capping
 - Exposure Adjustments
 - Same predictive variables are selected for all the three models (*Frequency Model, Severity Model and Loss Cost Model*)
 - The modeling data is credible enough to support the severity model

Numerical Example 2

GLM Setup – In Training Dataset

- | | | |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <ul style="list-style-type: none"> • Frequency Model <ul style="list-style-type: none"> – Target = Frequency – Target = Claim Count /Exposure – Link = Log – Distribution = Poison – Weight = Exposure – Variable = <ul style="list-style-type: none"> • Territory • Agegrp • Deductable • Vehage_group • Credit_Score • AFA | <ul style="list-style-type: none"> • Severity Model <ul style="list-style-type: none"> – Target = Severity – Target = Loss/Claim Count – Link = Log – Distribution = Gamma – Weight=Claim Count – Variable = <ul style="list-style-type: none"> • Territory • Agegrp • Deductable • Vehage_group • Credit_Score • AFA | <ul style="list-style-type: none"> • Severity Model (Reduced) <ul style="list-style-type: none"> – Target = Severity – Target = Loss/Claim Count – Link = Log – Distribution = Gamma – Weight = Claim Count – Variable = <ul style="list-style-type: none"> • Territory • Agegrp • Vehage_group • AFA |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

Type 3 Statistics

	DF	ChiSq	Pr > Chisq
territory	2	5.9	0.2066
agegrp	2	25.36	<.0001
vehage_group	4	294.49	<.0001
Deductable	2	41.07	<.0001
credit_score	2	64.1	<.0001
AFA	2	15.58	0.0004

Type 3 Statistics

	DF	ChiSq	Pr > Chisq
territory	2	15.92	0.0031
agegrp	2	2.31	0.3151
vehage_group	4	36.1	<.0001
Deductable	2	1.64	0.4408
credit_score	2	2.16	0.7059
AFA	2	11.72	0.0028

Type 3 Statistics

	DF	ChiSq	Pr > Chisq
Territory	2	15.46	0.0038
agegrp	2	2.34	0.3107
vehage_group	4	35.36	<.0001
AFA	2	11.5	0.0032

Numerical Example 2

GLM Output (Models Built in Training Data)

		Frequency Model Rating		Severity Model Rating		Frq * Sev Rating	Loss Cost Model (p=1.3) Rating	
		Estimate	Factor	Estimate	Factor	Factor	Estimate	Factor
Territory	T1	0.03	1.03	-0.17	0.84	0.87	-0.15	0.86
Territory	T2	0.02	1.02	-0.11	0.90	0.92	-0.09	0.91
Territory	T3	0.00	1.00	0.00	1.00	1.00	0.00	1.00
.....						
Deductable	100	0.33	1.38			1.38	0.36	1.43
Deductable	250	0.25	1.28			1.28	0.24	1.27
Deductable	500	0.00	1.00			1.00	0.00	1.00
CREDIT_SCORE	1	0.82	2.28			2.28	0.75	2.12
CREDIT_SCORE	2	0.52	1.68			1.68	0.56	1.75
CREDIT_SCORE	3	0.00	1.00			1.00	0.00	1.00
AFA	0	-0.25	0.78	-0.19	0.83	0.65	-0.42	0.66
AFA	1	-0.03	0.97	-0.19	0.83	0.80	-0.21	0.81
AFA	2+	0.00	1.00	0.00	1.00	1.00	0.00	1.00

Numerical Example 2

Model Comparison In Testing Dataset

- In the testing dataset, generate two sets of loss cost Scores corresponding to the two sets of loss cost estimates
 - Score_fs (based on the FS modeling parameter estimates)
 - Score_lc (based on the LC modeling parameter estimates)
- Compare goodness of fit (GF) of the two sets of loss cost scores in the testing dataset
 - Log-Likelihood

Numerical Example 2

Model Comparison In Testing Dataset - Cont

*GLM to Calculate GF Stat of
Score_fs*

Data: Testing Dataset

Target: Loss Cost

Predictive Var: Non

Error: tweedie

Link: log

Weight: Exposure

P: 1.15/1.20/1.25/1.30/1.35/1.40

Offset: log(Score_fs)

*GLM to Calculate GF Stat of
Score_lc*

Data: Testing Dataset

Target: Loss Cost

Predictive Var: Non

Error: tweedie

Link: log

Weight: Exposure

P: 1.15/1.20/1.25/1.30/1.35/1.40

Offset: log(Score_lc)

Numerical Example 2

Model Comparison In Testing Dataset - Cont

*GLM to Calculate GF Stat
Using Score_fs as offset*

Log likelihood from output

P=1.15 log-likelihood=-3749

P=1.20 log-likelihood=-3699

P=1.25 log-likelihood=-3673

P=1.30 log-likelihood=-3672

P=1.35 log-likelihood=-3698

P=1.40 log-likelihood=-3755

*GLM to Calculate GF Stat
Using Score_lc as offset*

Log likelihood from output

P=1.15 log-likelihood=-3744

P=1.20 log-likelihood=-3694

P=1.25 log-likelihood=-3668

P=1.30 log-likelihood=-3667

P=1.35 log-likelihood=-3692

P=1.40 log-likelihood=-3748

The loss cost model has better goodness of fit.

Numerical Example 2

Findings from the Model Comparison

- In many cases, the frequency model and the severity model will end up with different sets of variables. More than likely, less variables will be selected for the severity model
 - Data credibility for middle size or small size companies
 - For certain low frequency coverage, such as Bl...
- As a result
 - F_S approach shows more insights, but needs additional effort to roll up the frequency estimates and severity estimates to LC relativities
 - In these cases, frequently, the LC model shows better goodness of fit

A Frequently Applied Methodology

Loss Cost Refit

- Loss Cost Refit
 - Model frequency and severity separately
 - Generate frequency score and severity score
 - $LC\ Score = (Frequency\ Score) \times (Severity\ Score)$
 - Fit a LC model to the LC score to generate LC Relativities by Rating Variables
 - Originated from European modeling practice
- Considerations and Suggestions
 - Different regulatory environment for European market and US market
 - An essential assumption – The LC score is unbiased.
 - Validation using a LC model

Constrained Rating Plan Study

- Update a rating plan with keeping certain rating tables or certain rating factors unchanged
- One typical example is to create a rating tier variable on top of an existing rating plan
 - Catch up with marketing competitions to avoid adverse selection
 - Manage disruptions

Constrained Rating Plan Study - Cont

- Apply GLM offset techniques
- The offset factor is generated using the unchanged rating factors.
- Typically, for creating a rating tier on top of an existing rating plan, the offset factor is given as the rating factor of the existing rating plan.
- All the rating factors are on loss cost basis. It is natural to apply the LC modeling approach for rating tier development.

How to Select Modeling Approach?

- Data Related Considerations
- Modeling Efficiency Vs. Business Insights
- Quality of Modeling Deliverables
 - Goodness of Fit (on loss cost basis)
 - Other model comparison scenarios
- Dynamics on Modeling Applications
 - Class Plan Development
 - Rating Tier or Score Card Development
- Post Modeling Considerations
- Run a LC model to double check the parameter estimates generated based on a F-S approach

An Exhibit from a Brazilian Modeler

Exposição > 0		Erro Geral
Tweedie		-2%
Bin. Neg. e Gamma		8%
Poisson e Gamma		7%
Bin. e Gamma		7%
Rosa dos Ventos		23%
Exposição > 50		Erro Geral
Tweedie		3%
Bin. Neg. e Gamma		10%
Poisson e Gamma		9%
Bin. e Gamma		9%
Rosa dos Ventos		35%
Exposição > 100		Erro Geral
Tweedie		2%
Bin. Neg. e Gamma		8%
Poisson e Gamma		7%
Bin. e Gamma		7%
Rosa dos Ventos		34%
Exposição > 200		Erro Geral
Tweedie		0%
Bin. Neg. e Gamma		5%
Poisson e Gamma		4%
Bin. e Gamma		4%
Rosa dos Ventos		32%