And The Winner Is…? How to Pick a Better Model

Part 2 – Goodness-of-Fit and Internal Stability

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Goodness-of-Fit

- Trying to answer question: How well does our model fit the data?
- Can be measured on training data or on holdout data
- By identifying areas of poor model fit, we may be able to improve our model
- A few ways to measure goodness-of-fit
  - Squared or absolute error
  - Likelihood/log-likelihood
  - AIC/BIC
  - Deviance/deviance residuals
  - Pearson Chi-Squared
  - Plot of actual versus predicted target

Squared Error & Absolute Error

- For each record, calculate the squared or absolute difference between actual and predicted target variable

- Easy and intuitive, but generally inappropriate for insurance data, and can lead to selection of wrong model

- Squared error appropriate for Normal data, but insurance data generally not Normal

Likelihood

- The probability, as predicted by our model, that what actually did occur would occur
- A GLM calculates the parameters that maximize likelihood
- Higher likelihood → better model fit (very simple terms)
- Problem with likelihood – adding a variable always improves likelihood

AIC & BIC

- Akaike Information Criterion (AIC) =

  -2*(Log Likelihood) + 2*(Number of Parameters in Model)

- Bayesian Information Criterion (BIC) =

  -2*(Log Likelihood) + (Number of Parameters in Model)*ln(Number of Records in Dataset)

- Penalized measures of fit

- Good rule for deciding which variables to include – unless a variables improves AIC or BIC, don't include it

- BIC often too restrictive

Deviance

- Saturated model – the model with the highest possible likelihood
  - One indicator variable for each record, so model fits data perfectly
- Deviance = 2*(loglikelihood of saturated model – loglikelihood of fitted model)
- GLMs minimize deviance
- Like squared error, but reflects shape of assumed distribution
- We generally fit skewed distributions to insurance data (Tweedie, gamma, etc), and thus deviance is more appropriate than squared error

Deviance – in Math

- Poisson: $$2\sum_i w_i\left(y_i \ln\frac{y_i}{\mu_i} - y_i + \mu_i\right)$$
- Gamma: $$2\sum_i w_i\left(-\ln\frac{y_i}{\mu_i} + \frac{y_i - \mu_i}{\mu_i}\right)$$
- Tweedie: $$2\sum_i w_i\left(y_i\frac{{y_i}^{1-p} - \mu_i^{1-p}}{1-p} - \frac{{y_i}^{2-p} - \mu_i^{2-p}}{2-p}\right)$$
- Normal: $$\sum_i w_i(y_i - \mu_i)^2$$

Residuals

- Raw residual = $y_i - \mu_i$, where y is actual value of target variable and μ is predicted value

- In simple linear regression, residuals are supposed to be Normally distributed, and departure from Normality indicates poor fit

- For insurance data, raw residuals are highly skewed and generally not useful

Deviance Residuals

- Square root of (weighted) deviance times the sign of actual minus predicted
- Measures amount by which the model missed, but reflects the assumed distribution
- Should be approximately Normally distributed, and far departure from Normality indicates that incorrect distribution has been chosen
- Ideally, there should be no discernable pattern in deviance residuals
  – Model should miss randomly, not systemically

Deviance Residual Diagnostics

- Histogram of deviance residuals – look for approximate Normality (bell-shape)
  - Far departure from Normality generally indicates that incorrect distribution has been chosen
    - Can also indicate poor fit

- Scatter plot of deviance residuals versus predicted target variable
  - Should be uninformative cloud
  - Pattern in this plot indicates incorrect distribution

Deviance Residual Diagnostics

- Scatter plot of deviance residuals versus weight
  - If weight statement is appropriate, then plot should be uninformative cloud
- Plot deviance residual for each record and look for outliers
- Feed deviance residuals into tree algorithm
  - If deviance residuals are random, then tree should find no significant splits

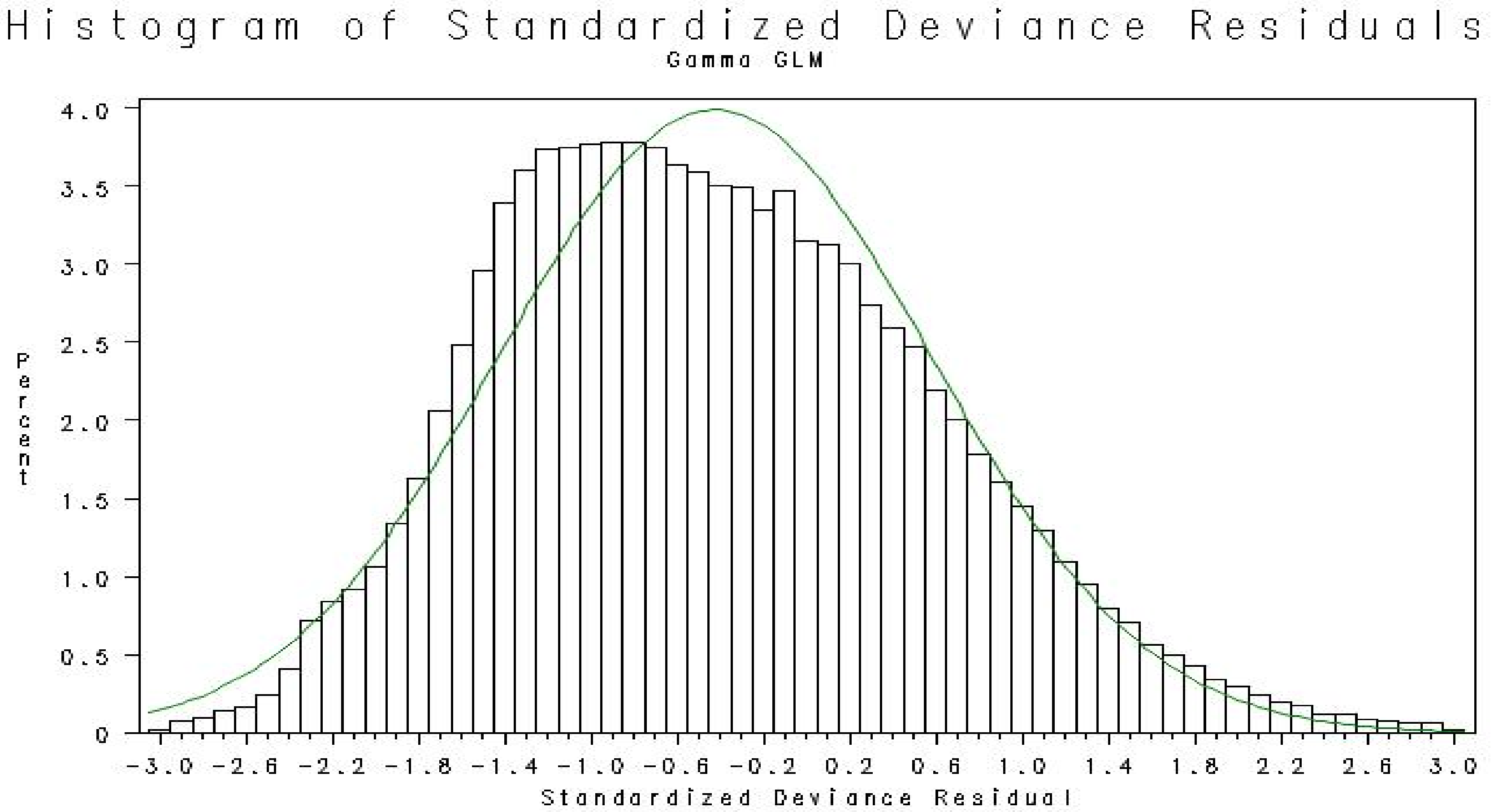
Example: Selecting Severity Model

- Goal is to select a distribution to model severity

- Two common choices – Gamma and Inverse Gaussian
  - Gamma: $V(\mu) = \mu^2$
  - Variance of severity is proportional to mean severity squared
  - Inverse Gaussian: $V(\mu) = \mu^3$
  - Variance of severity is proportional to mean severity cubed

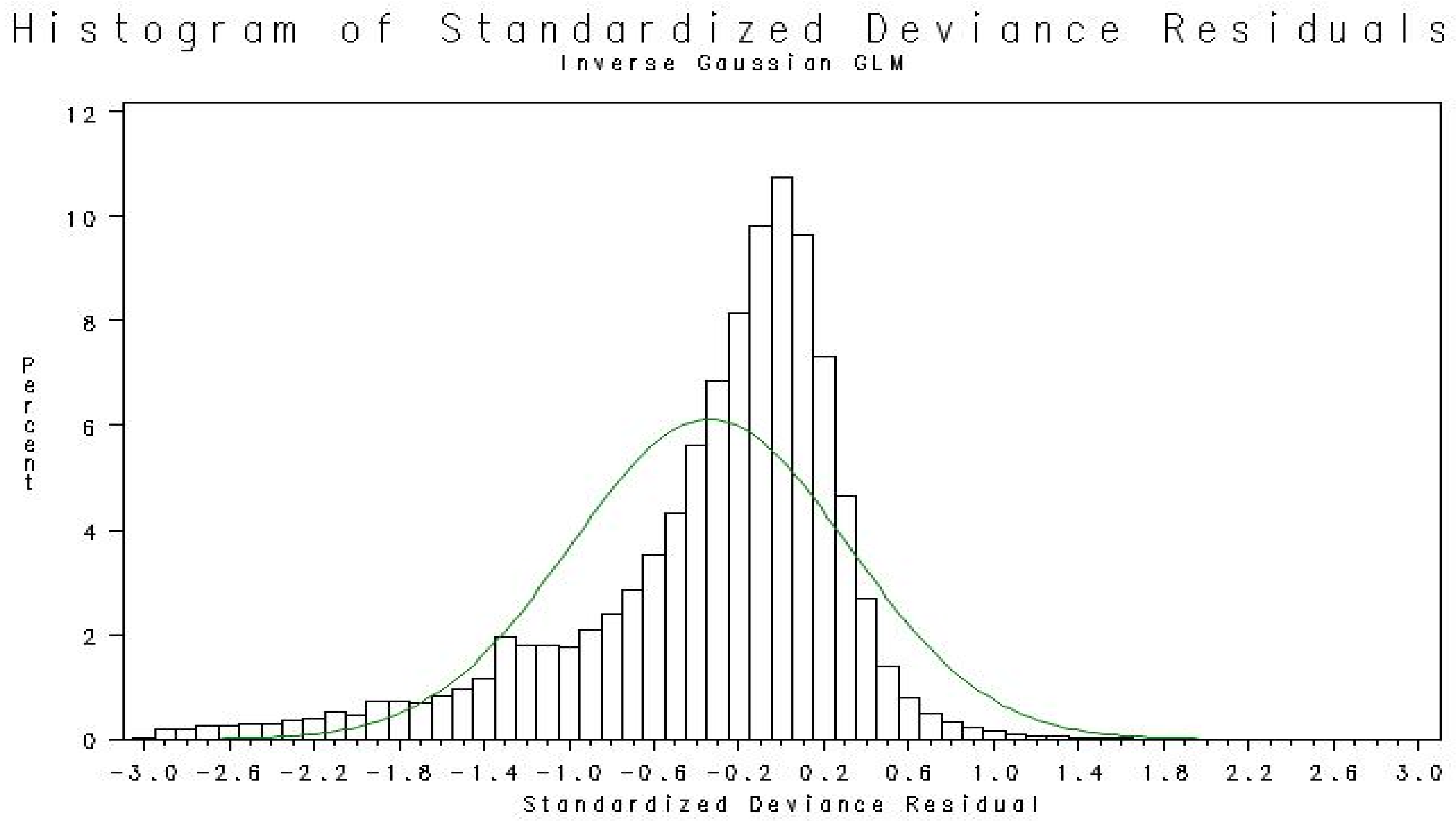
- Two lines of business
  - LOB1 is high-frequency, low-severity
  - LOB2 is low-frequency, high-severity

Deviance Residual Histogram



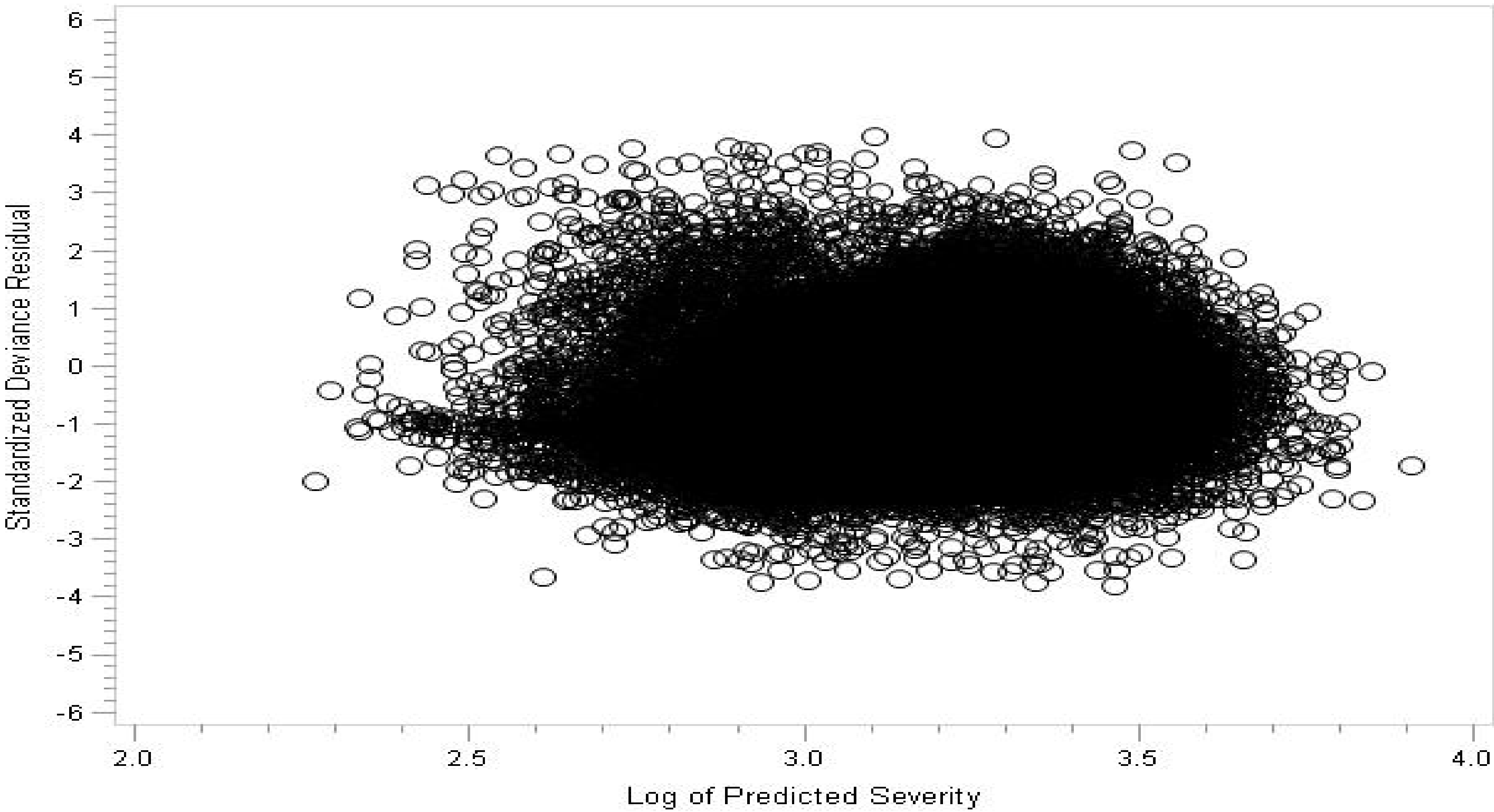
LOB1, Gamma GLM

Deviance Residual Histogram



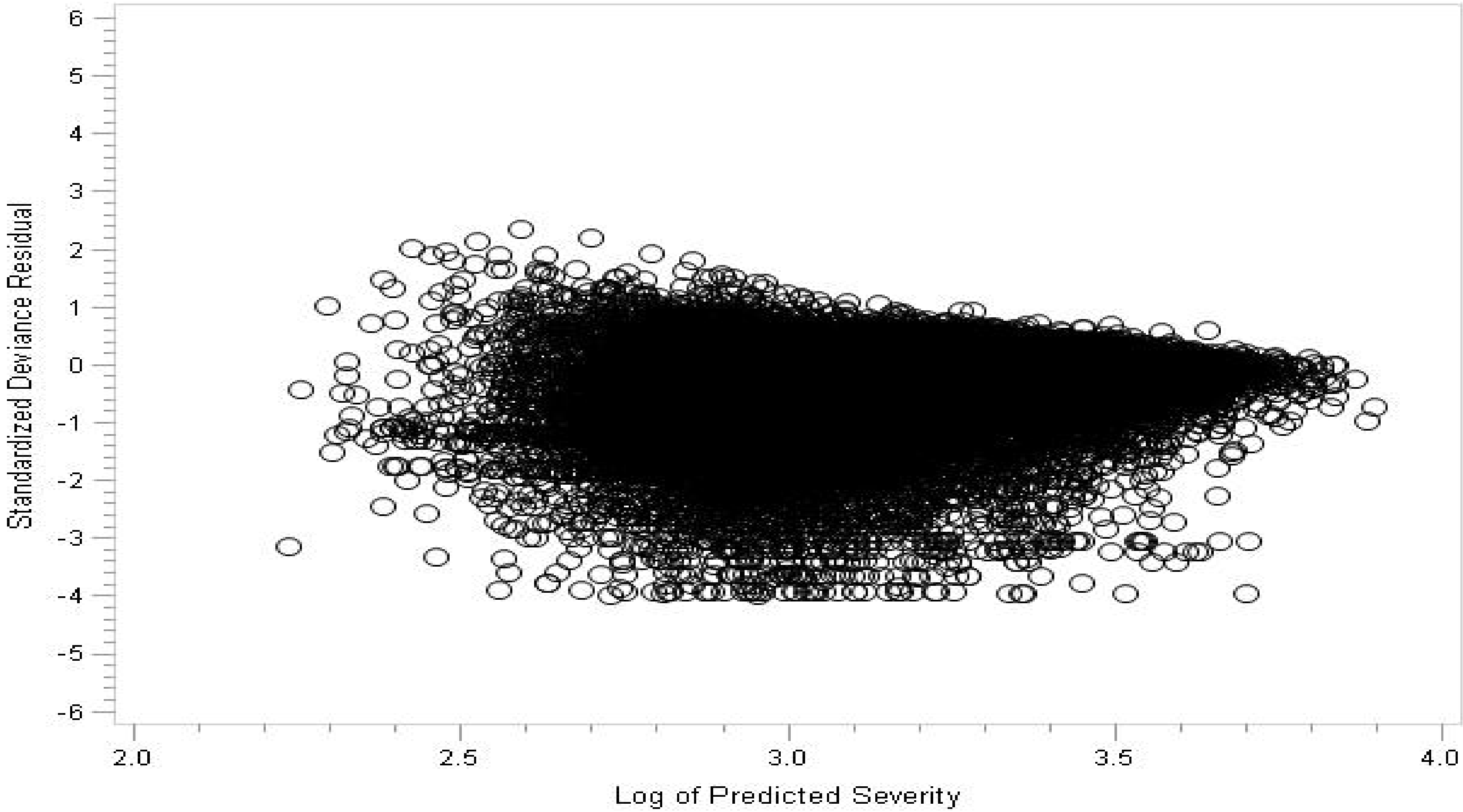
LOB1, IG GLM

Deviance Residual Histogram



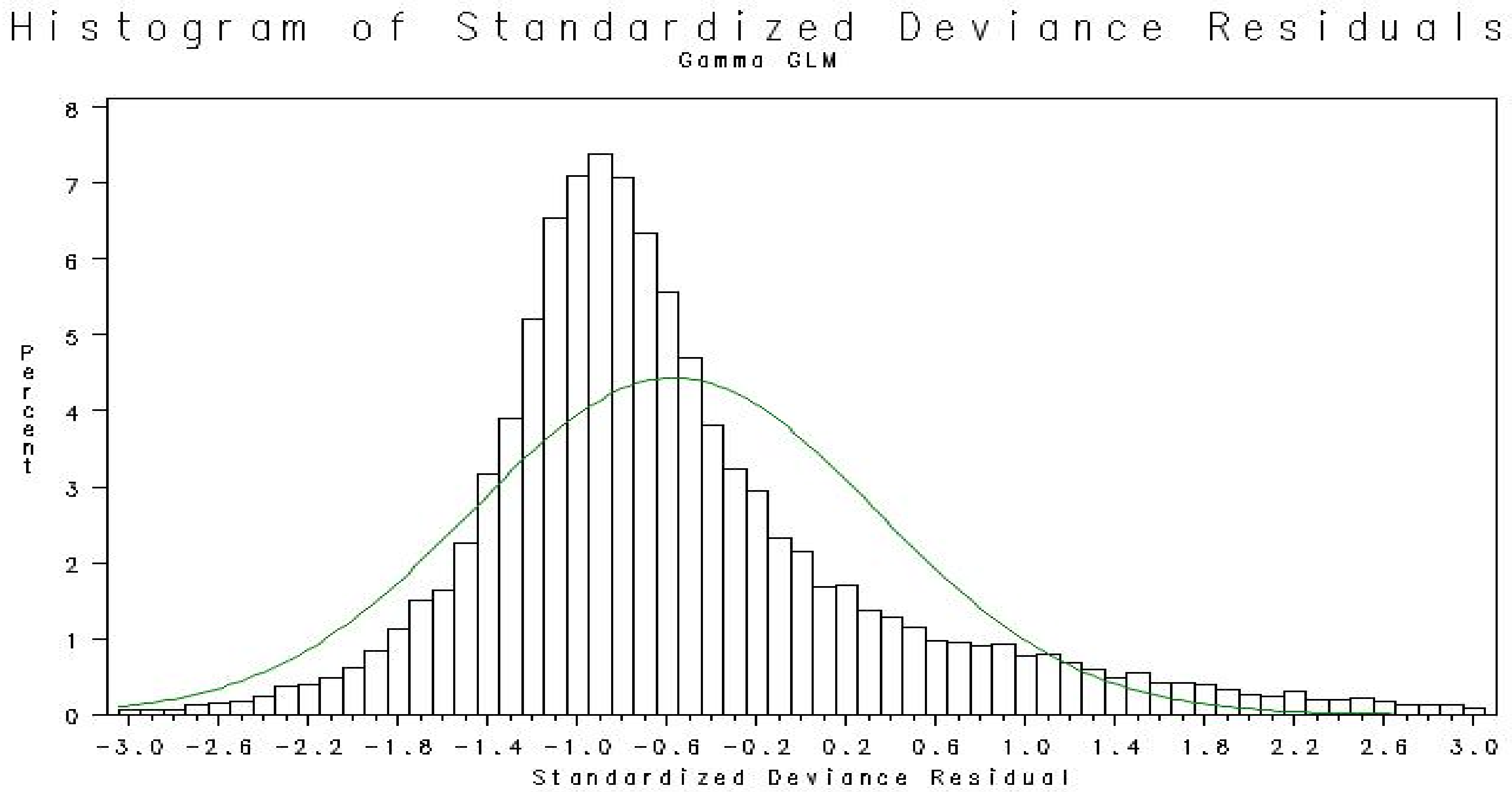
LOB1, Gamma GLM

Deviance Residual Histogram



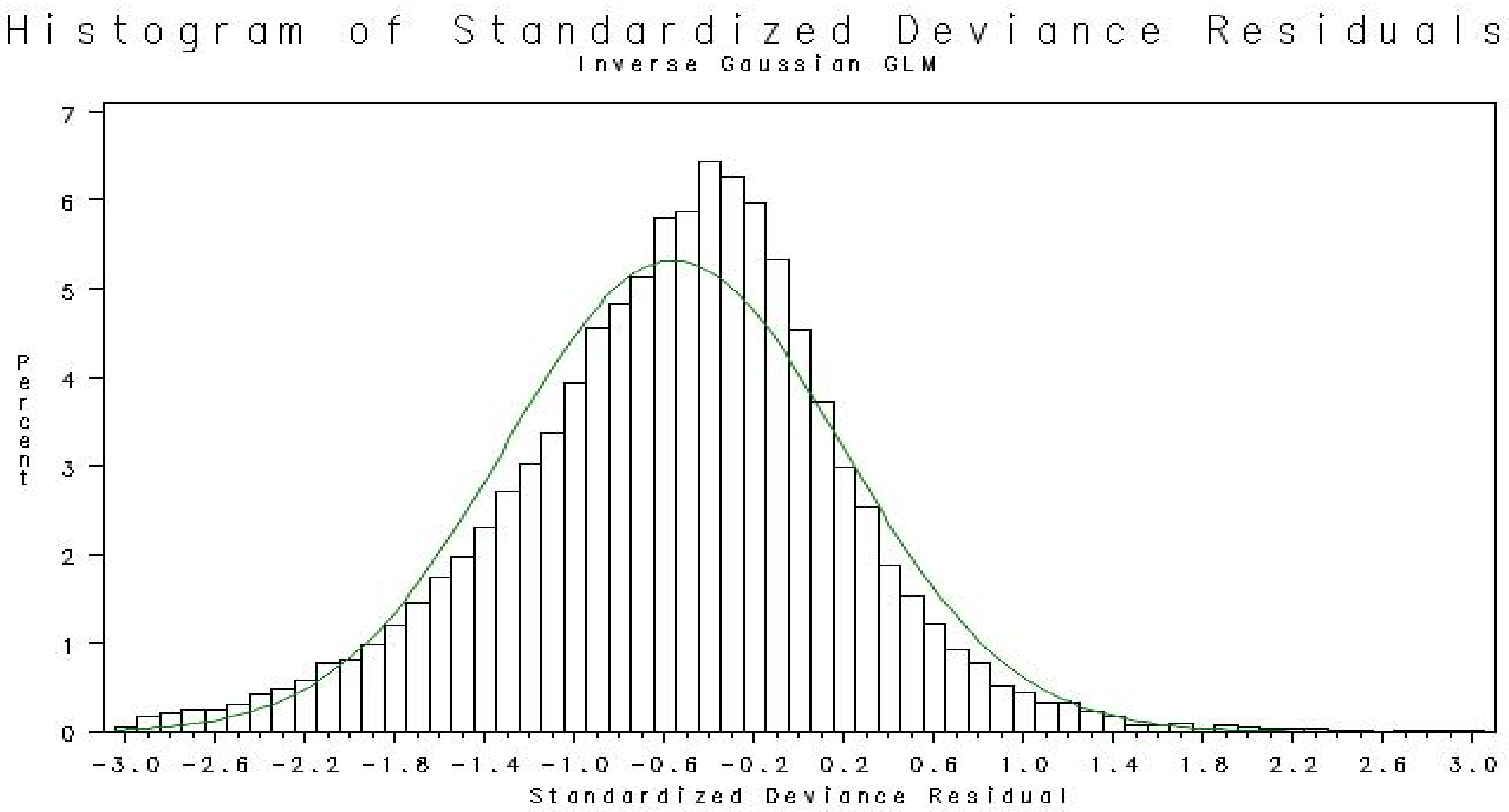
LOB1, IG GLM

Deviance Residual Histogram



LOB2, Gamma GLM

Deviance Residual Histogram



LOB2, IG GLM

Deviance Residuals Caution

- Analysis of deviance residuals only applicable to continuous or somewhat-continuous data

- If building a frequency model, and every record has either 0 or 1 claim, then deviance residuals will be bimodal

- If can aggregate discrete data to make it somewhat continuous, then deviance residual diagnostics may be appropriate

Actual vs Predicted Target

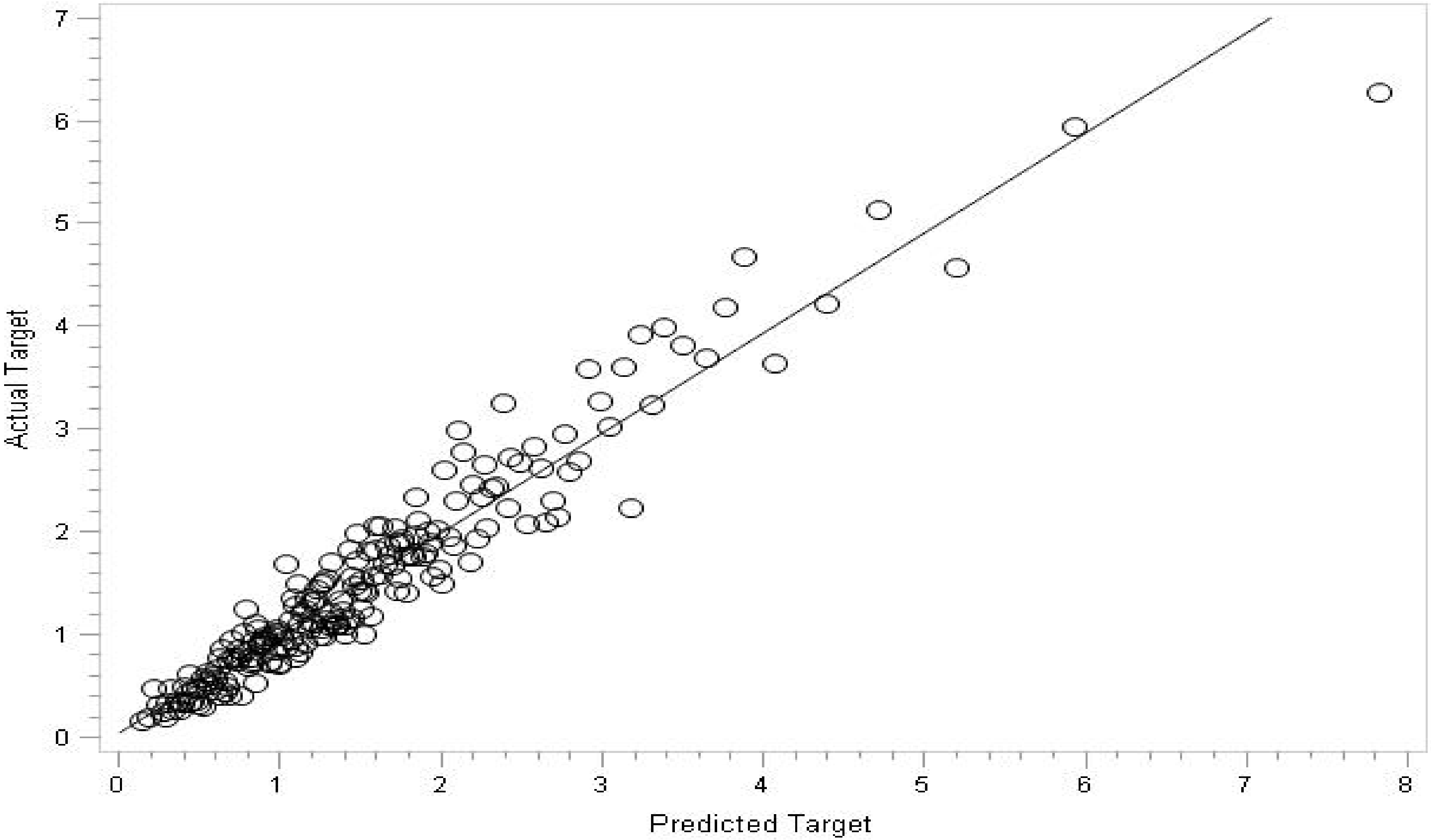
- Scatter plot of actual target variable (on y-axis) versus predicted target variable (on x-axis)

- If model fits well, then plot should produce a straight line, indicating close agreement between actual and predicted
  - Focus on areas where model seems to miss

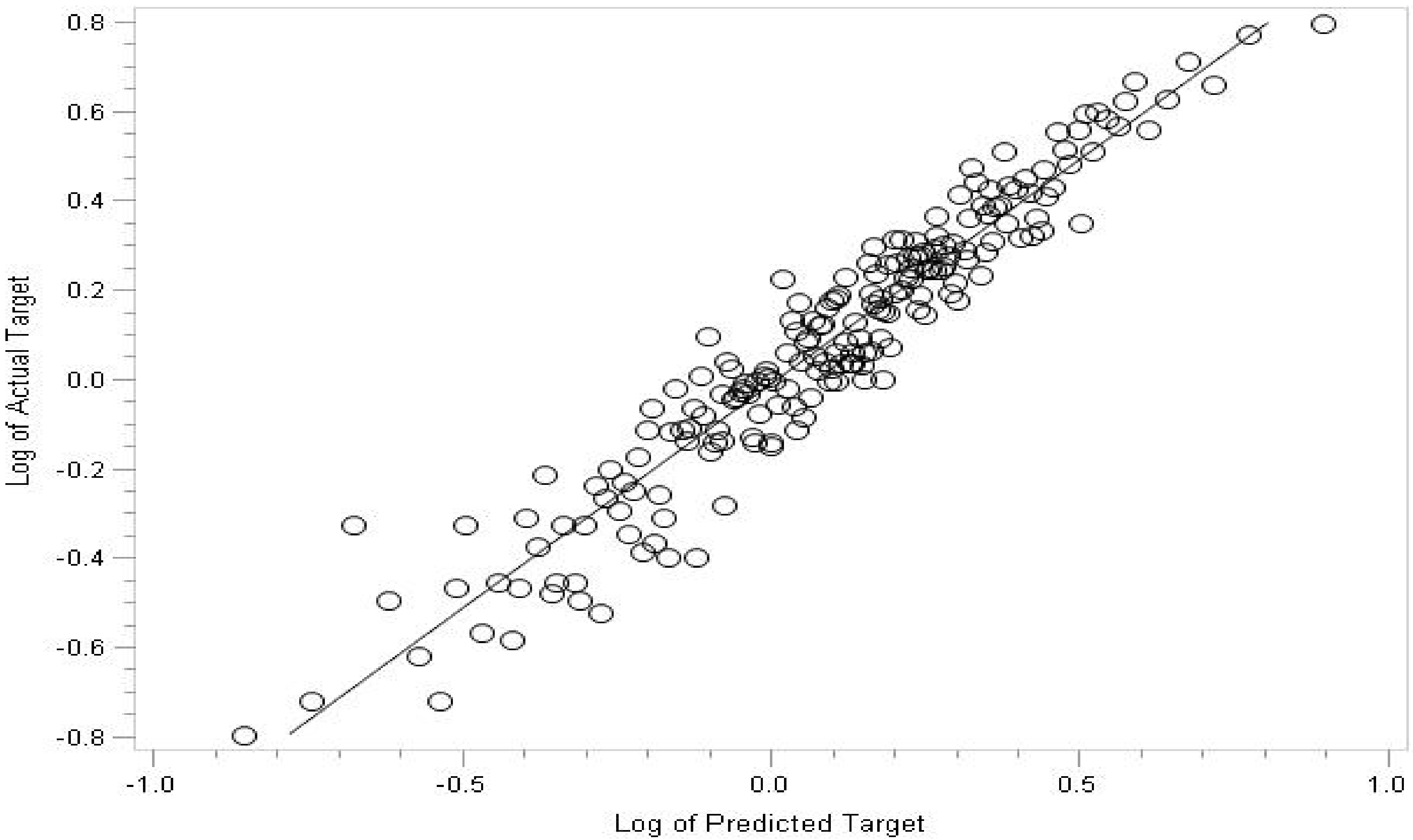
- If have many records, may need to bucket (such as into percentiles)

- Depending on scale, may need to plot on a log-log scale

Example of Actual vs Predicted



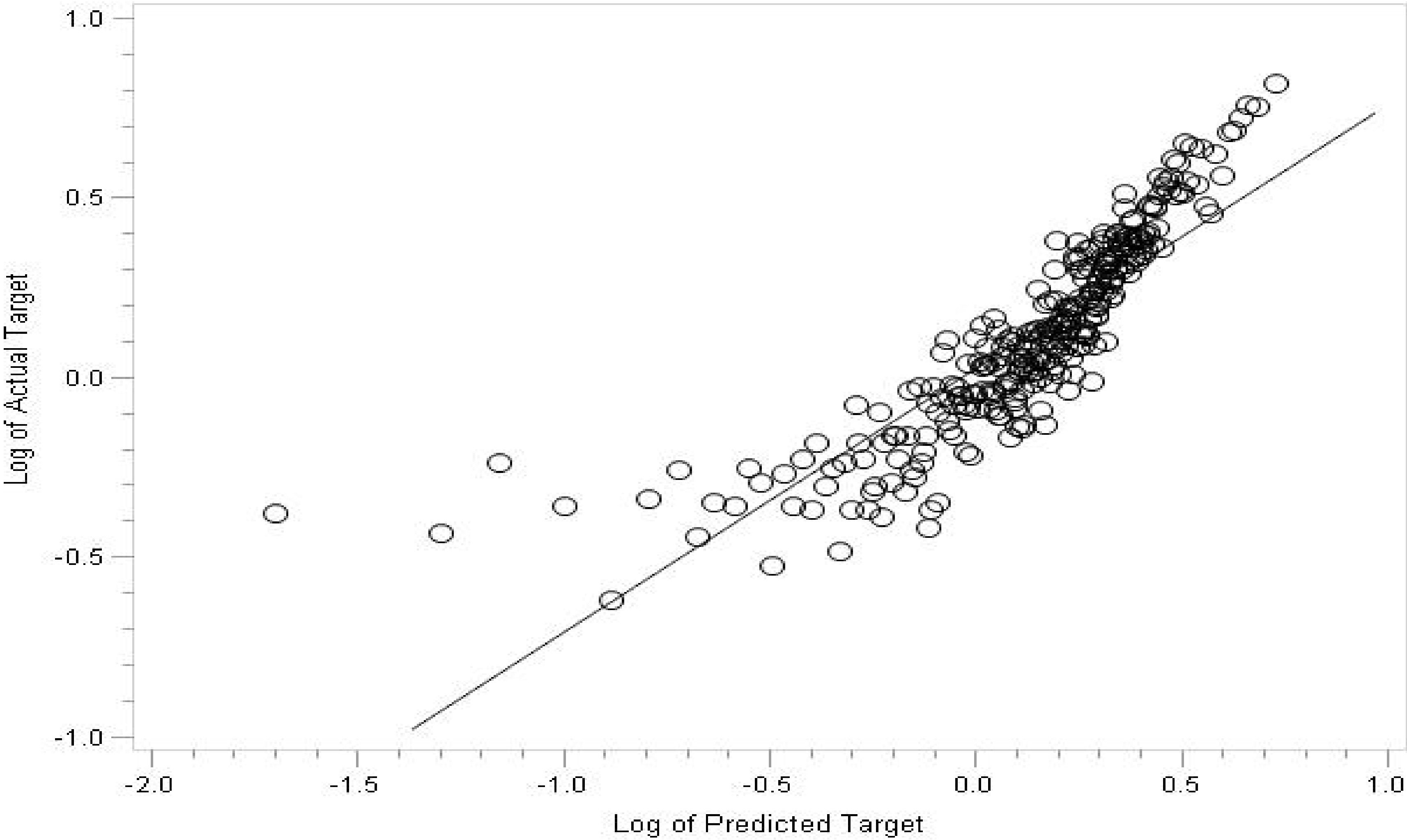
Example of Log of Actual vs Log of Predicted



Benefit of Deviance over Squared Error

- Since squared error is the deviance of a regression model with a Normal distribution, using squared error for non-Normal data can lead to incorrect model being chosen
- We run two models on our dataset – one with a Tweedie distribution and one with a Normal distribution
- Data is far from Normal, but using squared error as a metric, the Normal GLM wins
  - Even absolute error shows the Normal winning

Log of Actual vs Log of Pred Target with Normal Linear Regression



Measuring Internal Stability

- Process of determining how robust our model results are

- Useful measures:
  - Out-of-sample (out-of-time) validation
  - Cross-validation
  - Plotting actual versus predicted target variable on holdout data
  - Measures of influence (e.g. Cook's Distance)
  - Bootstrapping

Out-of-Sample Validation

- Important to assess model fit on data that was not used in model construction

- Two approaches:
  - Initially split dataset into training and test, build model on training, and measure fit on test
  - Cross-validate – repeatedly use one subset to build and one to test

- Can randomly split dataset, or can split based on a control variable (like year)

Assessing Stability over Time

- Generally want model results to be stable over time

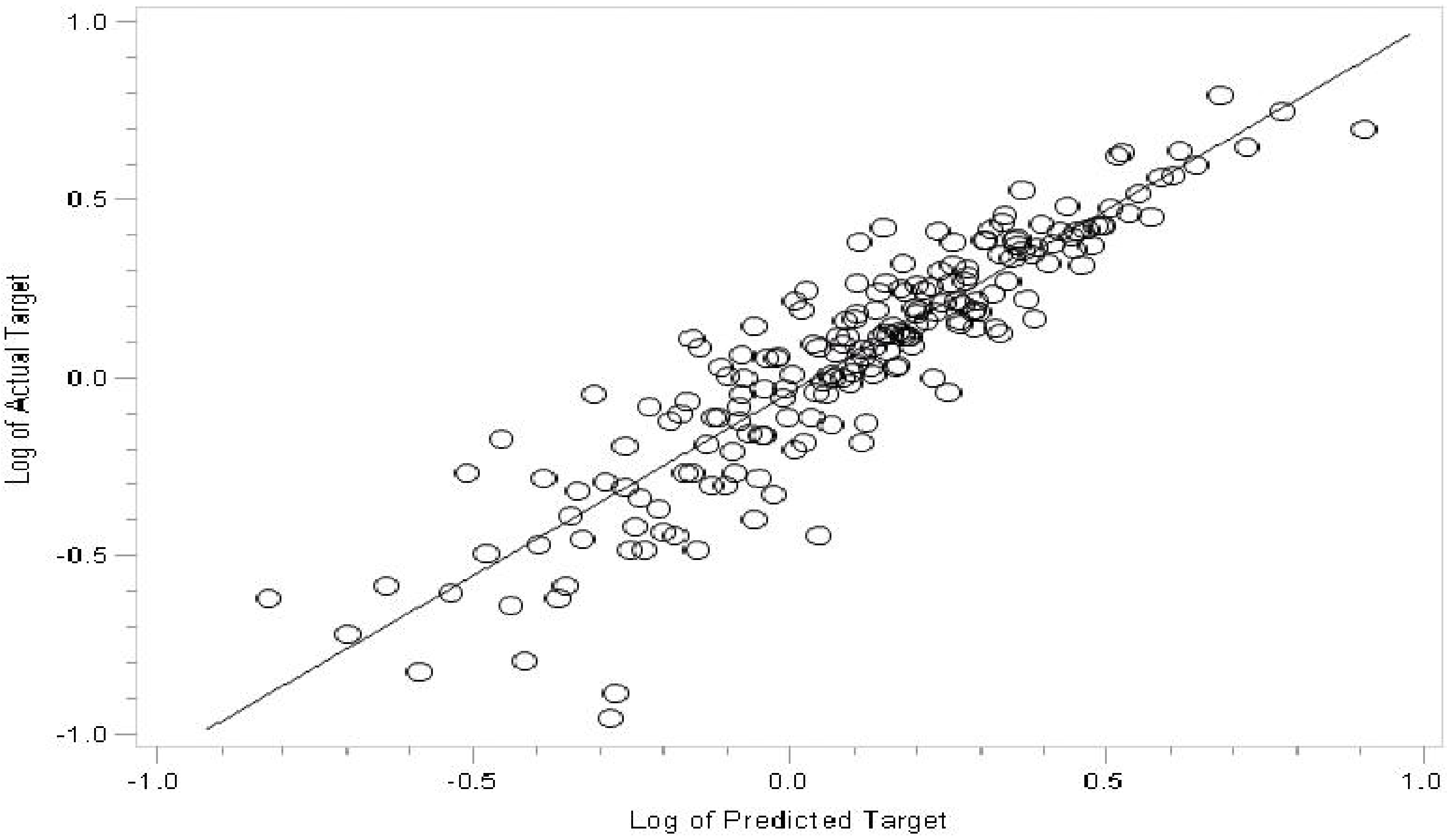
- To assess temporal stability, can run the model on individual years and look for variability
  - For example, if have 5 years, can run model on just years 1 and 2, then on just years 2 and 3, etc
  - Ideally, the parameter estimates don't change significantly across subsets

Plot of Actual vs Predicted on Holdout

- Produce scatter plot of actual target variable versus predicted target variable as before, but use one set to build model and another set to plot

- Very simple diagnostic to produce and understand, and tells a powerful story
  - Easy to explain to non-technical audience

Example of Plot of Actual vs Predicted on Holdout



Bootstrapping

- Re-sampling technique that allows us to get more out of our data
- Start with a dataset and sample from it with replacement
  - Some records will get pulled multiple times, and some will not get pulled at all
- Generally, we create a dataset with the same number of records as our original dataset
- Can create many bootstrap datasets, and each dataset can be thought of as an alternate reality
  - Since each bootstrap is an alternate reality, we can use bootstrapping to construct confidence intervals

Bootstrap CIs for Parameter Estimates

- GLMs produce confidence intervals for parameter estimates, but it is valuable to get a second opinion
- Create many bootstrap datasets, re-run the GLM on each dataset, and construct a confidence interval based on the resulting parameter estimates
- If bootstrap confidence interval is significantly wider than that produced by GLM, it is a sign that our results are overly-influenced by a few records

Confidence Intervals for Lift Measures

- Can use bootstrapping to put confidence intervals around lift measures, like Gini indices
- In measuring lift, we seek to answer the question: Does Model A outperform Model B?
- If the answer is yes, then the second question is: How significant is the win?
- Say Model A has a Gini index of 15.90 and Model B has a Gini index of 15.40
  - Model A has a Gini index that is 0.50 higher, but is that difference significant?
- Can also bootstrap quantile plots and double lift charts

References

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