CAS Ratemaking and Product Management Seminar

Pricing Options for Risk Exposure Accumulation

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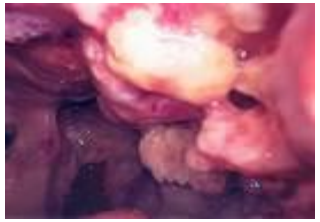
Risk Exposure Accumulation - Definition

- Risk of large aggregate losses from a single event or peril due to the concentration of insured risk exposed to that single event or peril
  - Hurricane

  - Earthquake/ Fire following

  - Asbestos

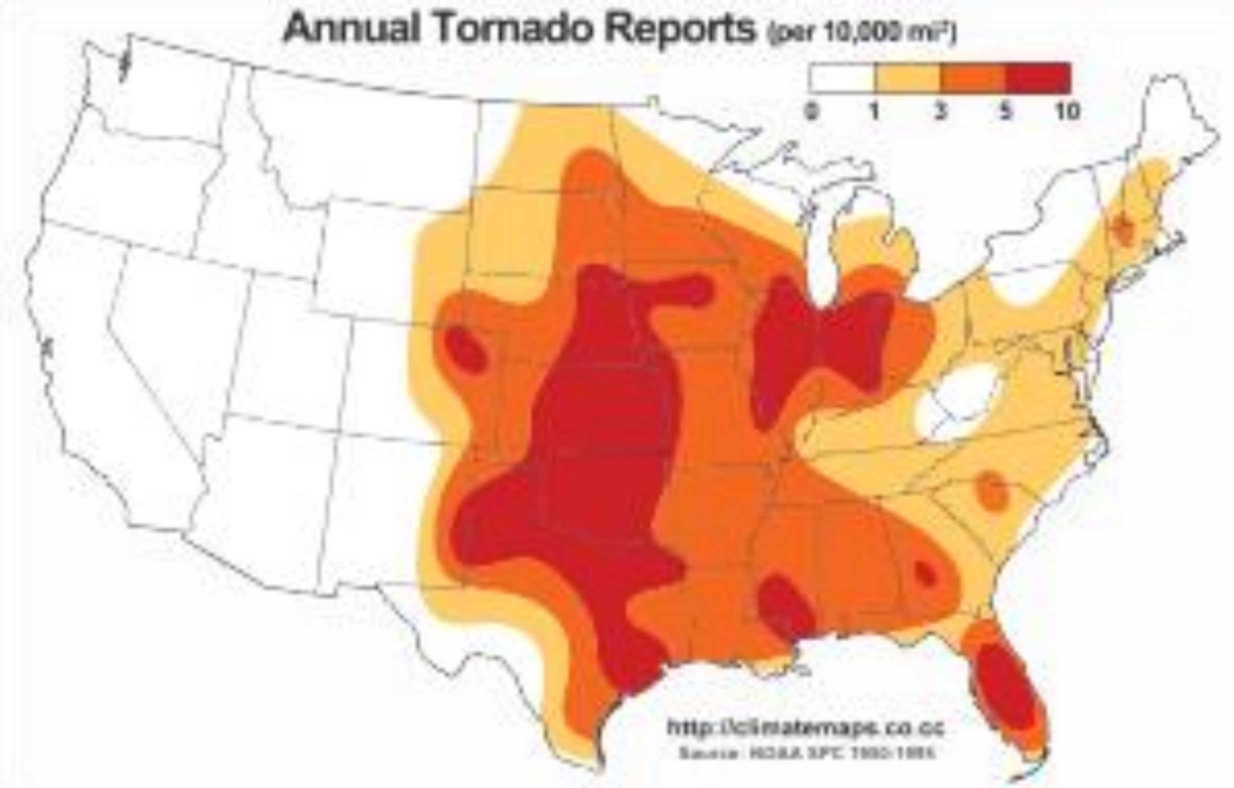


  - Wildfire

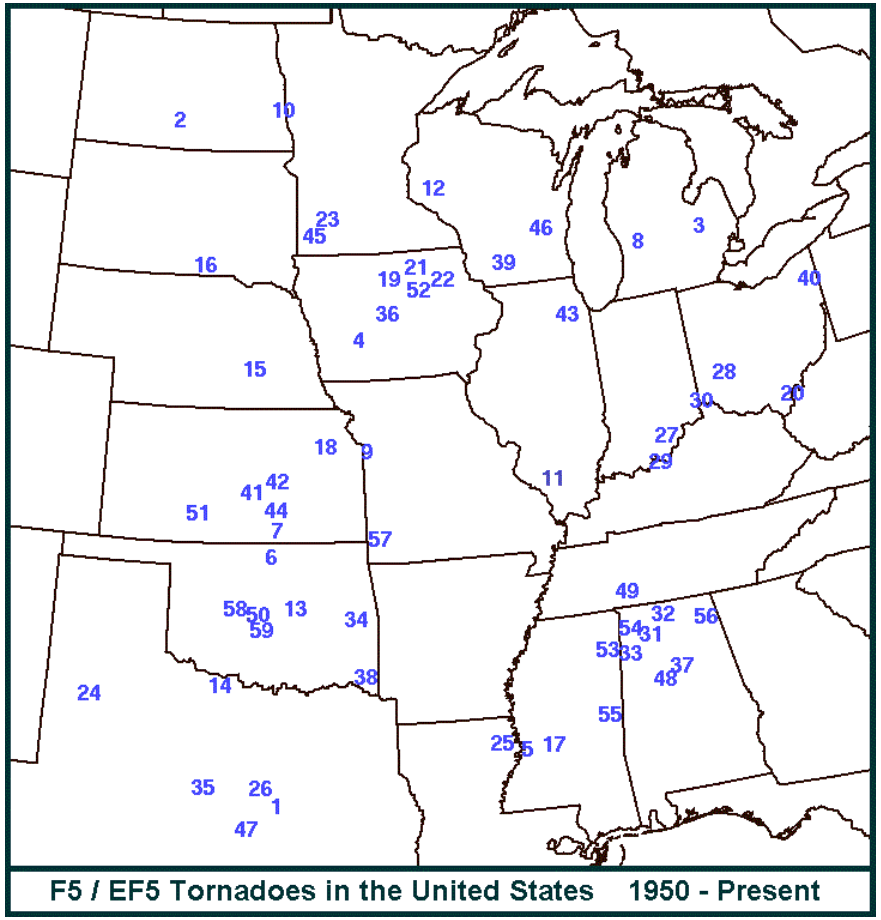
  - Tornado

  - Pollution

Tornado risk is not limited to a coast or a fault line. It is concentrated in a multi-state region.



For perspective, Oklahoma is 69,960 square miles



Source: http://www.spc.noaa.gov/faq/tornado/f5torns.html

Last storm listed is the May 20, 2013 Moore, OK tornado of 2013

Tornadoes: How bad can they get?

- EF5 tornadoes have wind gusts of over 200 MPH -
  - May 31 2013, El Reno, Oklahoma tornado had speeds near 295 MPH

- Tornadoes can have tracks over 100 miles long
  - The El Reno Oklahoma tornado storm track was 16.2 miles long and 2.6 miles wide at its widest point: (Manhattan is 2.3 miles wide at its widest point)

- An EF5 tornado once lifted and threw a 160,000 pound tanker several hundred feet.

Tornadoes: How bad can they get?

	DATE	LOCATION(S)	ACTUAL $	INFLATION ADJUSTED* $
1	22 May 2011	Joplin, MO	2,800,000,000	2,907,000,000
2	27 April 2011	Tuscaloosa, AL	2,450,000,000	2,543,690,000
3	8 Jun 1966	Topeka, KS	250,000,000	1,797,810,000
4	11 May 1970	Lubbock, TX	250,000,000	1,502,960,000
5	3 May 1999	Oklahoma City, OK	1,000,000,000	1,401,730,000
6	27 Apr 2011	Hackleburg, AL	1,290,000,000	1,339,330,000
7	3 Apr 1974	Xenia, OH	250,000,000	1,183,600,000
8	6 May 1975	Omaha, NE	250,603,000	1,084,430,000
9	10 Apr 1979	Wichita Falls, TX	277,841,000	893,853,000
10	3 Jun 1980	Grand Island, NE	285,050,000	807,953,000

http://www.spc.noaa.gov/faq/tornado/damage$.htm

* 2013 dollars, using the U.S. Federal Reserve Bank's Consumer Price Index calculations available online.

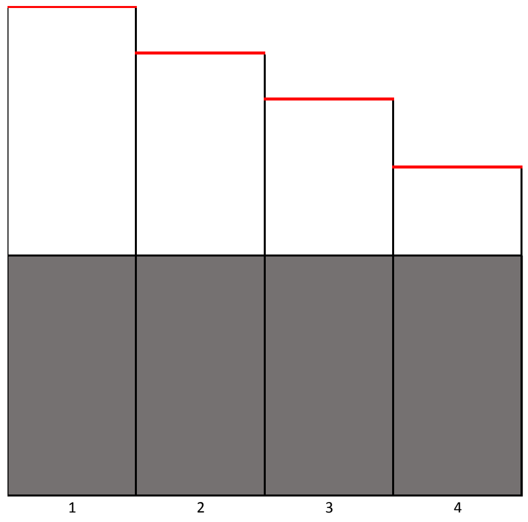
Risk Exposure Accumulation: Management Options

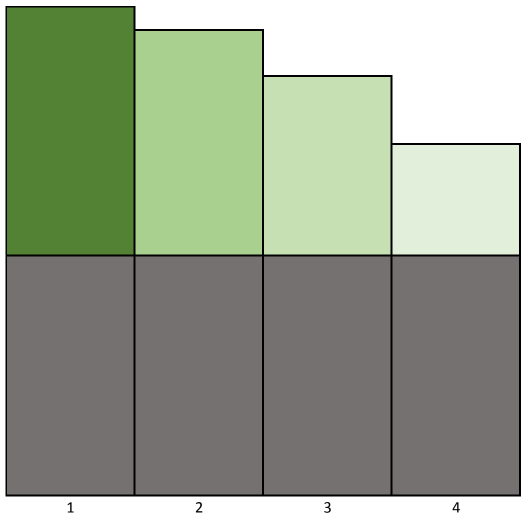
- Exclude the Risk through *Marketing* and *Underwriting Rules*
- Measure the risk of adding one more policyholder to a territory
  - Marginal VaR
  - Marginal CTE

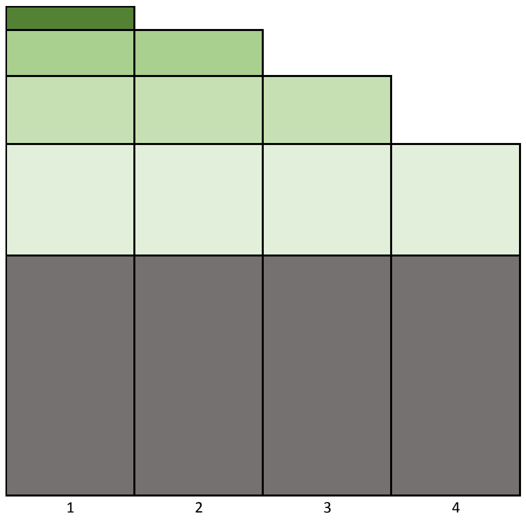
- Transfer the Risk through *Reinsurance* and *Alternative Risk Transfer*
- **Price** the average cost of capital to a reinsurer of the current book
  - Capital Consumption model

- Price for the retained risk through *Risk loads*
- **Price** the average cost of capital to an investor of the current book
  - Risk loads based on CAT bond pricing

- Reduce the risk through *Property level mitigation credits* and *inspections*







What is the Risk Load?

- Starting with an Ordinary Premium Equation where is the risk load?

$$P(terr) = \frac{LLAE(terr)+F}{1-V-p} + risk\ load?$$

What is the Risk Load?

- According to ASOP 30: Treatment of Profit and Contingency Provisions and the Cost of Capital in Property Casualty Insurance Ratemaking

2.3 Cost of Capital – The rate of return that capital could be expected to earn in alternative investments of equivalent ***risk***; also known as opportunity cost (italics, bold and color added)

3.1 Estimating the Cost of Capital and Underwriting Profit Provision – Property/casualty insurance rates should provide for all expected costs, including an appropriate ***cost of capital*** associated with the specific ***risk transfer***. This cost of capital can be provided for by estimating that cost and translating it into an underwriting profit provision, after taking leverage and investment income into account. Alternatively, the actuary may develop an underwriting profit provision and test that profit provision for consistency with the cost of capital. The actuary may use any appropriate method, as long as such method is consistent with the considerations of this standard. … (truncated. italics, bold and color added)

What is the Risk Load?

- By-Peril Premium Equation Separates Premium into Perils.

$$P(terr) = P_{Non-CAT}(terr) + P_{CAT}(terr)$$

- Where Each Peril has its own Profit Load.

$$P(terr) = \frac{E(LLAE_{Non-CAT}(terr)) + F}{1 - V - p_{Non-CAT}} + \frac{E(LLAE_{CAT}(terr)) + F}{1 - V - p_{CAT}(terr)}$$

What is the Risk Load?

- Risk Load is embedded in the CAT Premium.

$$P_{CAT}(terr) = \frac{E(LLAE_{CAT}(terr)) + F}{1 - V - p_{CAT}(terr)}$$

$$P_{CAT}(terr) = \frac{E(LLAE_{CAT}(terr)) + F}{1 - V} + risk\ load(terr)$$

- Risk Load is a function of CAT Profit Load, variable expenses and CAT Premium.

$$risk\ load(terr) = \frac{P_{CAT}(terr)p_{CAT}(terr)}{(1 - V)}$$

What is the Risk Load?

Simplified Process for *territories* (rather than lines of business) based on Appendix B of Don Mango's Capital Consumption paper:

- Generate Modeled Scenarios of Losses for all territories.
- For each scenario, calculate capital depletion costs
  - Apply a risk-averse utility function to aggregate depleted capital.
- For each scenario, allocate capital depletion costs back to territory
  - Allocate proportionally to all territories having an underwriting loss.
- Risk load by territory is expected value of depletion costs.

A Good Risk Averse Utility Function?

- Expected Excess Return (Risk Load) = (Yield – Risk Free rate) – Expected Default Loss
  = Yield Spread - Expected Default Loss

- $100 capital investment with $10 return, and 2% chance of $50 losses has
  - Yield spread of 10% - 5% = 5%
  - Expected Loss of (2% x $50)/$100 = 1%
  - Expected excess return (risk load) of 5% - 1% = 4%
  - Profit multiple of 4% / 1% = 4

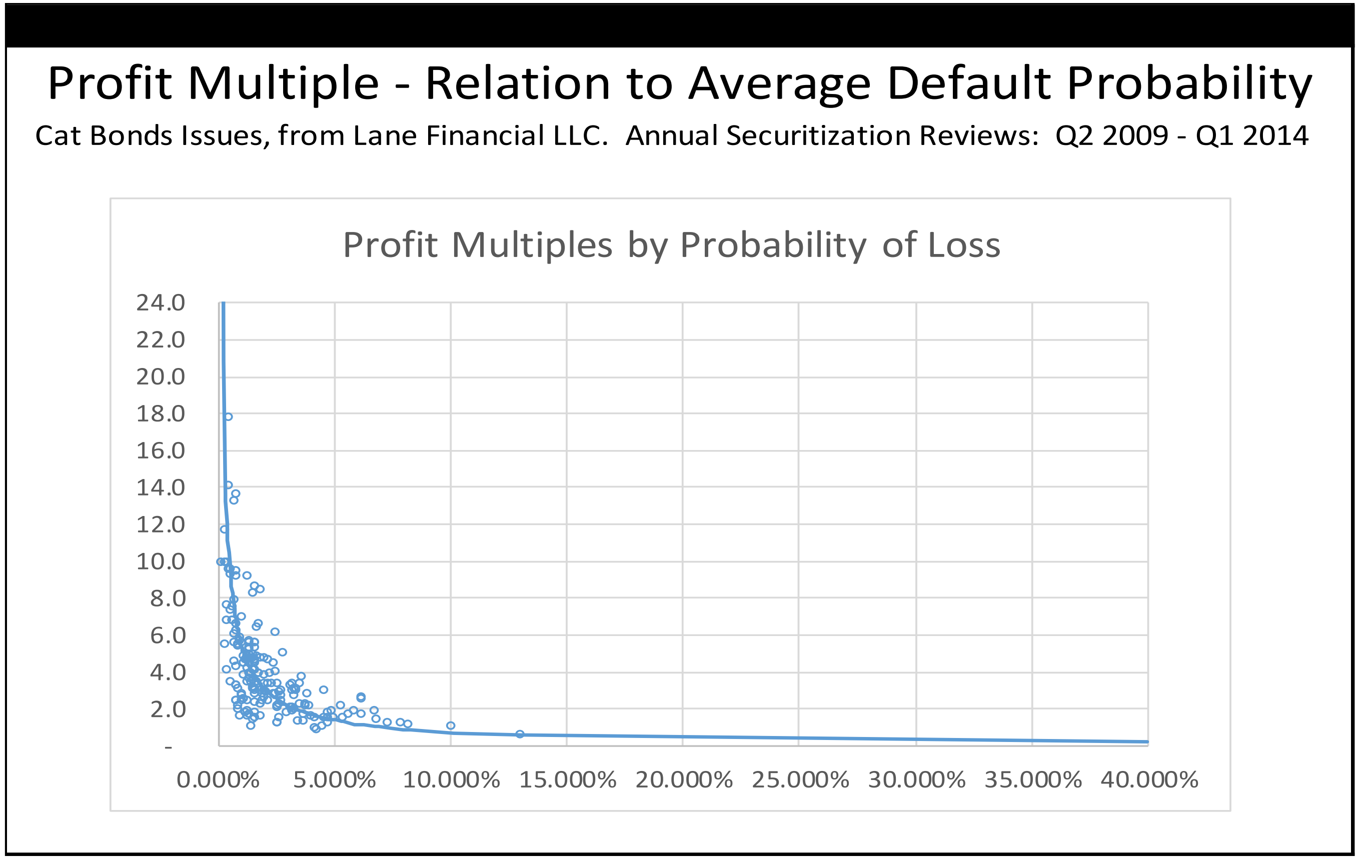
- $100 capital investment with $30 return, and 20% chance of $50 losses has
  - Yield spread of 30% - 5% = 25%
  - Expected Loss of (20% x $50)/$100 = 10%
  - Expected excess return (risk load) of 25% - 10% = 15%
  - Profit multiple of $15 / $10 = 1.5

A Good Risk Averse Utility Function?

- The profit multiple is an expression of a risk averse utility function.
  - The Capital Consumption Method estimates excess return based on the Utility of a capital call. With simple assumptions, the Capital Call Charges could also be converted to Profit Multiples as described by Chernick and Anderson:

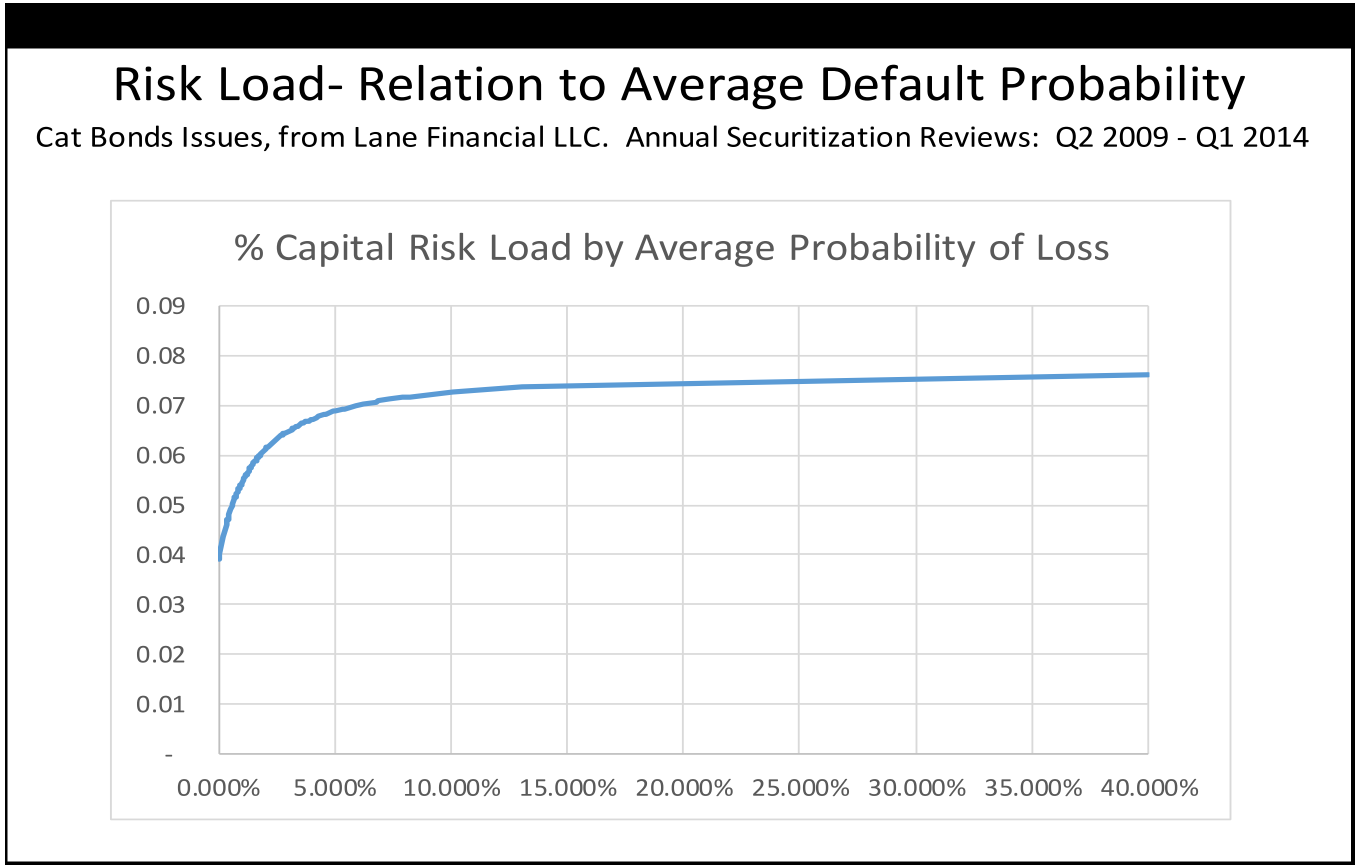
Capital Call Range		Capital Call Charge	Profit Multiple
0	5000	1.25	1.25
5001	10000	1.5	2.1
10001	20000	2	3.25
20001		4	7.06

A Good Risk Averse Utility Function?



- Chernick and Anderson described excess return based on Cat Bonds.
- They calculate profit multiples from those excess returns and fit a curve to them by probability of loss.
- It is well-known that the bond market expresses risk aversion.

A Good Risk Averse Utility Function?



- Chernick and Anderson described excess return based on Cat Bonds.
- They calculate profit multiples from those excess returns and fit a curve to them by probability of loss.
- But does regression alone result in the desired degree of risk aversion?

A Good Risk Averse Utility Function?

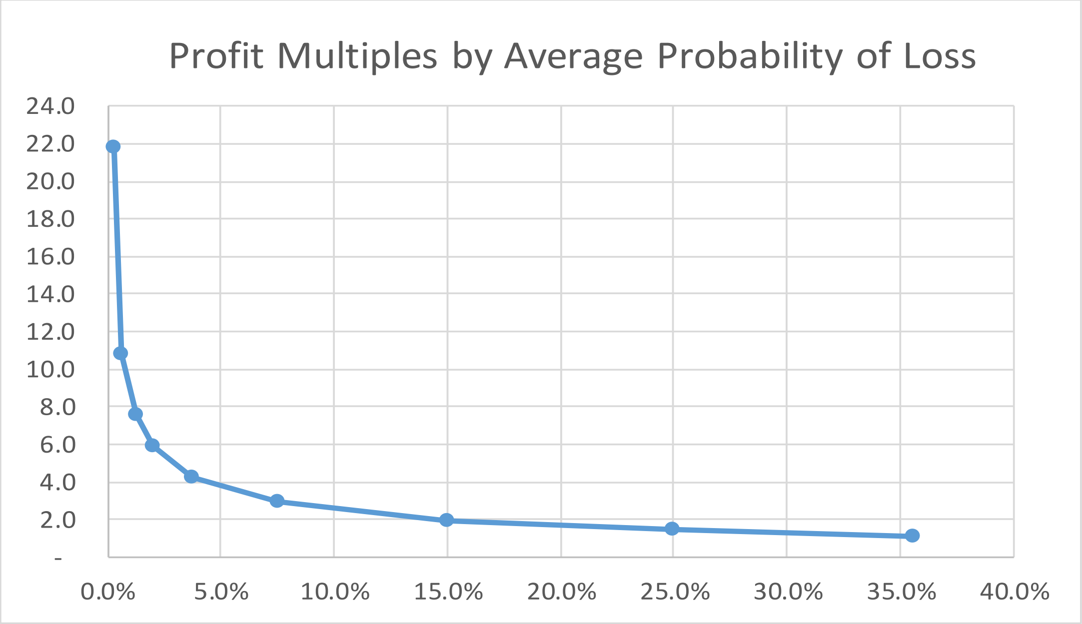
Average Default Probability	0.1%	0.6%	1.3%	2.0%	3.8%	7.5%	15.0%	25.0%	35.5%
CAT Bond-based Profit Multiples	31.3	8.0	4.5	3.1	1.8	1.0	0.5	0.3	0.2
Layer Risk Load (Layer Excess Return)	6,527	4,983	1,732	2,231	3,842	4,504	4,731	2,968	2,355
Standard Deviation of Layer Loss	5,229	7,778	3,307	4,936	10,497	15,878	21,678	16,260	14,233
Cat Bond Pseudo-Sharpe Ratios	125%	64%	52%	45%	37%	28%	22%	18%	17%

- Basing Profit Multiples on regression using historical CAT bond values can result in uncertainty in the tail, and a sparse number of right and left tail data points can have a leveraged impact on the curve.
- The key profit multiples are at low average default probabilities, where there is greater uncertainty.
- With such a limited number of low Default Probability CAT bonds, the functional form selected will have a large impact on the final profit multiples.

A Good Risk Averse Utility Function?

Profit Multiple - Relation to Average Default Probability

Based on Moody's BAA Rated Corporate Bonds & Assuming a Fixed Sharpe Ratio

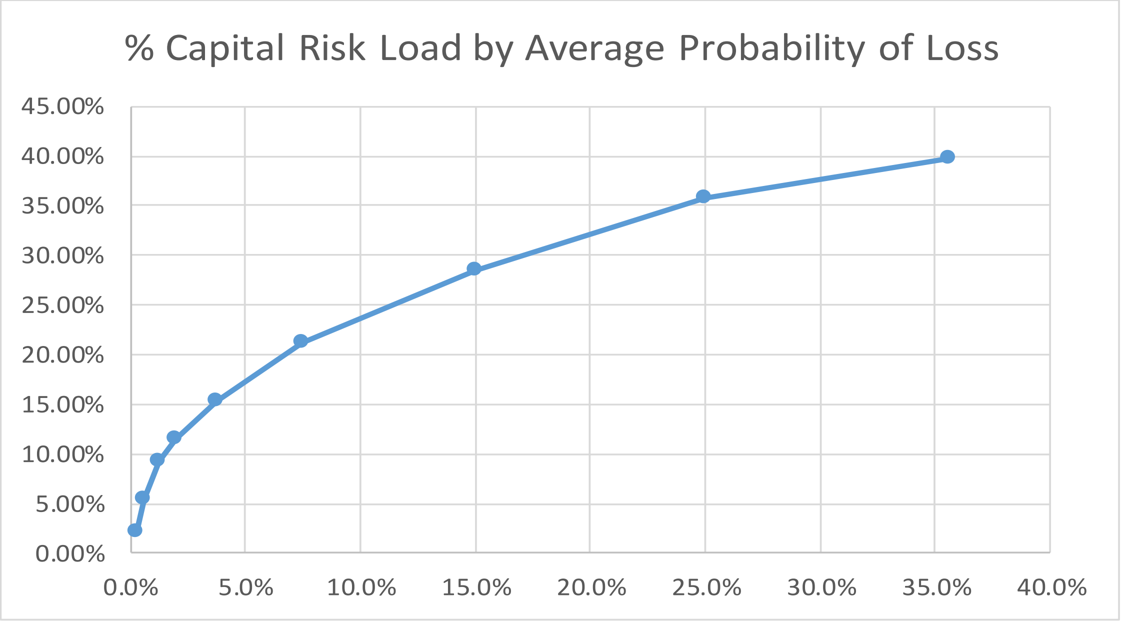


- 20-year Average Historical Bond Yields and long-term default rates for BAA rated bonds and treasuries give a risk load and profit multiple.
- The assumption of a constant pseudo-Sharpe ratio clearly expresses risk aversion and yields a profit multiple curve very similar to those of the CAT bonds.

A Good Risk Averse Utility Function?

Risk load - Relation to Average Default Probability

Based on Moody's BAA Rated Corporate Bonds & Assuming a Fixed Sharpe Ratio



- 20-year Average Historical Bond Yields and long-term default rates for BAA rated bonds and treasuries give a risk load and profit multiple.
- Risk load naturally ascends from zero risk load at zero probability of loss.

Three related risk load pricing methods

Capital Consumption method:

total capital call: Risk averse utility function based on its total capital magnitude

CAT Bond Risk Load method

Tranched capital: Risk load based on CAT bond prices by default probability

Corporate Bond, Constant Pseudo-Sharpe Ratio method

Tranched capital: Charges based on Corporate Bonds and constant Sharpe Ratio

Example: Triangular Tornado State

- Set-up of example:
  - Properties in a set of 10 territories
  - Simulation of the impact on those properties of random weather events
  - Uniform tornado risk in all territories
  - Capital adequate to cover all risks in the simulation

Example: Triangular Tornado State

- Scenario
  - Triangular Tornado State with equal sized (latitude/longitude) territories 1-10
  - # of Insured Homes & Average Insured Values are listed below.

Triangular State - Territory Numbers

1	2	4	7
	3	5	8
		6	9
			10

Triangular State - Insured Home Counts

41,577	3,016	2,413	2,059
	17,839	2,548	5,063
		5,099	919
			1,243

Triangular State - Average Insured Values

218,191	213,680	722,031	126,706
	107,161	98,780	219,714
		225,545	102,921
			293,849

Example: Triangular Tornado State

- For our example, Cholesky decomposition of the matrix below of correlations (then rescaling) was used to convert 10 independent uniform random variables between 0 and 1 to 10 correlated uniform random variables from 0 to 1. These variables represent percentiles of random lognormal "percentage loss" variables with mu = -4 and sigma = 0.5.

Triangular State - Matrix of Territorial Correlations of Uniformly Distributed Random Variables

	1	2	3	4	5	6	7	8	9	10
1	1.00	0.42	0.09	0.01	0.01	0.00	0.00	0.00	0.00	-
2	0.42	1.00	0.42	0.42	0.09	0.01	0.01	0.01	0.00	0.00
3	0.09	0.42	1.00	0.09	0.42	0.09	0.01	0.01	0.01	0.00
4	0.01	0.42	0.09	1.00	0.42	0.01	0.42	0.09	0.01	0.00
5	0.01	0.09	0.42	0.42	1.00	0.42	0.09	0.42	0.09	0.01
6	0.00	0.01	0.09	0.01	0.42	1.00	0.01	0.09	0.42	0.09
7	0.00	0.01	0.01	0.42	0.09	0.01	1.00	0.42	0.01	0.00
8	0.00	0.01	0.01	0.09	0.42	0.09	0.42	1.00	0.42	0.01
9	0.00	0.00	0.01	0.01	0.09	0.42	0.01	0.42	1.00	0.42
10	-	0.00	0.00	0.00	0.01	0.09	0.00	0.01	0.42	1.00

*Correlations selected based on distance from distance between centroids, with non-diagonals scaled to be positive definite

Example: Triangular Tornado State

- For each iteration / for each of territories 1 to 10, the CAT losses are calculated as
  CAT Losses = (Insured home count) x (Average Insured Value) x (Percentage Losses)
- Average CAT losses by territory across all scenarios are listed below. The total across all territories is $333,110

Triangular State - Territory Numbers

1	2	4	7
	3	5	8
		6	9
			10

Triangular State - Average Losses in '000's

187,109	12,543	33,612	5,016
	37,267	4,905	20,839
		22,810	1,869
			7,140

Example: Triangular Tornado State

BAA Bond Default Rate	0.27%
BAA Risk Load / Capital	2.31%
Tranche A Default Rate	0.27%
Tranche A Capital	213,603
Tranche A Risk Load	4,924
Tranche A Std Dev	5,229
Pseudo-Sharpe Ratio	**94%**

- To determine the Sharpe Ratio, we first aligned the 90-year Moody's BAA rated corporate bond default rate with the Tranche A default rate.
- We then applied the BAA rated bond's Risk Load as a % of Capital i.e. Expected Excess Return / Capital to our Tranche A Capital to determine the Tranche A risk load.
- We then calculated the standard deviation of simulated losses (excess of the mean) within the Tranche A band.
- The final Sharpe Ratio is the excess return divided the risk (i.e. **Tranche A Risk Load / Tranche A Standard Deviation**).

Example: Triangular Tornado State

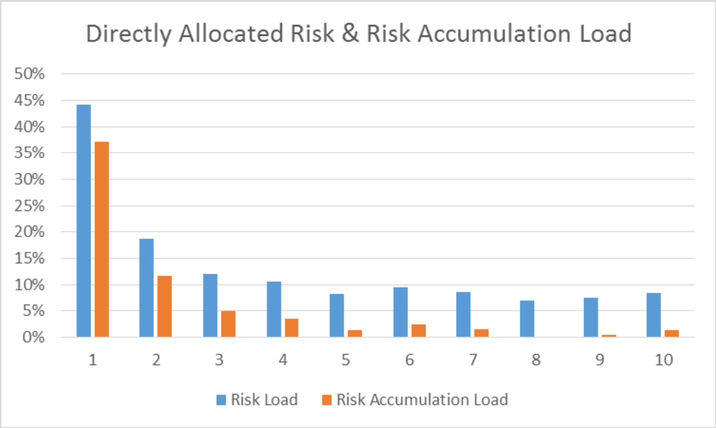
			Tranches								
		Calculation/Source	A	B	C	D	E	F	G	H	I
(1)	Loss Prob Low	Selected	0.00%	0.27%	1.00%	1.50%	2.50%	5.00%	10.00%	20.00%	30.00%
(2)	Loss Prob High	Selected	0.27%	1.00%	1.50%	2.50%	5.00%	10.00%	20.00%	30.00%	40.93%
(3)	Avg Default Prob	[(1)+(2)]/2	0.1%	0.6%	1.3%	2.0%	3.8%	7.5%	15.0%	25.0%	35.5%
(4)	E(Loss)	Between (6) and (7)	209	626	382	729	2,159	4,735	9,556	9,818	10,959
(5)	StDev(Loss)	Between (6) and (7)	5,229	7,778	3,307	4,936	10,497	15,878	21,678	16,260	14,233
(6)	Lower Limit	From simulation	786,397	663,463	632,312	595,139	535,230	469,995	403,646	364,208	333,110
(7)	Upper Limit*	From simulation	1,000,000	786,397	663,463	632,312	595,139	535,230	469,995	403,646	364,208
(8)	Capital	(7)-(6)	213,603	122,934	31,151	37,173	59,909	65,235	66,349	39,438	31,097
(9)	Risk Load	Sharpe Ratio x (5)	4,924	7,324	3,114	4,648	9,885	14,951	20,412	15,311	13,402
(10)	risk load/Capital	(9)/(8)	2.31%	5.96%	10.00%	12.50%	16.50%	22.92%	30.76%	38.82%	43.10%
(11)	Profit Multiple	(9)/(4)	23.6	11.7	8.1	6.4	4.6	3.2	2.1	1.6	1.2
(12)	Recovery Rate	[(2)-(4)/(8)]/(2)	63.80%	49.10%	18.22%	21.56%	27.91%	27.41%	27.99%	17.02%	13.90%

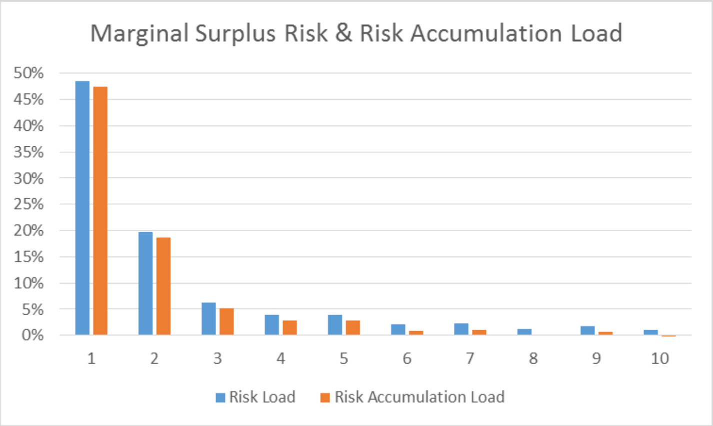
*Upper Limit for Tranch 1 of 1,000,000 is selected to be above the highest simulated losses.

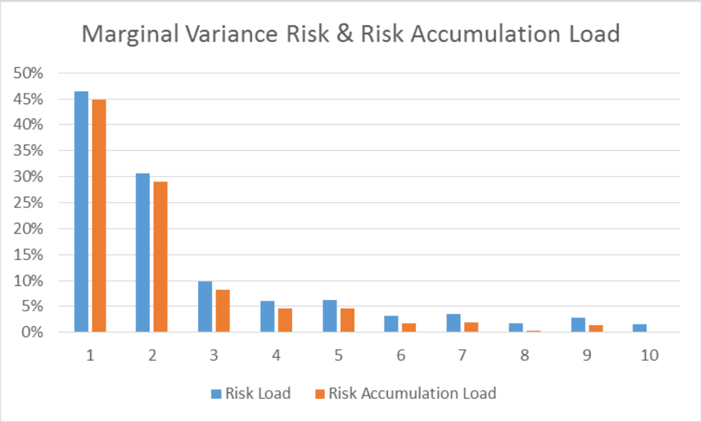
- The Sharpe Ratio calculated on the previous page is used to calculate the Risk Load (9).
- Lower Limit (6) corresponds to a 1-(2) Value at risk.
- The Lower Limit (6) at Tranche I of 333,110 equal to the all scenario mean of 333,110.

Risk Accumulation Loads as a Percentage of Losses

Profit Multiple Based on Corporate Bonds & Constant Sharpe Ratio







- Directly allocated Risk Loads
  - The Risk Load for a given Territory/scenario is calculated as the sumproduct() of the losses and the profit multiple by tranch.
  - The Risk Load for a given Territory is the average of the Loads for all Scenarios of that Territory
- Marginal Surplus Method Risk Loads
  - The Load for a given Territory is the usual Marginal Surplus Method. Risk load for Territory N is allocated based on the standard deviation of aggregate losses for all territories, minus the standard deviation of aggregate losses less the losses for Territory N.
- Marginal Variance Method Risk Loads
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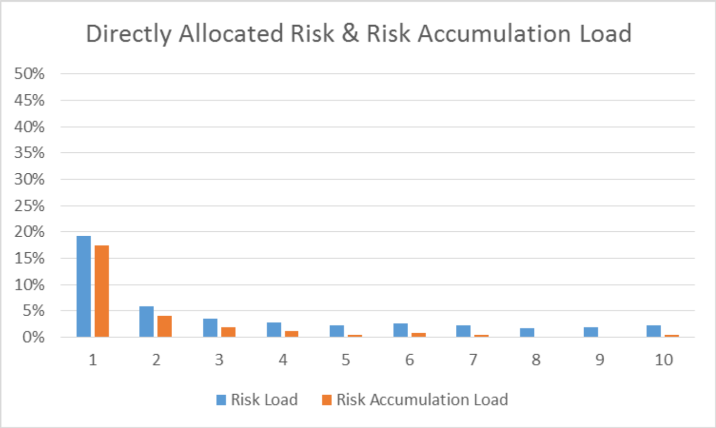
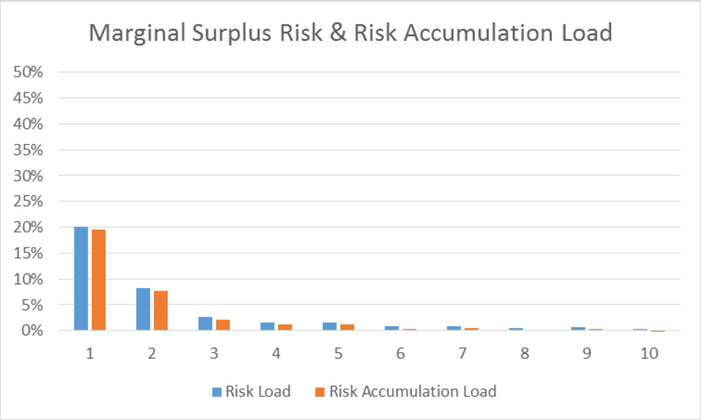
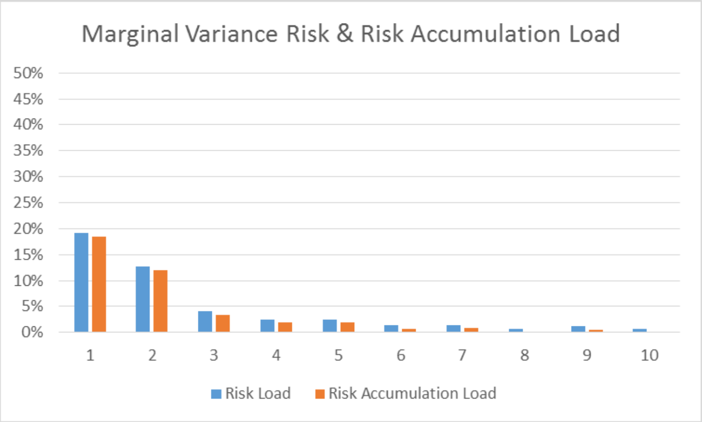
- Risk Accumulation Load for a Territory is estimated as Risk Load for that Territory minus the smallest Risk Load of all Territories.
- Is the Magnitude of the Risk Accumulation load appropriate? Will it impact retention and close ratios?

Risk Accumulation Loads as a Percentage of Losses

Profit Multiple Based Regression on CAT Bond Data







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Closing Steps

- Calculate territorial CAT premium: $P_{CAT}(terr) = \frac{E(LLAE_{CAT}(Terr))+F}{1-V} + risk\ load$
- Calculate territorial Profit load: $p_{CAT}(Terr) = \frac{risk\ load(1-V)}{P_{CAT}(terr)}$
- Now we can calculate variable, territorial risk loads for each policyholder's individual premium amount.

$$P(Terr) = \frac{E(LLAE_{Non-CAT}(Terr)) + F}{1 - V - p_{Non-CAT}} + \frac{E(LLAE_{CAT}(Terr)) + F}{1 - V - p_{CAT}(Terr)}$$

Questions?

Key References:

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