CAS Ratemaking and Product Management Seminar
Session RR4: Capital Allocation - A Hands-on Case Study

The purpose of this exercise is to have you apply a variety of capital allocation methods to a single set of data, evaluate the results of the different methods and, for different assigned roles, select and justify a particular allocation method and parameterization. To make this process less cumbersome, we will focus on just two lines of business (Homeowners and Private Passenger Auto) and one category of investments. For the lines of business underwriting gain/loss will be used to allocate capital. For investments, net investment gain/loss will be used. The Ruhm-Mango-Kreps algorithm will be used to calculate the capital allocations. Although for some methods this is only an approximation to the exact values, it provides a reasonable enough result to demonstrate the effect of the different methods.

There are five roles to play in this case. Before starting the calculations, assign the roles of VP-Homeowners, VP-Auto, VP-Investments, CRO and CEO. Work on all of the calculations together, but you are each to select one allocation method (the one that serves your self-interest the best) and develop a convincing argument as to why that allocation method should be the one the company uses for all purposes, including evaluating management performance.

The hypothetical company writes approximately $130 million in Homeowners premium and approximately $350 million in Private Passenger Auto premium.   The company has $100 million in capital in total. Clearly this is a very highly leveraged company, but the figures were selected so the different capital allocation methods can be applied.

To perform the calculations, start with the Case Study data set that was provided as an Excel file. This includes 10,000 iterations of the economic capital model the company uses. Next sort the entire file from smallest to largest based on the Aggregate Gain/Loss column. Make sure that you sort all of the columns based on that value so that, once sorted, the values in each row add to the aggregate values.

Once you have the sorted file, you can start to perform the calculations necessary to determine the capital allocation. Add a row that shows the average (Expected Value) of each of the columns, Homeowners and Private Passenger Auto underwriting gain/loss and Investment Gain/Loss. Add another row (blank at first) that will show the Risk Weighted Expected Value for each column. Add a third row that shows the Risk Measure. This will be the Risk Weighted Expected Value minus the Expected Value once you have a Risk Weighted Expected Value. Then add a final row that shows the Capital Allocation as a percentage. This will be the Risk Measure for each column divided by the Risk Measure for the Aggregate column. Finally, add a column on the right side of the table for the Riskiness Leverage Factor (RLF). This is the value that you will change depending on the capital allocation method you are using.

Start with one of the easiest calculations, TVaR. Calculate the capital allocation for three different levels of confidence, 95%, 99% and 99.9%. For the 95% TVaR, the RLF will be 1 for the top 500 iterations (5% of the 10,000 iterations) and 0 for the rest. Since you already sorted the results from smallest to largest, the top 500 on the table will be the 5% worst results.

To calculate the Risk Weighted Expected Value for each column, sum the product of the RLF times the gain/loss value and divide by the total of the RLF column. It may help to perform this calculation in several steps (calculate the products, then sum the products and sum the RLFs, then divide the sum of the products by the sum of the RLFs) However, it is possible to perform that calculation as a single step by defining the cell properly.

Once you have the Risk Weighted Expected Value, the Risk Measure is this value minus the Expected Value and the Capital Allocation is the Risk Measure for each column divided by the total Capital Allocation.

For the 99% TVaR, the RLF will be 1 for the top 100 iterations, 0 otherwise. For the 99.9% TVaR, the RLF will be 1 for only the top 10 iterations, 0 otherwise. Write the

capital allocations in the chart below. Then take some time to discuss the differences and try to understand why the results vary the way they do.

Capital Allocation Percentages Based on TVaR

Based on:	Homeowners	PP Auto	Investments
TVaR(95%)			
TVaR(99%)			
TVaR(99.9%)			

Next calculate the capital allocation based on 95% Value-at-Risk with $\varepsilon = 0.5\%$. The 95% VaR is the value for which 5% (or 500 in this case) of the simulations have losses in excess of that level. This would be the 501st lowest simulation once the results have been ranked from smallest to largest. Since $\varepsilon = 0.5\%$, the RLF should be 1 for the 50 simulations on either side of the 95% VaR level. Thus, including the 95% VaR simulation there will be 101 cases where the RLF is 1, and the remaining values will be 0. The remaining calculations will be the same as described above. Also, calculate the capital allocation for the 99% VaR. In this case, the RLFs will be 1 for the 101 cases that bracket the aggregate result that is exceeded on 1% (100 simulations out of 10,000).   List the results below.

Capital Allocation Percentages Based on VaR

Based on:	Homeowners	PP Auto	Investments
VaR(95%)			
VaR(99%)			

Next calculate the capital allocation based on semi-variance. In this case, the RLF should equal $\mu - x_i$ if $x_i < \mu$, otherwise 0, where x_i is the aggregate value for simulation i, and μ is the average of all the aggregate values. Calculate the RLF for each row by comparing the cell value in the aggregate gain/loss column with mean aggregate gain/loss. For this method, approximately half of the 10,000 iterations will be unique values, and the rest 0.

It will not be exactly 50% since the distribution of aggregate results is not symmetric. Calculate the capital allocations as above and list here.

Capital Allocation Percentages Based on Semi-Variance

Based on:	Homeowners	PP Auto	Investments
Semivariance			

In order to determine the capital allocation based on the marginal capital approach of Myers-Read, determine when the company would be insolvent and look at the impact of adding a marginal amount of underwriting or investment risk around that point. For this case, assume that the insurer has $100 million in capital. Since the spreadsheet shows results in thousands, then this would be $100,000 on the table. Therefore, for any iteration that produced a loss in excess of $100,000 the firm would be insolvent an unable to pay its claims in full. Set the RLFs within ε of that iteration at 1, and the rest at 0. Use three different levels for ε, 1%, 0.5% and 0.1%. For example, for an ε of 1.0%, then the 100 rows above and 100 rows below the case where the aggregate losses are $100,000 would be 1, and all the rest of the values would be 0, (for a total of 200 RLFs of 1). Perform the calculations as described above and list the results below.

Capital Allocation Percentages Based on Myers-Read

Based on:	Homeowners	PP Auto	Investments
$\varepsilon = 1.0\%$			
$\varepsilon = 0.5\%$			
$\varepsilon = 0.1\%$			

This ends the calculation stage of this case. Next you are to select the capital allocation approach and parameterization level (if applicable) that you think benefits your role's best interest and develop a convincing argument as to why the company should adopt that approach. Test out your position by discussing it with the other group members and listen to their positions to help strengthen your case. The person playing the role of CRO

should select a position that reflects the best interests of the firm as a whole. Other than the CRO, each person receives a significant portion of their total compensation as a performance bonus. For the VPs (Homeowners, Auto or Investments) the bonus is determined based on risk adjusted results for the year within their division. This is determined by taking the actual results divided by the allocated capital. The CEO's bonus, on the other hand, is a percentage of the aggregate results over $36.4 million (or 36,354 on the table, which would occur if the insurer's results were in the top 25% of the modeled values), with no financial penalty for underperformance, although the CEO would be out of a job if the company were to become insolvent. You do not need to write out your case in full, but be prepared to present that position in class discussion.