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Basic Ratemaking Workshop: Intro to Increased Limit Factors

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Agenda

- Background and Notation
- Overview of Basic and Increased Limits
- Increased Limits Ratemaking
- Deductible Ratemaking
- Mixed Exponential Procedure (Overview)

Basic Ratemaking Workshop: Intro to Increased Limit Factors

Background and Notation

Loss Severity Distributions

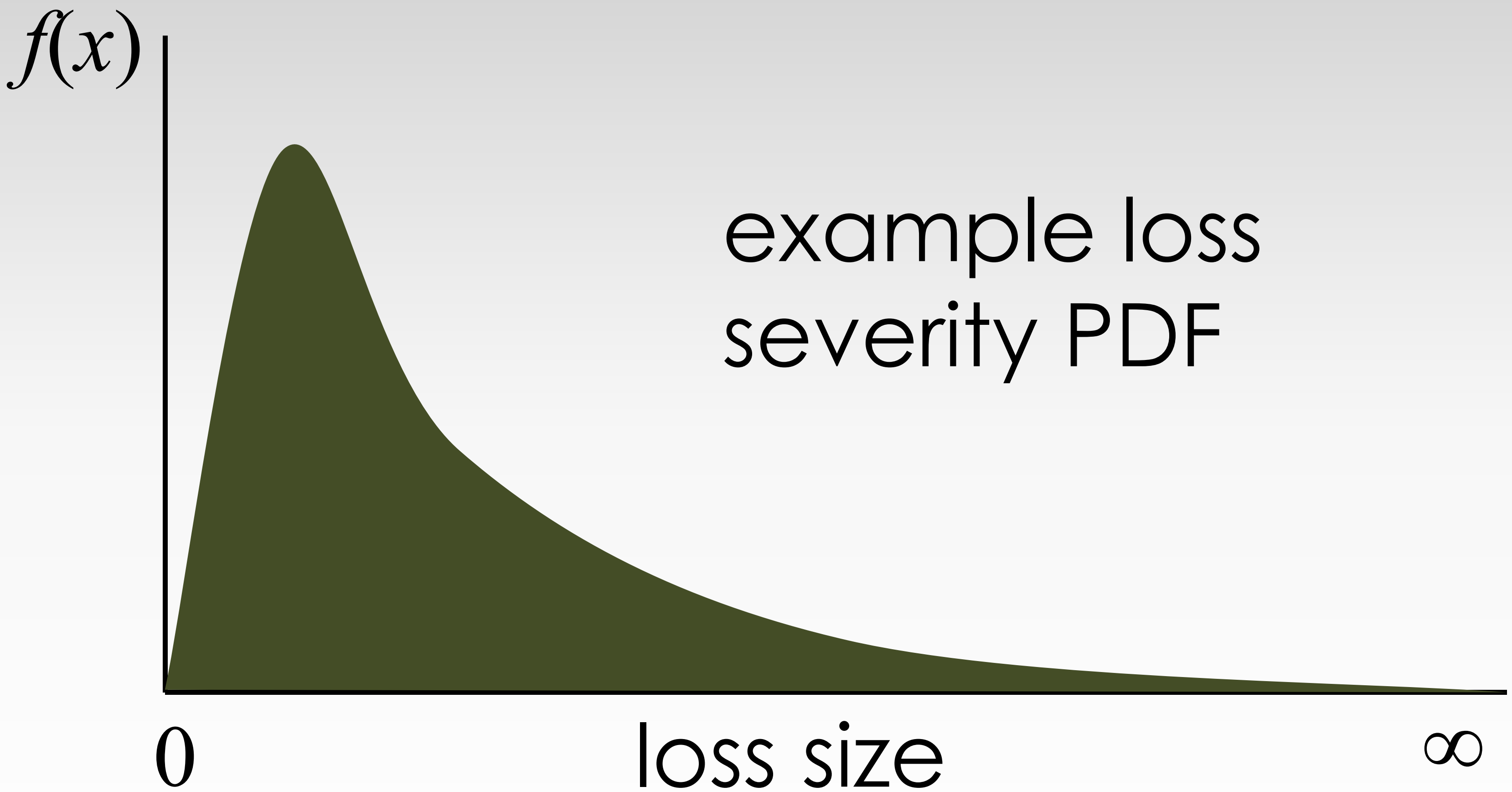
Probability Density Function (PDF) – $f(x)$

- describes the probability density of the outcome of a random variable X
- theoretical equivalent of a histogram of empirical data

Loss severity distributions are skewed

- a few large losses make up a significant portion of the total loss dollars

Loss Severity Distributions



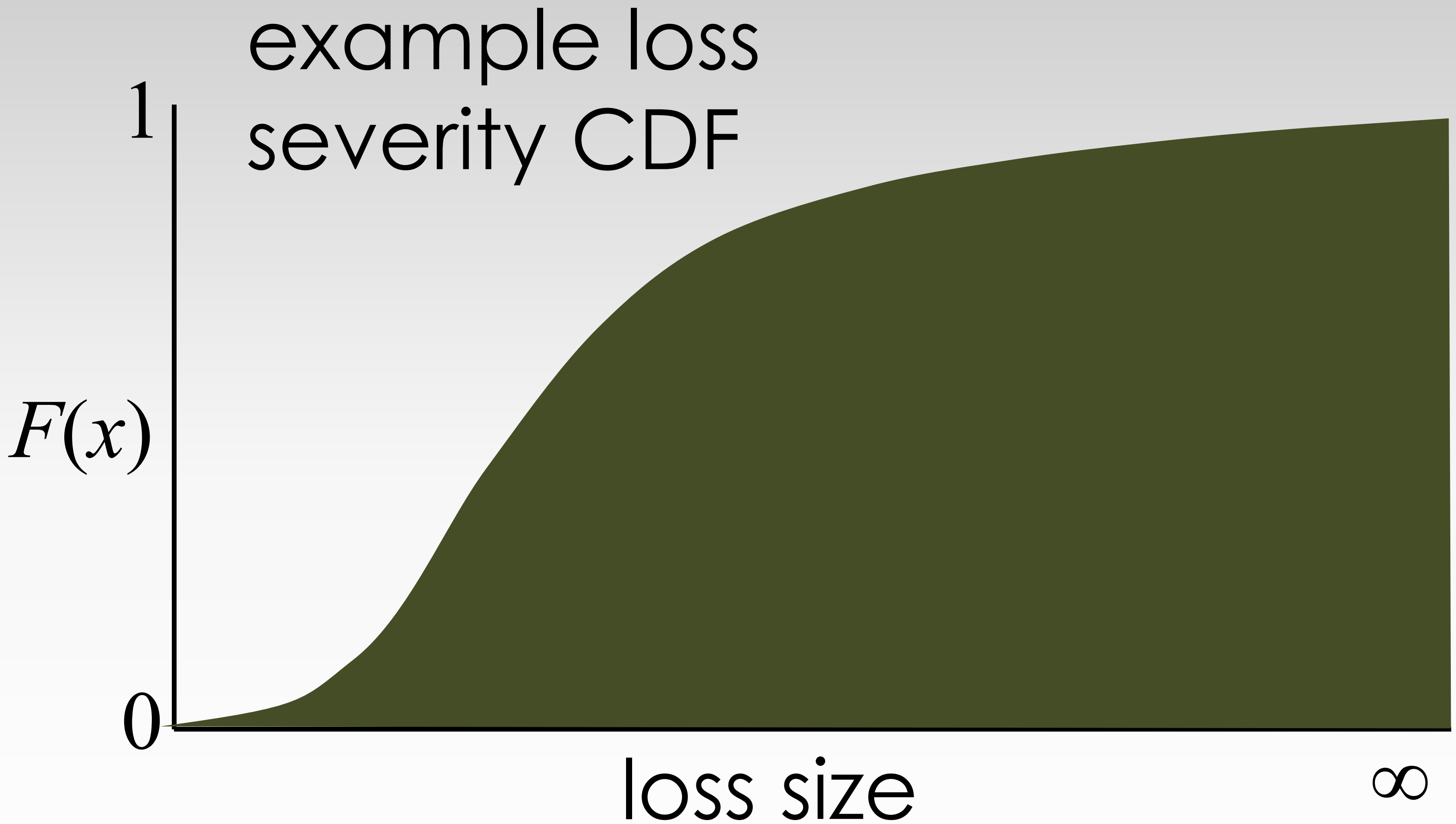
Loss Severity Distributions

Cumulative Distribution Function (CDF)

- describes the probability that a random variable X takes on values less than or equal to x

$$F(x) = \Pr\left[X \le x\right] = \int_0^x f(t)\,dt$$

Loss Severity Distributions



Mathematical Notation

Expected Value (mean, μ, first raw moment)

- average value of a random variable

$$E[X] = \int_0^{\infty} x f(x) dx$$

$$= \int_0^{\infty} S(x) dx, \text{ where } S(x) = 1 - F(x)$$

Mathematical Notation

Limited Expected Value (at k)

- expected value of the random vairable limited to a maximum value of k
- often referred to as the limited average severity (LAS) when working with losses

$$X \wedge k = \begin{cases} X, \text{where } X \leq k \\ k, \text{where } X > k \end{cases}$$

$$E[X \wedge k] = \int_0^k x f(x)dx + k(1 - F(k)) = \int_0^k S(x)dx$$

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Overview of Basic and Increased Limits

Basic and Increased Limits

Different insureds have different coverage needs, so third-party liability coverage is offered at different limits.

Typically, the lowest level of insurance offered is referred to as the basic limit and higher limits are referred to as increased limits.

Basic and Increased Limits

Basic Limit loss costs are reviewed and filed on a regular basis (perhaps annually)

- a larger volume of losses capped at the basic limit can be used for a detailed experience analysis
- experience is more stable since large, volatile losses are capped and excluded from the analysis

Higher limits are reviewed less frequently

- requires more data volume
- fewer policies are written at higher limits
- large losses are highly variable

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Increased Limits Ratemaking

Increased Limits Ratemaking

Basic Limit data aggregation

- losses are restated as if all policies were purchased at the basic limit
- basic limit is usually the financial responsibility limit or a commonly selected limit
- ALAE is generally uncapped

Increased Limits data aggregation

- losses are limited to a higher limit
- ALAE generally remains uncapped

Increased Limits Ratemaking

- the process of developing charges for expected losses at higher limits of liability
- usually results in a multiplicative factor to be applied to the basic limit loss cost, i.e. the increased limit factor (ILF)

$$\text{ILF}(k) = \frac{\text{expected pure premium at policy limit } k}{\text{expected pure premium at basic limit } b}$$

Increased Limits Ratemaking

A key assumption of IL ratemaking is that claim frequency is independent of claim severity

- claim frequency does not depend on policy limit
- only claim severity is needed to calculate ILFs

Increased Limits Ratemaking

$$\text{ILF}(k) = \frac{\text{expected pure premium at policy limit } k}{\text{expected pure premium at basic limit } b}$$

$$= \frac{E\left[\text{frequency}_k\right] \times E\left[\text{severity}_k\right]}{E\left[\text{frequency}_b\right] \times E\left[\text{severity}_b\right]}$$

$$= \frac{E\left[\text{frequency}\right] \times E\left[\text{severity}_k\right]}{E\left[\text{frequency}\right] \times E\left[\text{severity}_b\right]}$$

$$= \frac{E\left[\text{severity}_k\right]}{E\left[\text{severity}_b\right]} = \frac{E\left[X \wedge k\right]}{E\left[X \wedge b\right]}$$

Increased Limits Ratemaking

For practical purposes, the expected costs include a few components:

- limited average severity
- allocated loss adjustment expenses
- unallocated loss adjustment expenses
- risk load

We will focus mostly on LAS, with some discussion of ALAE.

Calculating an ILF using Empirical Data

The basic limit is \$100k. Calculate ILF(\$1000k) given the following set of ground-up, uncapped losses.

Recall ILF(k)=E[X^k]/E[X^b].

Losses x
\$50,000
\$75,000
\$150,000
\$250,000
\$1,250,000

Calculating an ILF using Empirical Data

Losses x	min{x, \$100k}	min{x, \$1000k}
\$50,000	\$50,000	\$50,000
\$75,000	\$75,000	\$75,000
\$150,000	\$100,000	\$150,000
\$250,000	\$100,000	\$250,000
\$1,250,000	\$100,000	\$1,000,000

ILF(k)=E[X^k]/E[X^b]

E[X^\$100k] = \$425,000/5 = \$85,000

E[X^\$1000k] = \$1,525,000/5 = \$305,000

ILF(\$1000k) = E[X^\$1000k]/E[X^\$100k] = 3.59

Calculating an ILF using Empirical Data

The basic limit is $25k. Calculate ILF($125k) given the following set of losses.

Losses x
$5,000
$17,500
$50,000
$162,500
$1,250,000

Calculating an ILF using Empirical Data

Losses x	min{x, \$25k}	min{x, \$125k}
\$5,000	\$5,000	\$5,000
\$17,500	\$17,500	\$17,500
\$50,000	\$25,000	\$50,000
\$162,500	\$25,000	\$125,000
\$1,250,000	\$25,000	\$125,000

E[X^\$25k] = \$97,500/5 = \$19,500

E[X^\$125k] = \$322,500/5 = \$64,500

ILF(\$125k) = E[X^\$125k]/E[X^\$25k] = 3.31

Aggregating and Limiting Losses

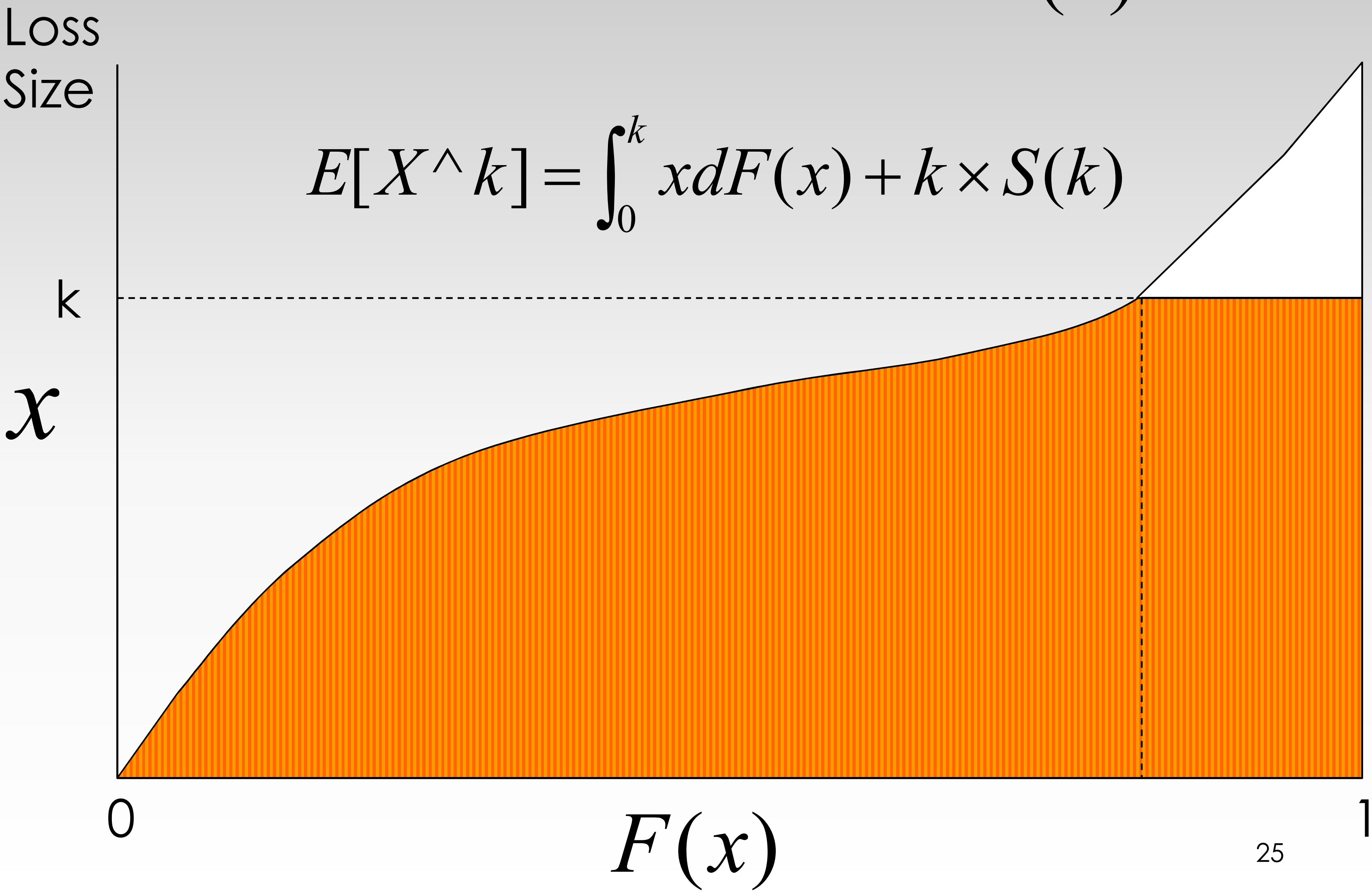
Size of Loss method

- individual losses are grouped by size into predetermined intervals
- the aggregate loss within each interval is limited, if necessary, to the limit being reviewed
- ALAE is added to the aggregate limited loss

Aggregating and Limiting Losses

$$*\,S(x) = 1 - F(x)$$

$$E[X \wedge k] = \int_0^k x\,dF(x) + k \times S(k)$$



Aggregating and Limiting Losses

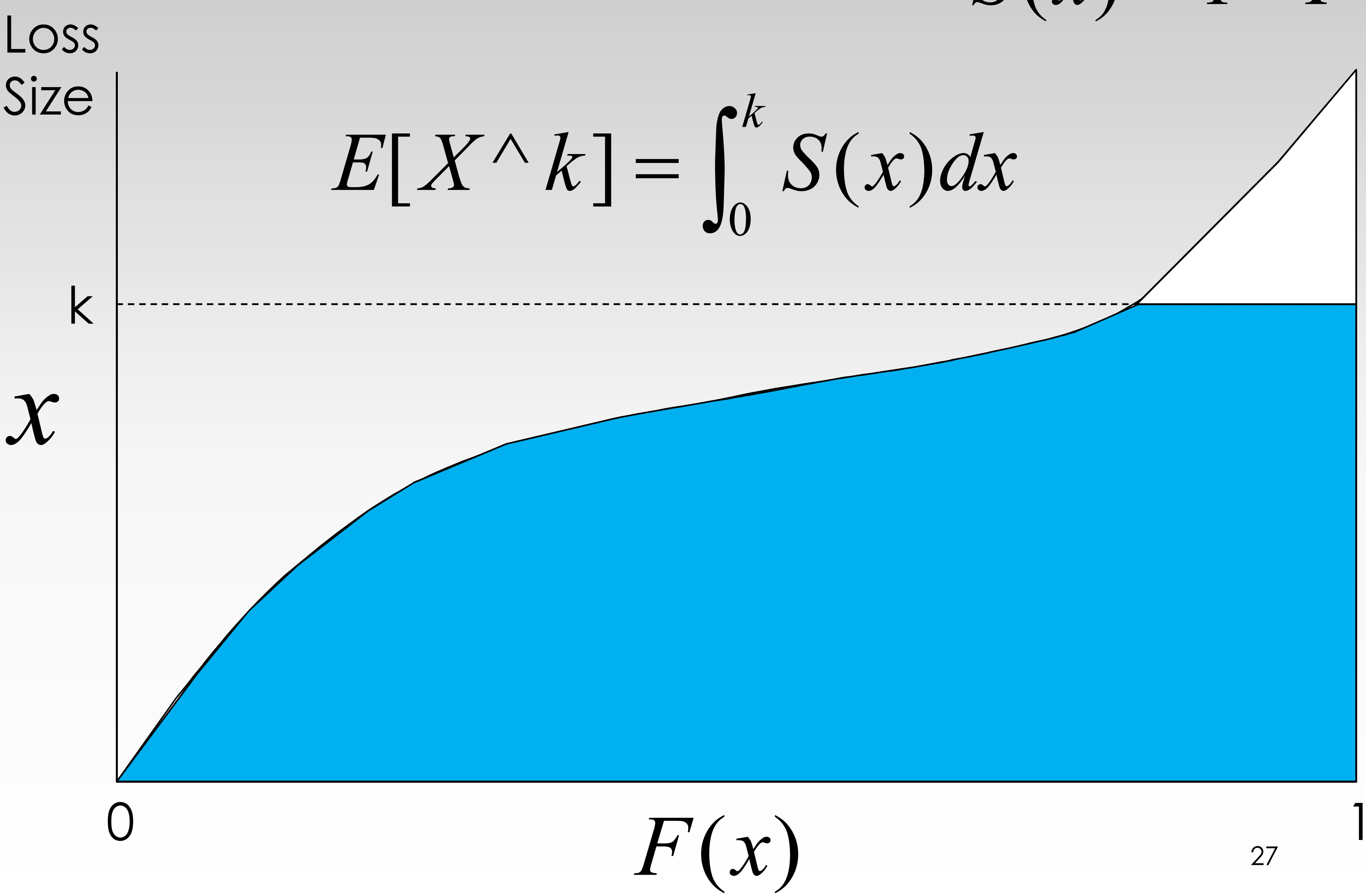
Layer method

- individual losses are sliced into layers based on predetermined intervals
- for each loss, the amount of loss corresponding to each layer is added to the aggregate for that layer
- the aggregate loss for each layer up to the limit is added together
- ALAE is added to the aggregate limited loss

Layer Method

$$* S(x) = 1 - F(x)$$

$$E[X \wedge k] = \int_0^k S(x)dx$$



Size Method vs Layer Method

	Size Method	Layer Method
Advantages	•conceptually straightforward •data can be used in calculations immediately •more complicated integral is actually generally easier to calculate	•computationally simple for calculating sets of increased limit factors •no integration disadvantage when data is given numerically, which is generally the practical case
Disadvantages	•computationally intensive for calculating sets of increased limit factors	•unintuitive •data must be processed so that it can be used in calculations •$S(x)$ is generally a more difficult function to integrate

Calculating an ILF using the Size Method

Individual Loss Intervals (basic limit is $100k)		Aggregate Losses in Interval	Number of Claims in Interval
Lower Bound	Upper Bound		
$1	$100,000	$25,000,000	1,000
$100,001	$250,000	$75,000,000	500
$250,001	$500,000	$60,000,000	200
$500,001	$1,000,000	$30,000,000	50
$1,000,001	∞	$15,000,000	10

$$E[X \wedge k] = \frac{\text{losses on claims up to } k + k \times \text{number of claims exceeding } k}{\text{total number of claims}}$$

Calculating an ILF using the Size Method

Individual Loss Intervals (basic limit is $100k)		Aggregate Losses in Interval	Number of Claims in Interval
Lower Bound	Upper Bound		
$1	$100,000	$25,000,000	1,000
$100,001	$250,000	$75,000,000	500
$250,001	$500,000	$60,000,000	200
$500,001	$1,000,000	$30,000,000	50
$1,000,001	∞	$15,000,000	10

Calculate ILF($1000k).

Calculating an ILF using the Size Method

Individual Loss Intervals (basic limit is $100k)		Aggregate Losses in Interval	Number of Claims in Interval
Lower Bound	Upper Bound		
$1	$50,000	$8,400,000	200
$50,001	$100,000	$46,800,000	600
$100,001	$250,000	$64,000,000	400
$250,001	$500,000	$38,200,000	100
$500,001	∞	$17,000,000	20

Calculate ILF($250k) and ILF($500k).

E[X^$100k] = [$55.2M + 520 × $100k]/1,320 = $81,212

E[X^$250k] = [$119.2M + 120 × $250k]/1,320 = $113,030

ILF($250k) = E[X^$250k]/E[X^$100k] = 1.39

E[X^$500k] = [$157.4M + 20 × $500k]/1,320 = $126,818

ILF($500k) = E[X^$500k]/E[X^$100k] = 1.56

Calculating an ILF using the Size Method with ALAE

Individual Loss Intervals (basic limit is \$100k)		Aggregate Losses in Interval	Agg. ALAE on Claims in Interval	Number of Claims in Interval
L. Bound	U. Bound			
\$1	\$100,000	\$16,000,000	\$100,000	200
\$100,001	\$300,000	\$42,000,000	\$500,000	350
\$300,001	\$500,000	\$36,000,000	\$800,000	90
\$500,001	∞	\$3,000,000	\$200,000	5

$$E[X \wedge k] = \frac{\text{losses up to } k + k \times \text{claims exceeding } k + \text{total ALAE}}{\text{total claims}}$$

Calculating an ILF using the Size Method with ALAE

Individual Loss Intervals (basic limit is $100k)		Aggregate Losses in Interval	Agg. ALAE on Claims in Interval	Number of Claims in Interval
L. Bound	U. Bound			
$1	$100,000	$16,000,000	$100,000	200
$100,001	$300,000	$42,000,000	$500,000	350
$300,001	$500,000	$36,000,000	$800,000	90
$500,001	∞	$3,000,000	$200,000	5

Calculate ILF($500k).

E[X^$100k] = [$16M + 445 × $100k + $1600k]/645 = $96,279
E[X^$500k] = [$94M + 5 × $500k + $1600k]/645 = $152,093
ILF($500k) = E[X^$500k]/E[X^$100k] = 1.58

Calculating an ILF using the Layer Method

Loss Layer (basic limit is $50k)		Aggregate Losses in Layer	Claims Reaching Layer
Lower Bound	Upper Bound		
$1	$50,000	$3,800,000	100
$50,001	$100,000	$2,000,000	50
$100,001	$250,000	$2,500,000	25
$250,001	∞	$4,000,000	10

$$E[X \wedge k] = \frac{\text{sum of all losses in each layer up to } k}{\text{total claims}}$$

Calculating an ILF using the Layer Method

Loss Layer (basic limit is $50k)		Aggregate Losses in Layer	Claims Reaching Layer
Lower Bound	Upper Bound		
$1	$50,000	$3,800,000	100
$50,001	$100,000	$2,000,000	50
$100,001	$250,000	$2,500,000	25
$250,001	∞	$4,000,000	10

Calculate ILF($250k).

E[X^$50k] = $3,800,000 / 100 = $38,000

E[X^$250k] = ($3.8M + $2.0M + $2.5M)/100 = $83,000

ILF($250k) = E[X^$250k]/ E[X^$50k] = 2.18

Calculating an ILF using the Layer Method with ALAE

Loss Layer (basic limit is $50k)		Aggregate Losses in Layer (ALAE = $1.1M)	Claims Reaching Layer
Lower Bound	Upper Bound		
$1	$50,000	$39,500,000	1,000
$50,001	$100,000	$32,000,000	800
$100,001	$250,000	$9,500,000	100
$250,001	∞	$14,200,000	10

$$E[X \wedge k] = \frac{\text{sum of all losses in each layer up to } k + \text{total ALAE}}{\text{total claims}}$$

Calculating an ILF using the Layer Method

Loss Layer (basic limit is $50k)		Aggregate Losses in Layer (ALAE = $1.1M)	Claims Reaching Layer
Lower Bound	Upper Bound		
$1	$50,000	$39,500,000	1,000
$50,001	$100,000	$32,000,000	800
$100,001	$250,000	$9,500,000	100
$250,001	∞	$14,200,000	10

Calculate ILF($250k).

E[X^$50k] = ($39.5M + $1.1M) / 1000 = $40,600

E[X^$250k]=($39.5M+$32.0M+$9.5M +$1.1M)/1000=$82,100

ILF($250k) = E[X^$250k]/ E[X^$50k] = 2.02

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Consistency Rule

Consistency Rule

The marginal premium per dollar of coverage should decrease as the limit of coverage increases.

- ILFs should increase at a decreasing rate
- expected costs per unit of coverage should not increase in successively higher layers

Inconsistency can indicate the presence of anti-selection

- higher limits may influence the size of a suit, award, or settlement

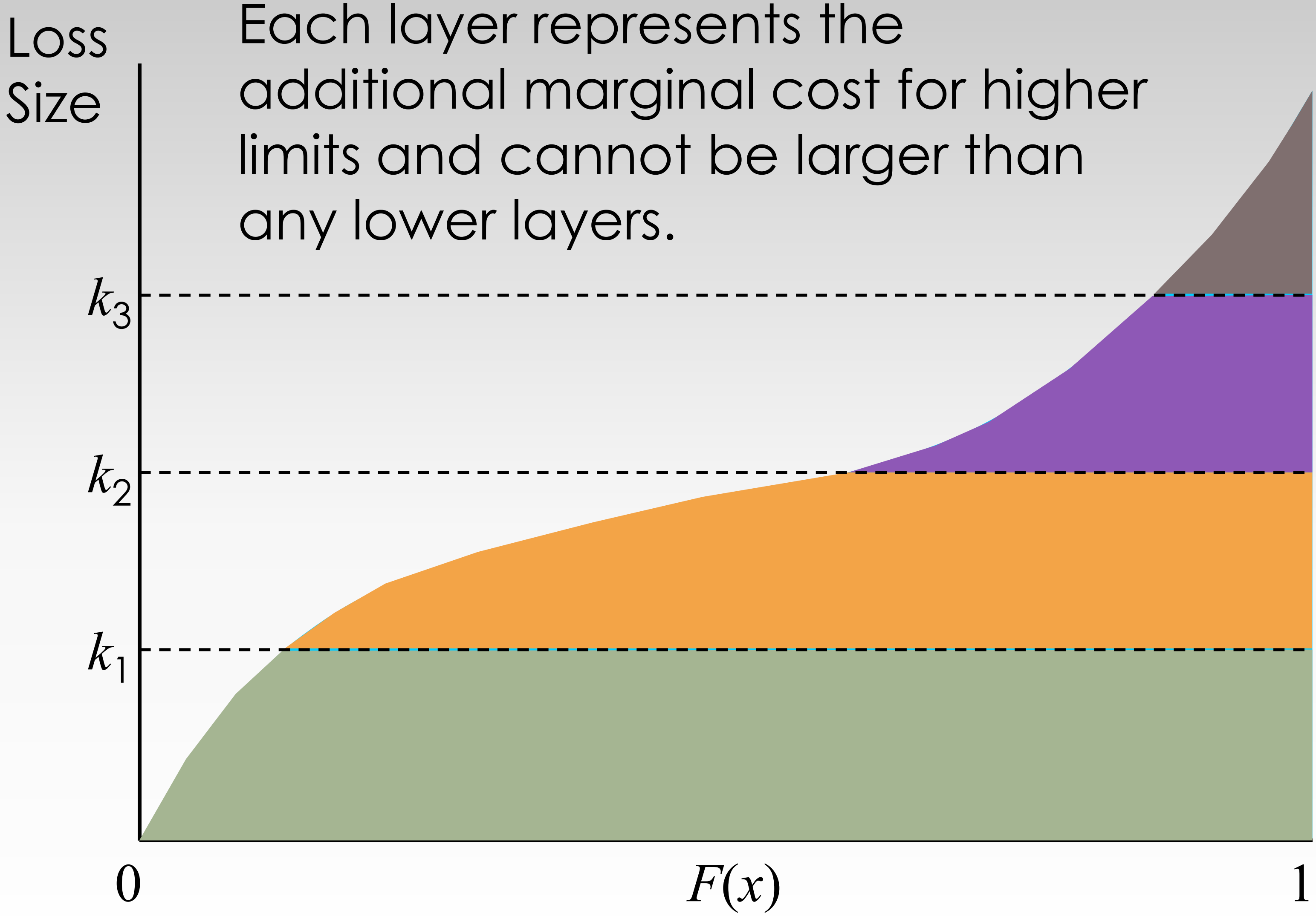
Consistency Rule

Limit ($000s)	ILF	ΔILF/Δlimit
25	1.00	–
50	1.60	0.0240
100	2.60	0.0200
250	6.60	0.0267
500	10.00	0.0136

inconsistency at $250k limit

Consistency Rule

Each layer represents the additional marginal cost for higher limits and cannot be larger than any lower layers.



Consistency Rule

Limit ($000s)	ILF	ΔILF/Δlimit
10	1.000	–
25	1.195	0.0130
35	1.305	0.0110
50	1.385	0.0053
75	1.525	***0.0056***
100	1.685	***0.0064***
125	1.820	***0.0054***
150	1.895	0.0030
175	1.965	0.0028
200	2.000	0.0014
250	2.060	0.0012
300	2.105	0.0009
400	2.245	***0.0014***
500	2.315	0.0007

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Deductible Ratemaking

Deductibles

Deductible ratemaking is closely related to increased limits ratemaking

- based on the same idea of loss layers
- difference lies in the layers considered

We will focus on the fixed dollar deductible

- most common
- simplest
- same principles can be applied to other types of deductibles

Deductibles

Loss Elimination Ratio (LER)

- savings associated with use of deductible
- equal to proportion of ground-up losses eliminated by deductible

Expected ground-up loss

- full value property or total limits liability = E[X]

Expected losses below deductible j

- limited expected loss = E[X^j]

Example: LER(j) = E[X^j] / E[X]

Deductibles

The LER is used to derive a deductible relativity (DR)

- deductible analog of an ILF
- factor applied to the base premium to reflect a deductible

Factor depends on:

- LER of the base deductible
- LER of the desired deductible

Deductibles

Example:

- base deductible is full coverage (i.e. no deductible)
- insurance policy with deductible j benefits from a savings equal to LER(j)
- in this case, DR(j) = 1 – LER(j)

Deductibles

If the full coverage premium for auto physical damage is $1,000 and the customer wants a $500 deductible, we can determine the $500 deductible premium if we know LER($500). Assume LER($500) = 31%.

- DR($500) = 1 – 0.31 = 0.69
- $500 deductible premium = 0.69 × $1,000 = $690

Calculating a Deductible Relativity using Empirical Data

Calculate the $5,000 and $10,000 deductible relativities using the following ground-up losses for unlimited policies with no deductibles.

Losses x
$2,000
$9,500
$18,000
$30,500
$75,000

Calculating a Deductible Relativity using Empirical Data

Losses x	min{x, \$5k}	min{x, \$10k}
\$2,000	\$2,000	\$2,000
\$9,500	\$5,000	\$9,500
\$18,000	\$5,000	\$10,000
\$30,500	\$5,000	\$10,000
\$75,000	\$5,000	\$10,000

E[X] = \$135,000 / 5 = \$27,000
E[X^\$5k] = \$22,000 / 5 = \$4,400
E[X^\$10k] = \$41,500 / 5 = \$8,300
LER(\$5k) = E[X^\$5k] / E[X] = 0.163
DR(\$5k) = 1 – LER(\$5k) = 0.837
LER(\$10k) = E[X^\$10k] / E[X] = 0.307
DR(\$10k) = 1 – LER(\$10k) = 0.693

Deductibles

The prior examples were simplistic because the base deductibles were full coverage.

A more generalized formula can be used to calculate deductible relativities where the bases deductible is non-zero.

We divide out the effect of the base deductible and multiply by the effect of the desired deductible. In other words, go back to the full coverage case and work from there.

Deductibles

The deductible relativity from the base deductible d to another deductible j can be expressed as:

$$DR_d(j) = \frac{1 - LER(j)}{1 - LER(d)}$$

Example:

- base deductible is \$500 and LER(\$500) = 0.24
- \$250 deductible is desired and LER(\$250) = 0.19
- $DR_{\$500}$(\$250) = (1 – 0.19) / (1 – 0.24) = 1.066

Deductibles

The base deductible for this coverage is $500 and the unlimited average severity is $5,000. Calculate the $0, $250, $500, and $1000 deductible relativities.

j	E[X^*j*]	LER(*j*)	$DR_{\$500}(j)$
$0	$0	$0 / $5000 = 0.000	(1 – 0.000) / (1 – 0.094) = 1.104
$250	$240	$240 / $5000 = 0.048	(1 – 0.048) / (1 – 0.094) = 1.051
$500	$470	$470 / $5000 = 0.094	(1 – 0.094) / (1 – 0.094) = 1.000
$1,000	$900	$900 / $5000 = 0.180	(1 – 0.180) / (1 – 0.094) = 0.905

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Mixed Exponential Procedure

Problems Associated with Calculating ILFs and DRs

- censorship – loss amounts are known but their values are limited
  - right censorship (from above) occurs when a loss exceeds the policy amount, but its value is recorded as the policy limit amount
- truncation – events are undetected and their values are completely unknown
  - left truncation (from below) occurs when a loss below the deductible is not reported

Problems Associated with Calculating ILFs and DRs

- data sources include several accident years
  - trend
  - loss development
- data is sparse at higher limits

Fitted Distributions

Data can be used to fit the severity function to a probability distribution

Addresses some concerns

- ILFs can be caluclated for all policy limits
- empirical data can be smoothed
- trend
- payment lag

ISO has used different distributions, but currently uses the mixed exponential model

Mixed Exponential Procedure (Overview)

- Use paid (settled) occurrences from statistical plan data and excess and umbrella data
- Fit a mixed exponential distribution to the lag-weighted occurrence size distribution from the data
- Produces the limited average severity component from the resulting distribution

Mixed Exponential Procedure (Overview)

Advantages of the Mixed Exponential Model:

- continuous distribution
  - calculation of LAS for all possible limits
  - smoothed data
  - simplified handling of trend
  - calculation of higher moments used in risk load
- provides a good fit to empirical data over a wide range of loss sizes, is flexible, and easy to use

Mixed Exponential Procedure (Overview)

- trend
- construction of the empirical survival distribution
- payment lag process
- tail of the distribution
- fitting a mixed exponential distribution
- final limited average severities

Questions and Answers

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