# And The Winner Is…? How to Pick a Better Model

## Part 2 – Goodness-of-Fit and Internal Stability

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## Goodness-of-Fit

- Trying to answer question: How well does our model fit the data?
- Can be measured on training data or on holdout data
- By identifying areas of poor model fit, we may be able to improve our model
- A few ways to measure goodness-of-fit
  - Squared or absolute error
  - Likelihood/log-likelihood
  - AIC/BIC
  - Deviance/deviance residuals
  - Pearson Chi-Squared
  - Plot of actual versus predicted target



## Squared Error & Absolute Error

- For each record, calculate the squared or absolute difference between actual and predicted target variable
- Easy and intuitive, but generally inappropriate for insurance data, and can lead to selection of wrong model
- Squared error appropriate for Normal data, but insurance data generally not Normal

## Likelihood

- The probability, as predicted by our model, that what actually did occur would occur
- A GLM calculates the parameters that maximize likelihood
- Higher likelihood → better model fit (very simple terms)
- Problem with likelihood – adding a variable always improves likelihood



## AIC & BIC

- Akaike Information Criterion (AIC) =
  -2*(Log Likelihood) + 2*(Number of Parameters in Model)
- Bayesian Information Criterion (BIC) =
  -2*(Log Likelihood) + (Number of Parameters in Model)*ln(Number of Records in Dataset)
- Penalized measures of fit
- Good rule for deciding which variables to include – unless a variables improves AIC or BIC, don't include it
- BIC often too restrictive



## Deviance

- Saturated model – the model with the highest possible likelihood
  - One indicator variable for each record, so model fits data perfectly
- Deviance = 2*(loglikelihood of saturated model – loglikelihood of fitted model)
- GLMs minimize deviance
- Like squared error, but reflects shape of assumed distribution
- We generally fit skewed distributions to insurance data (Tweedie, gamma, etc), and thus deviance is more appropriate than squared error

## Deviance – in Math

- Poisson: $2\sum_i w_i\left(y_i \ln\frac{y_i}{\mu_i} - y_i + \mu_i\right)$
- Gamma: $2\sum_i w_i\left(-\ln\frac{y_i}{\mu_i} + \frac{y_i-\mu_i}{\mu_i}\right)$
- Tweedie: $2\sum_i w_i\left(y_i\frac{{y_i}^{1-p}-\mu_i^{1-p}}{1-p} - \frac{{y_i}^{2-p}-\mu_i^{2-p}}{2-p}\right)$
- Normal: $\sum_i w_i(y_i-\mu_i)^2$



## Residuals

- Raw residual = $y_i - \mu_i$, where y is actual value of target variable and μ is predicted value
- In simple linear regression, residuals are supposed to be Normally distributed, and departure from Normality indicates poor fit
- For insurance data, raw residuals are highly skewed and generally not useful



## Deviance Residuals

- Square root of (weighted) deviance times the sign of actual minus predicted
- Measures amount by which the model missed, but reflects the assumed distribution
- Should be approximately Normally distributed, and far departure from Normality indicates that incorrect distribution has been chosen
- Ideally, there should be no discernable pattern in deviance residuals
  - Model should miss randomly, not systemically

## Deviance Residual Diagnostics

- Histogram of deviance residuals – look for approximate Normality (bell-shape)
  - Far departure from Normality generally indicates that incorrect distribution has been chosen
    - Can also indicate poor fit
- Scatter plot of deviance residuals versus predicted target variable
  - Should be uninformative cloud
  - Pattern in this plot indicates incorrect distribution



## Deviance Residual Diagnostics

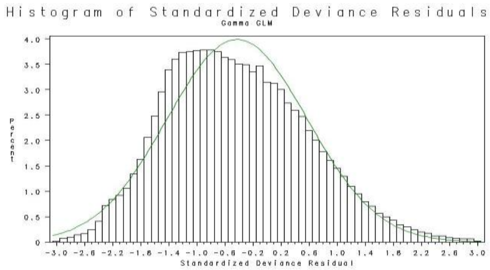
- Scatter plot of deviance residuals versus weight
  - If weight statement is appropriate, then plot should be uninformative cloud
- Plot deviance residual for each record and look for outliers
- Feed deviance residuals into tree algorithm
  - If deviance residuals are random, then tree should find no significant splits



## Example: Selecting Severity Model

- Goal is to select a distribution to model severity
- Two common choices – Gamma and Inverse Gaussian
  - Gamma: $V(\mu) = \mu^2$
  - Variance of severity is proportional to mean severity squared
  - Inverse Gaussian: $V(\mu) = \mu^3$
  - Variance of severity is proportional to mean severity cubed
- Two lines of business
  - LOB1 is high-frequency, low-severity
  - LOB2 is low-frequency, high-severity

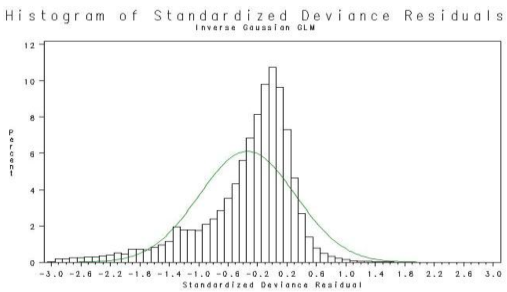
## Deviance Residual Histogram




LOB1, Gamma GLM



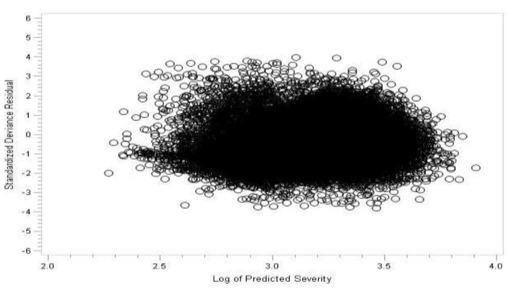
## Deviance Residual Histogram




LOB1, IG GLM

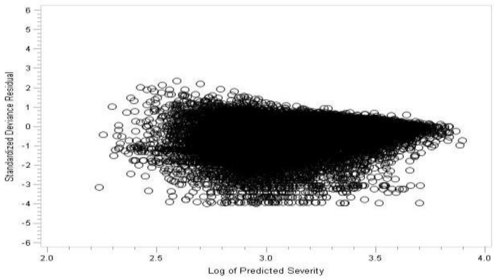


## Deviance Residual Histogram




LOB1, Gamma GLM

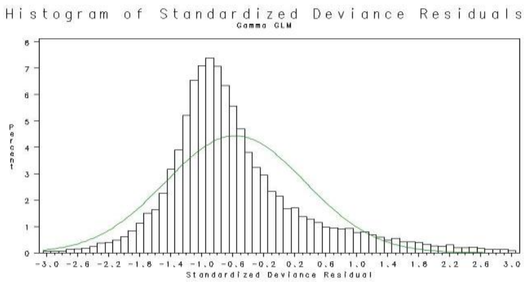
## Deviance Residual Histogram




LOB1, IG GLM



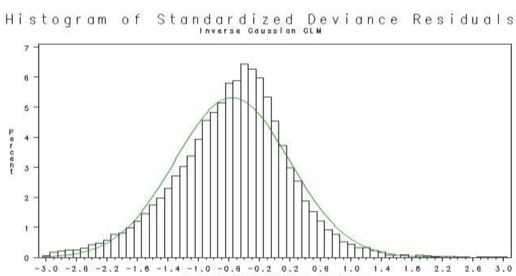
## Deviance Residual Histogram




LOB2, Gamma GLM



## Deviance Residual Histogram




LOB2, IG GLM

## Deviance Residuals Caution

- Analysis of deviance residuals only applicable to continuous or somewhat-continuous data
- If building a frequency model, and every record has either 0 or 1 claim, then deviance residuals will be bimodal
- If can aggregate discrete data to make it somewhat continuous, then deviance residual diagnostics may be appropriate

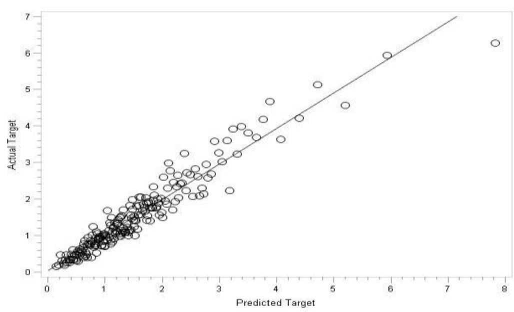


## Actual vs Predicted Target

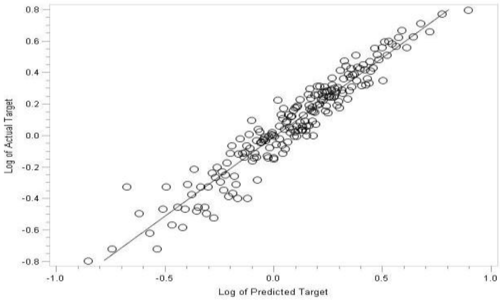
- Scatter plot of actual target variable (on y-axis) versus predicted target variable (on x-axis)
- If model fits well, then plot should produce a straight line, indicating close agreement between actual and predicted
  - Focus on areas where model seems to miss
- If have many records, may need to bucket (such as into percentiles)
- Depending on scale, may need to plot on a log-log scale



## Example of Actual vs Predicted



## Example of Log of Actual vs Log of Predicted



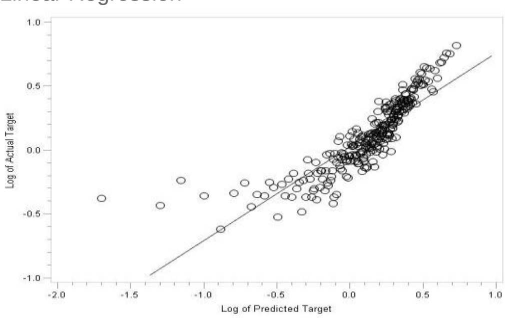



## Benefit of Deviance over Squared Error

- Since squared error is the deviance of a regression model with a Normal distribution, using squared error for non-Normal data can lead to incorrect model being chosen
- We run two models on our dataset – one with a Tweedie distribution and one with a Normal distribution
- Data is far from Normal, but using squared error as a metric, the Normal GLM wins
  - Even absolute error shows the Normal winning



## Log of Actual vs Log of Pred Target with Normal Linear Regression



## Measuring Internal Stability

- Process of determining how robust our model results are
- Useful measures:
  - Out-of-sample (out-of-time) validation
  - Cross-validation
  - Plotting actual versus predicted target variable on holdout data
  - Measures of influence (e.g. Cook's Distance)
  - Bootstrapping



## Out-of-Sample Validation

- Important to assess model fit on data that was not used in model construction
- Two approaches:
  - Initially split dataset into training and test, build model on training, and measure fit on test
  - Cross-validate – repeatedly use one subset to build and one to test
- Can randomly split dataset, or can split based on a control variable (like year)



## Assessing Stability over Time

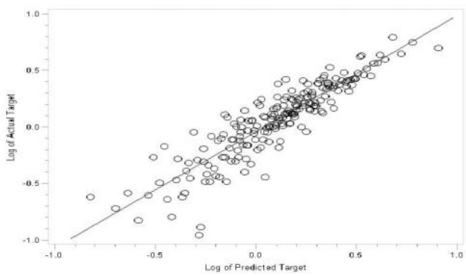
- Generally want model results to be stable over time
- To assess temporal stability, can run the model on individual years and look for variability
  - For example, if have 5 years, can run model on just years 1 and 2, then on just years 2 and 3, etc
  - Ideally, the parameter estimates don't change significantly across subsets

## Plot of Actual vs Predicted on Holdout

- Produce scatter plot of actual target variable versus predicted target variable as before, but use one set to build model and another set to plot
- Very simple diagnostic to produce and understand, and tells a powerful story
  - Easy to explain to non-technical audience



## Example of Plot of Actual vs Predicted on Holdout






## Bootstrapping

- Re-sampling technique that allows us to get more out of our data
- Start with a dataset and sample from it with replacement
  - Some records will get pulled multiple times, and some will not get pulled at all
- Generally, we create a dataset with the same number of records as our original dataset
- Can create many bootstrap datasets, and each dataset can be thought of as an alternate reality
  - Since each bootstrap is an alternate reality, we can use bootstrapping to construct confidence intervals

## Bootstrap CIs for Parameter Estimates

- GLMs produce confidence intervals for parameter estimates, but it is valuable to get a second opinion
- Create many bootstrap datasets, re-run the GLM on each dataset, and construct a confidence interval based on the resulting parameter estimates
- If bootstrap confidence interval is significantly wider than that produced by GLM, it is a sign that our results are overly-influenced by a few records



## Confidence Intervals for Lift Measures

- Can use bootstrapping to put confidence intervals around lift measures, like Gini indices
- In measuring lift, we seek to answer the question: Does Model A outperform Model B?
- If the answer is yes, then the second question is: How significant is the win?
- Say Model A has a Gini index of 15.90 and Model B has a Gini index of 15.40
  - Model A has a Gini index that is 0.50 higher, but is that difference significant?
- Can also bootstrap quantile plots and double lift charts



## References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