CAS Ratemaking and Product Management Seminar - March 2016

Severe Weather Part VIII - Alternative Uses of Weather Models

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Economic Capital Modeling
TransRe

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Agenda

- Nomenclature – What are we talking about?
  - CAT models vs Predictive Models
- CAT Result Basics
  - ELT, Return Period, Per Event vs AAL
- CAT Pricing
  - Algorithms and Risk Measures
  - Counterexamples
- Monitoring Rate Change
  - Rate change vs Change in Rate Adequacy
  - Model change, exposure change, layer change

CAT Models – Weather and More

- Weather Models are a subset of CAT Models
- Vendor models
  - EQ, Hurricane - usually
  - Tornado, Terrorism, WC - some
  - Flood - less often
  - US, Japan, EU, vs rest of the world

Are CAT Models Predictive Models?

- CAT Models are predictive models
  - CAT Models can be used to make statements about probabilities of losses of a given size occurring in the future over a given time frame from specified CAT perils in specified regions.
- CAT Models are not Predictive Models
  - GLM vendors started calling their GLMs Predictive Models. This started as a marketing tool, but now the terminology is embedded and goes beyond the Insurance Industry.
  - In the Insurance Industry, CAT Modeling and Predictive Modeling are done by different teams.
  - Predictive Modeling prices the mean of the cells. CAT modeling quantifies the aggregation of risk.

CAT Result Basics

- Event Loss Table
- Event Exceeding Probability Calculation
- Simulated years
- AEP and OEP TVaR Calculations

Event Loss Table

Event Rank	Peril	Region	Annual Prob	Specific Event Return period	Risk A Loss	Risk B Loss	Risk C Loss	…	Total Portfolio Loss
1	EQ	CA	0.021%	4,762	300	1,200	0	…	125,000
2	EQ	CA	0.040%	2,500	0	1,000	0	…	100,000
3	HU	FLA	0.080%	1,250	0	0	3,000	…	90,000
4	EQ	CA	0.070%	1,429	900	400	0	…	80,000
5	HU	LA	0.045%	2,222	0	0	2,100	…	75,000
6	EQ	CA	0.055%	1,818	700	0	700	…	70,000
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
998	HU	NC	0.015%	6,667	0	2	0	…	2
999	HU	FL	0.400%	250	0	2	1	…	2
1,000	HU	SC	0.200%	500	0	1	0	…	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
4,998	EQ	NM	0.100%	1,000	0	0	0	…	0
4,999	HU	FLA	0.400%	250	0	0	0	…	0
5,000	EQ	AK	0.500%	200	0	0	0	…	0

Portfolio Event Exceeding Probability Table

k Event Rank	Peril	Region	p(k) Annual Prob	Event Return period	EP(k) Exceeding Probability	Portfolio Event EP Return Period	Portfolio Event Loss
1	EQ	CA	0.021%	4,762	0.021%	4,762	125,000
2	EQ	CA	0.040%	2,500	0.061%	1,640	100,000
3	HU	FLA	0.080%	1,250	0.141%	710	90,000
4	EQ	CA	0.070%	1,429	0.211%	474	80,000
5	HU	LA	0.045%	2,222	0.256%	391	75,000
6	EQ	CA	0.055%	1,818	0.311%	322	70,000
. . .	. . .	. . .	. . .				
998	HU	NC	0.015%	6,667	24.000%	4	2
999	HU	FL	0.400%	250	24.304%	4	2
1,000	HU	SC	0.200%	500	24.455%	4	1
. . .	. . .	. . .	. . .				
4,998	EQ	NM	0.100%	1,000	83.000%	1	-
4,999	HU	FLA	0.400%	250	83.068%	1	-
5,000	EQ	AK	0.500%	200	83.153%	1	-

Exceeding Probability and Return Period

$$EP(k+1) = EP(k) + p(k+1)\left(1 - EP(k)\right)$$

- Exceeding Probability
- EP(k) = Probability that over one year there will be a loss bigger than or equal to the k^{th} largest loss in the event loss table
- Return period = 1/EP(k)
- The event associated with the 100 year return period has annual probability, p(k), less than 1/100

Simulation Trials

Trial Year	Event 1	Event 2	Event 3	...		Largest Event over the Year	Total Annual Loss
1	40,000	-	-	-		40,000	40,000
2	1	3,500	9	-	-	3,500	3,510
3	-	-	-			0	0
4	10	27,550	-			27,550	27,560
5	700	400	50			700	1,150
6	1,250	4	25			1,250	1,279
7	-	-	-			0	0
8	75	45	70,000			70,000	70,120
9	-	-	-			0	0
10	15	3,500	45			3,500	3,560
⋮	.	.	.			⋮	⋮
9,998	2	-	-			2	2
9,999	550	7,750	-			7,750	8,300
10,000	650	-	-			650	650

Annual Loss Rank Ordered Simulation Trials

Trial Year Rank	Ranking based on total annual loss	Largest Event	Total Annual Loss
1		125,000	175,000
2		125,000	170,000
3		90,000	155,000
4		100,000	137,500
5		100,000	135,000
6		100,000	130,000
7		90,000	125,000
8		90,000	115,000
9		100,000	105,000
10		90,000	102,500
⋮		⋮	⋮
99		21,250	37,500
100		21,000	36,675
101		35,000	35,950
⋮		⋮	⋮
9,998		-	0
9,999		-	0
10,000		-	0

100/10000 = 1.0%
100 year return period
AEP VaR = 36,675

Largest Event Rank Ordered Simulation Trials

Trial Year Rank	Ranking based on largest event loss	Largest Event	Total Annual Loss
1		125,000	175,000
2		125,000	170,000
3		100,000	137,500
4		100,000	135,000
5		100,000	130,000
6		100,000	100,000
7		95,000	97,500
8		92,500	102,000
9		90,000	155,000
10		90,000	125,000
:		:	:
99		35,125	35,250
100	100/10000 = 1.0% 100 year return period OEP VaR = 35,000	35,000	35,950
101		35,000	35,125
:		:	:
9,998		-	0
9,999		-	0
10,000		-	0

Pricing CAT Coverage

- Basic equations
- Definition of properties and coherence
- Pricing Algorithms
    - Reference Portfolio
- Risk Measures
- A few counterexamples

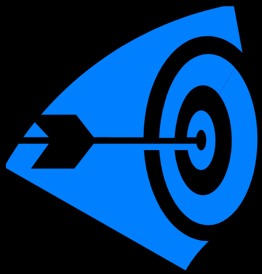
Basic Pricing Equations

- $P = E[X] + RL(X)$

  P = Indicated premium prior to expense loading

  X = CAT Loss

  RL(X) = Risk Load



- $RL(X) = r_{target} * C(X)$
- C(X) = Required Capital
- RORAC Approach used by many
- RAROC /Bond equivalent sued by some
- CAPM not used
  - since CATs independent of stock market, CAPM risk load should be zero

Premium – Basic Properties

1. ***Monotonic:*** ***If*** $X_1 \leq X_2$, ***then*** $P(X_1) \leq P(X_2)$
2. ***Pure:*** ***If*** $X \equiv \alpha$ ***then*** $P(X) = E[X]$
3. ***Bounded: If*** $X \leq k$, ***then*** $P(X) \leq k$
4. ***Continuous (Stable): P(X) is continuous***
   - small changes in X do not cause large changes in P(X)

Premium – Coherence Properties

1. ***Scalable:*** $P(\lambda X) = \lambda \cdot P(X)$
2. ***Translation Invariant:*** $P(X+\alpha) = P(X) + \alpha$

   when $0 \leq \alpha$.
3. ***Subadditive:*** $P(X_1 + X_2) \leq P(X_1) + P(X_2)$

   A failure of subadditivity means there is consolidation penalty instead of a benefit

Required Capital Algorithms

Standalone:

$C(X) = \rho(X)$

$\rho(X)$ is a risk measure.

Portfolio Incremental:

$C(X) = C(X|R)$

$= \rho(R+X) - \rho(R)$

Portfolio Allocation

$C(X) = C(X|R)$

$= A(X,R) * \rho(R+X)$

Capital Algorithm Properties

- Incremental
  - Order Dependent
  - Reference portfolio
  - Portfolio dependent capital $\leq$ Standalone capital
- Allocation
  - Automatically Calibrated $\sum \boldsymbol{C}(\mathbf{X}|\mathbf{R}) = \mathbf{C}(\mathbf{R})$
- Co-measures and Euler allocation

Risk Measures: VaR, TVaR

Value at Risk	$VaR(\theta) = sup\{x \mid F(x) \leq \theta\}$
Excess Value at Risk	$XVaR(\theta) = Var(\theta) - \mu$
Tail Value at Risk	*TVaR(θ) = conditional mean of x values in the tail, 1 - θ , of probability*
Excess Tail Value at Risk	$XTVaR(\theta) = TVaR(\theta) - \mu$

Examples and Counterexamples

Example of VaR, XVaR, TVaR, and XTVaR

Statistic	Value	Statistic	Value
Trials	10	Rank for VaR	3.0
Average	100.0	VaR	150.0
Percentage	70.00%	XVaR	50.0
		TVaR	250.0
		XTVaR	150.0

Ordered Loss Data

Rank	Loss	VaR Percentage	Conditional Tail Avg
1	400	90.0%	400
2	200	80.0%	300
3	***150***	***70.0%***	***250***
4	100	60.0%	213
5	80	50.0%	186
6	50	40.0%	163
7	15	30.0%	142
8	3	20.0%	125
9	2	10.0%	111
10	0	0.0%	100

Ranking Definition of VaR and TVaR

- Let $X_1 \geq X_2 \ldots \geq X_n$ *be an ordering of n trials of X*
- *Suppose k = (1 - θ)n, then*

$$VaR(\theta) = X_k$$

$$TVaR(\theta) = \frac{1}{k}\sum_{j=1}^{k} X_j$$

- Note TVaR is <u>not</u> necessarily equal to the Conditional Tail Expectation (CTE) when the data is discrete.

Example: TVaR and CTE are not the same

Statistic	Value
Trials	10
Pct	50%
Rank	5

Results	A	Ref	A+Ref
Mean	2.50	25.00	27.50
VaR	*2.00*	*31.00*	*31.00*
TVaR	4.80	33.80	34.20
CTE (>)	6.67	34.50	35.00
CTE (≥)	4.80	33.80	33.67

Loss Data by Trial

Trial	A	Ref	A+Ref
1	7.00	12.00	19.00
2	0.00	37.00	37.00
3	0.00	31.00	31.00
4	0.00	35.00	35.00
5	0.00	33.00	33.00
6	2.00	17.00	19.00
7	9.00	11.00	20.00
8	2.00	33.00	35.00
9	4.00	27.00	31.00
10	1.00	14.00	15.00

Separately Ordered Loss Data

Rank	A	Ref	A+Ref
1	9.00	37.00	37.00
2	7.00	35.00	35.00
3	4.00	33.00	35.00
4	2.00	33.00	33.00
5	2.00	31.00	31.00
6	1.00	27.00	31.00
7	0.00	17.00	20.00
8	0.00	14.00	19.00
9	0.00	12.00	19.00
10	0.00	11.00	15.00

VaR Subadditivity-Epic Fail

Statistic	Value
Trials	10
Percentage	50.00%
Rank	5

	Mean	VaR
Risk A	10.00	6.00
Reference Portfolio	100.00	124.00
Sum	110.00	130.00
Combined Portfolio	110.00	148.00
Consolidation Benefit	0.00	-18.00
Incremental VaR for A		**24.00**

Loss Data by Trial				**Separately Ordered Loss Data**			
Trial	A	Ref	A+Ref	Rank	A	Ref	A+Ref
1	6	40	46	1	26	148	170
2	0	148	148	2	24	144	154
3	26	144	170	3	18	140	150
4	14	140	154	4	14	132	148
5	18	132	150	5	6	124	148
6	4	68	72	6	6	92	94
7	0	64	64	7	4	68	72
8	24	124	148	8	2	64	64
9	2	92	94	9	0	48	54
10	6	48	54	10	0	40	46

Incremental VaR not scalable: A

Statistic	Value
Trials	10
Percentage	50.00%
Rank	5

	Mean	VaR
Risk A Standalone	10.00	**11.00**
Reference Portfolio	100.00	96.00
Sum	110.00	107.00
Combined Portfolio	110.00	105.00
Incremental VaR for A		**9.00**

Loss Data by Trial				**Separately Ordered Loss Data**			
Trial	A	Ref	A+Ref	Rank	A	Ref	A+Ref
1	11	52	63	1	28	148	149
2	1	148	149	2	20	140	144
3	0	140	140	3	16	128	140
4	0	128	128	4	13	124	128
5	4	96	100	5	11	96	105
6	28	68	96	6	7	92	100
7	16	64	80	7	4	88	96
8	20	124	144	8	1	68	95
9	7	88	95	9	0	64	80
10	13	92	105	10	0	52	63

Incremental VaR not scalable: 2*A

Statistic	Value		Mean	VaR
Trials	10	Risk 2A Standalone	20.00	**22.00**
Percentage	50.00%	Reference Portfolio	100.00	96.00
Rank	5	Sum	120.00	118.00
		Combined Portfolio	120.00	124.00
		Incremental VaR for 2A		**28.00**

Loss Data by Trial				**Separately Ordered Loss Data**			
Trial	2A	Ref	2A+Ref	Rank	2A	Ref	2A+Ref
1	22	52	74	1	56	148	164
2	2	148	150	2	40	140	150
3	0	140	140	3	32	128	140
4	0	128	128	4	26	124	128
5	8	96	104	5	22	96	124
6	56	68	124	6	14	92	118
7	32	64	96	7	8	88	104
8	40	124	164	8	2	68	102
9	14	88	102	9	0	64	96
10	26	92	118	10	0	52	74

Co-VaR Instability

Rank	VaR Percentage	Portfolio Loss	Risk A Loss
1			
98	99.02%	$422	$6
99	99.01%	$408	$0
100	99.00%	$405	$20
101	98.99%	$395	$0
102	98.98%	$390	$4
10,000			

- The 100 year return period Co-Var for A is $20
  - Slight portfolio change or new simulation could make it $0

Pricing Summary and Conclusions

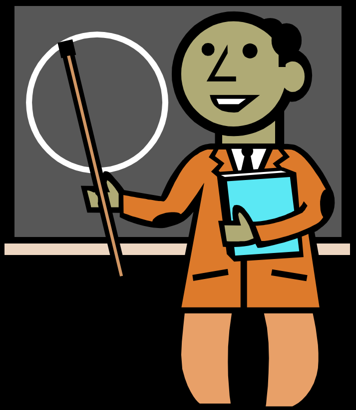
- Indicated pricing is based on target return on required capital.
- Debate is over required capital
- A profusion of methods and approaches
- Tail focus, portfolio dependence, absolute vs relative calibration are key areas where methods differ
- Some of key methods used in practice do not satisfy all the desired conceptual properties
- Try any method yourself on simple examples- understand how it works and how it fails.

Rate Change on CAT Business

- Renewal Rate Change
  - Definition and context
- Individual Account Calculation
  - Mix and Coverage Structure Adjustments
- Portfolio Average Rate Change
  - Weighting on renewal premiums is wrong!
  - Notional expiring weights and harmonic averages

Defining Renewal Rate Change

$$Rate = \frac{Prem}{Exposure}$$



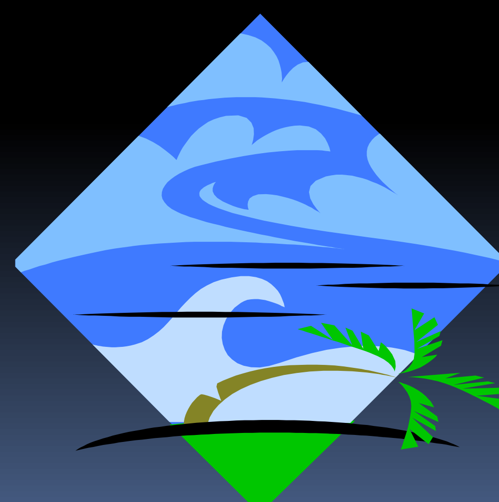
$$Nominal\ Renewal\ Rate\ Change = \frac{Renewal\ Rate}{Expiring\ Rate} - 1$$

Popularity of Renewal Rate Change

- Fair – based only on UWs own accounts
- Intuitive - easy to explain
- Data - available and timely

CAT Exposed Excess Property Business

- Commercial Property
- DIC or Specified Peril Coverage
- Excess and Surplus Lines – Wholesale
- Excess Layers
- Exposure to HC or EQ



Exposures

- Location Schedule
  - Addresses of all covered locations
  - Geo-coding accuracy important in CAT pricing
- Stated Values in Schedule - TIVs
  - Structure, Contents, and Time Element values
- Characteristics
  - Construction
  - Occupancy
  - Protection
  - Number of Stories
  - Age

Layers

- 100% Layer: Layer Limit xs of Attachment
  - $10.0m xs of $15.0m
- Share: Company Limit part of Layer Limit
  - $2.5m p/o $10.0m xs $15.0m
  - 25% share in $10.0m xs $15.0m layer

Account Nominal Rate Change

$$\Delta N = Nominal\ Rate\ Change = Change\ in\ 100\%\ Layer\ Rates$$

$$100\%\ Layer\ Rate = \frac{100\%\ Layer\ Premium}{\$100\ of\ TIV}$$

- 100% rates used to avoid confusing change in share with change in rate

Nominal Rate Change Example

Wayne's Widgets

	Expiring	Renewal
Premium	$50,000	$40,000
Coverage	$5m p/o $25m x $5m	$2.5m p/o $10m x $15m
Company Limit ($m)	$5.0	$2.5
Layer 100% Limit ($m)	$25.0	$10.0
Share	20.0%	25.0%
Attachment($m)	$5.0	$15.0
Exposure TIV ($m)	$30.0	$25.0
Layer 100% Premium	$250,000	$160,000
100% Rate per $100 TIV	$0.833	$0.640
Nominal Rate Change		-23%

Rate on Line

- Rate on Line = Premium Per Mill of Limit

Wayne's Widgets		
	Expiring	Renewal
Premium	$50,000	$40,000
Coverage	$5m p/o $25m x $5m	$2.5m p/o $10m x $15m
Company Limit ($m)	$5.0	$2.5
Layer 100% Limit ($m)	$25.0	$10.0
Share	20.0%	25.0%
Layer 100% Premium	$250,000	$160,000
ROL(Prem Per $M of Limit)	$10,000	$16,000
ROL change		60%

Nominal Renewal Rate Change Misleading?

- Ignores Location Mix changes
  - Dropping properties near the coast $\Rightarrow$ reduced rate
- Ignores Coverage Layer changes
  - Increase attachment/ reduce limit $\Rightarrow$ reduced rate
- Rate movements due to changes in coverage and location mix should not be counted as "real" rate changes

Property CAT Coverage and Schedule Changes

- Insureds
  - Affordability following CAT event
  - May reduce coverage, remove locations
- Insurance Company
  - Need to reduce aggregate CAT exposure
  - Change in UW strategy
- Broker
  - Introduce competition
  - Reduce price and keep the account

Quantify Impact on Technical Premium

- Strategy: Compute % Change in Technical Premium Rate
  - Back this change out of the nominal rate change
- Technical Premium
  - Machine generated premium
  - Includes loss and risk load provisions
  - No Schedule rating or market adjustments
- Stats from CAT Model
  - AAL = Annual Average Loss
  - PML = Probable Max Loss
- Demo: Tech Prem = 1.5*(AAL+.05*PML)

Account Rate Change Decomposition

Nominal Rate Change

Technical Rate Change due to Δin Coverage Layer
Technical Rate Change due to Δ in Location Schedule
(Effective) Renewal Rate Change

Coverage and Location Mix Adjustments

$$\text{Location Mix Adjustment Factor} = \text{MXA}F = \frac{\text{Tech Rate(Renewal Locs, Expiry Layer)}}{\text{Tech Rate(Expiry Locs, Expiry Layer)}}$$

$$\text{Coverage Structure Adjustment Factor} = \text{CSA}F = \frac{\text{Tech Rate(Renewal Locs, Renewal Layer)}}{\text{Tech Rate(Renewal Locs, Expiry Layer)}}$$

Adjustments with Technical Rates

Wayne's Widgets			
	Expiry Exposure Expiry Layer	Ren Exposure Expiry Layer	Ren Exposure Ren Layer
Coverage	$5m p/o $25m x $5m	$5m p/o $25m x $5m	$2.5m p/o $10m x $15m
Exposure TIV ($m)	$30.0	$25.0	$25.0
Company Share	20%	20%	25%
100% AAL	$50,000	$40,000	$15,000
100% PML ($m)	$15.0	$11.0	$05.0
Technical Premium	$300,000	$225,000	$97,500
Tech Rate ($100 TIV)	$1.00	$0.90	$0.39
Adjustment Factor		0.900	0.433
		MXAF	CSAF

Renewal Rate Change (ΔR)

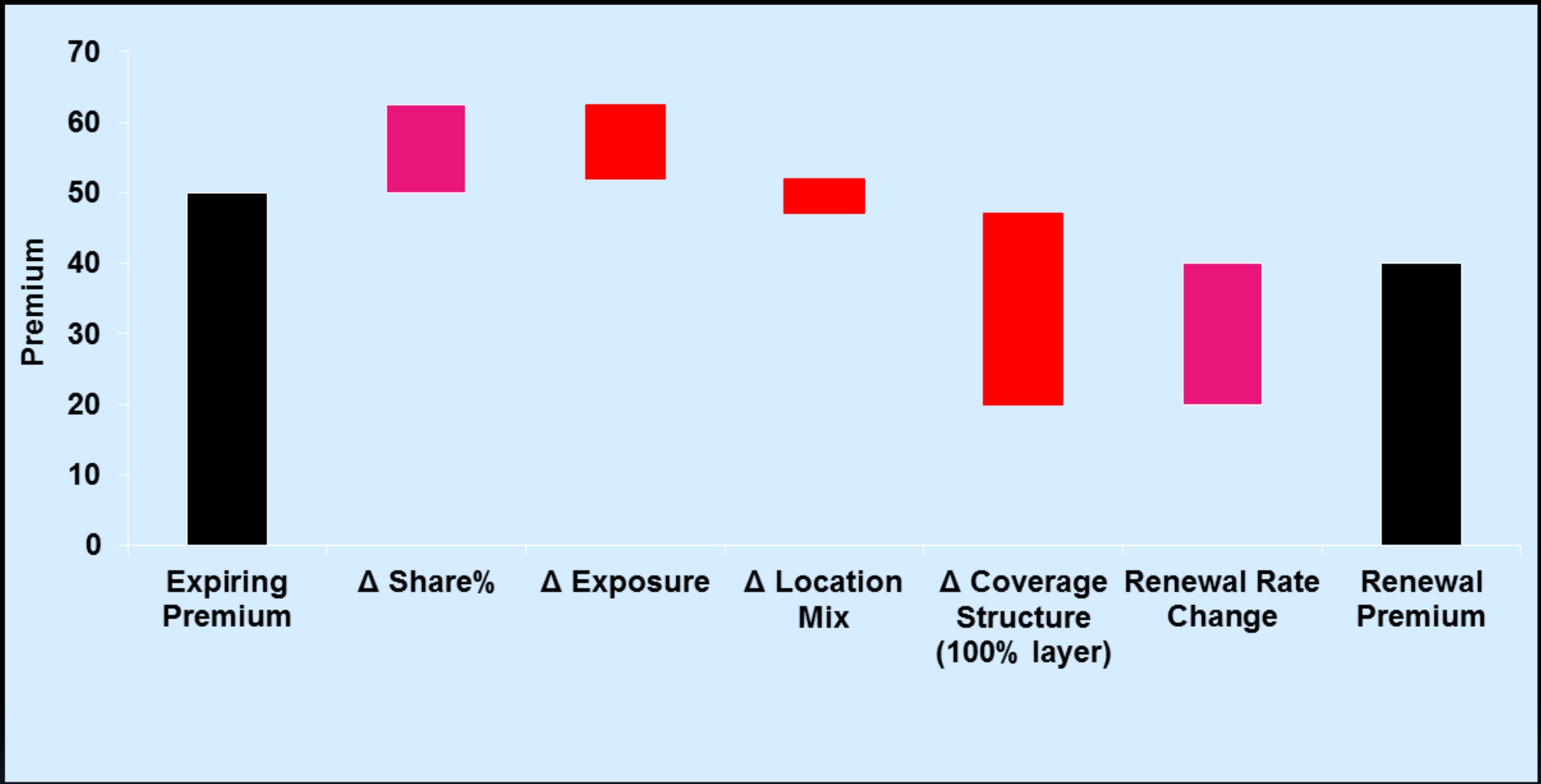
- ΔR = Renewal Rate Change
- ΔR = Nominal Rate Change net of Location Mix and Coverage Structure adjustment factors

$$\Delta R = \frac{(1+\Delta N)}{MXAF \cdot CSAF} - 1$$

Renewal Rate Change Example

Wayne's Widgets		
	Expiring	Renewal
Premium	$50,000	$40,000
Coverage	$5m p/o $25m x $5m	$2.5m p/o $10m x $15m
Exposure TIV ($m)	$30.0	$25.0
Company Share	20%	25%
Layer 100% Premium	$250,000	$160,000
Rate per $100 TIV	$0.833	$0.640
Nominal Rate Change		-23%
Location Mix (MXAF)		0.9000
Coverage (CSAF)		0.4333
Renewal Rate Change		97%

Premium Reconciliation



$$P_{REN} = P_{EXP} \cdot \frac{s_{REN}}{s_{EXP}} \cdot \frac{TIV_{REN}}{TIV_{EXP}} \cdot MXAF \cdot CSAF \cdot (1 + \Delta R)$$

Notional Expiring Premium

- What the Expiring Premium would have been if it were based on the same location schedule and layer of coverage as the Renewal policy

$$P_{NXP} = P_{EXP} \cdot \frac{TIV_{REN}}{TIV_{EXP}} \cdot \frac{s_{REN}}{s_{EXP}} \cdot MXAF \cdot CSAF$$

$$\Rightarrow P_{NXP} = \frac{P_{REN}}{(1 + \Delta R)}$$

Conclusion

- CAT Models are used for:
  - Pricing
  - Capital Allocation
  - Price Monitoring

- Questions?