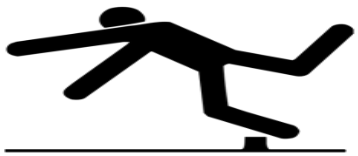
An Alternative Approach to Credibility for Large Account and Excess of Loss Treaty Pricing

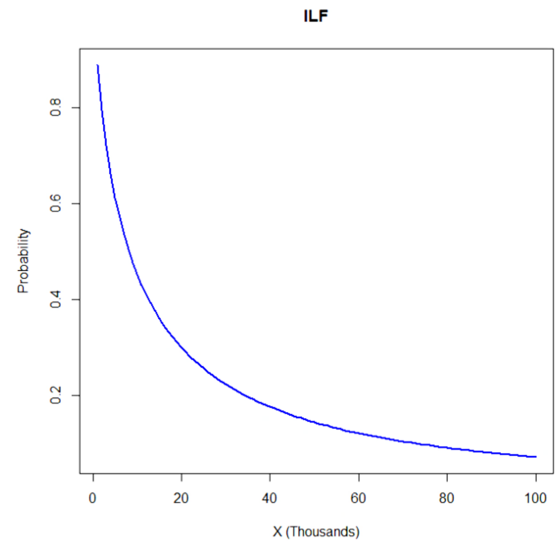
Uri Korn, FCAS, MAAA

Ratemaking and Product Management Seminar

March 27 - 29, 2017

How do you consider all available data?





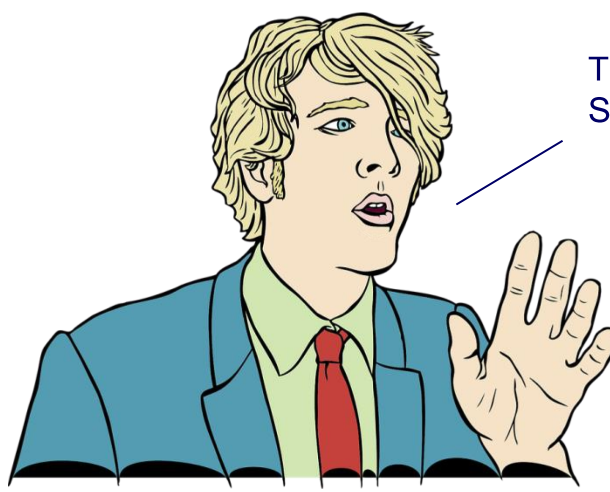
SIC Division Structure

A. Division A: Agriculture, Forestry, And Fishing
- Major Group 01: Agricultural Production Crops
- Major Group 02: Agriculture production livestock and animal specialties
- Major Group 07: Agricultural Services
- Major Group 08: Forestry
- Major Group 09: Fishing, hunting, and trapping

B. Division B: Mining
- Major Group 10: Metal Mining
- Major Group 12: Coal Mining
- Major Group 13: Oil And Gas Extraction
- Major Group 14: Mining And Quarrying Of Nonmetallic Minerals, Except Fuels

C. Division C: Construction
- Major Group 15: Building Construction General Contractors And Operative Builders
- Major Group 16: Heavy Construction Other Than Building Construction Contractors
- Major Group 17: Construction Special Trade Contractors

D. Division D: Manufacturing
- Major Group 20: Food And Kindred Products
- Major Group 21: Tobacco Products
- Major Group 22: Textile Mill Products



This account is best in class
So's that one, and that one…

Large Account/Treaty Pricing

- Terms:
  - Exposure Cost
  - Experience Cost
  - Burn Cost
  - Basic Limit
  - Large Loss Threshold

- When pricing an account, often limited loss experience is available. The actual data received depends on various factors, but the following is common:
  - Total sum of losses per year
  - Claim counts per year
  - Individual large losses greater than some threshold

- (Note that the following discussion is relevant to treaty pricing as well, but for brevity, references will be made to "accounts".)

Indication Options

Basic Limit Exposure Cost
X
ILF

Most stable, but less relevant

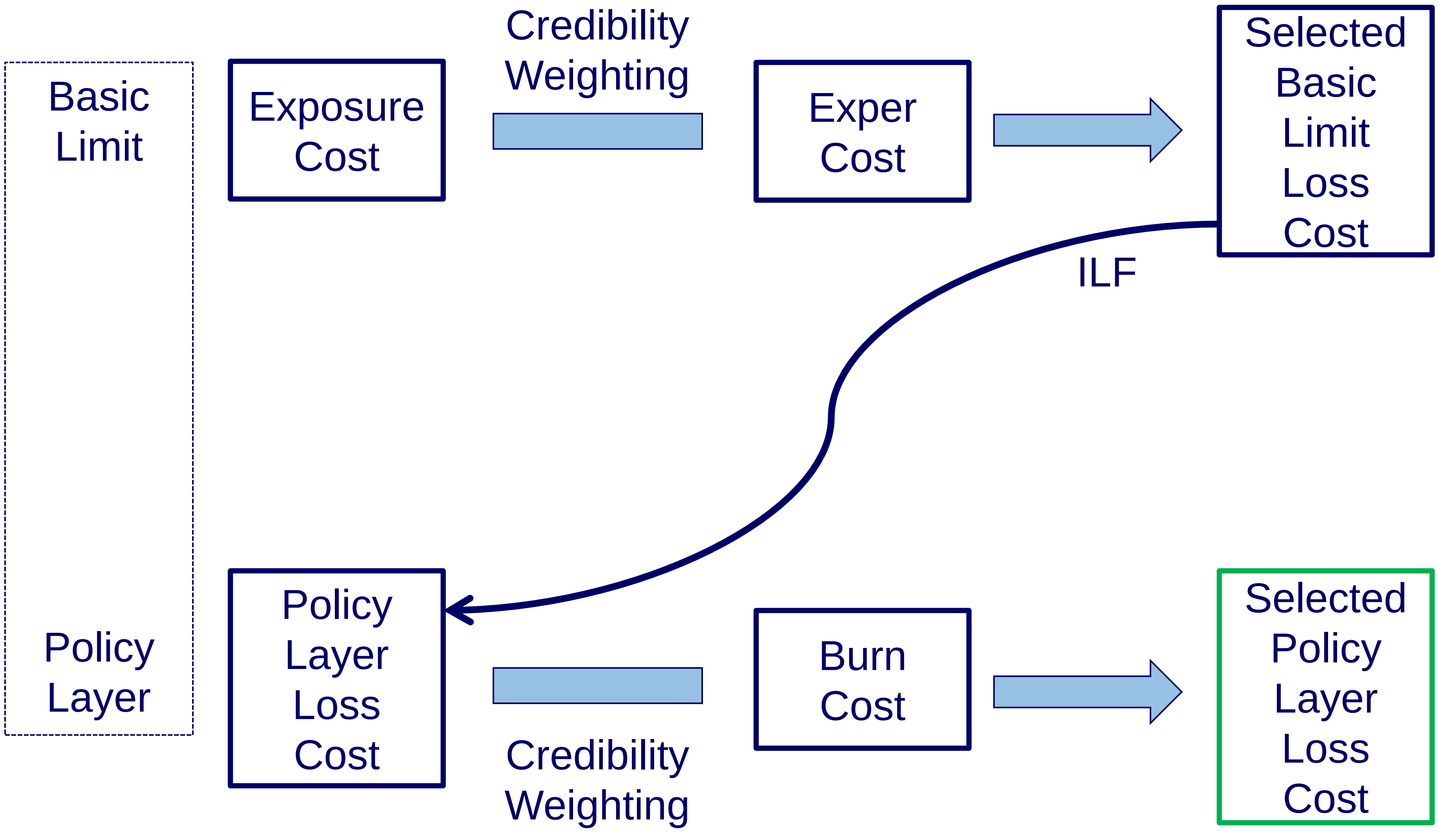
Basic Limit Experience Cost
(Capped, Exposure Adjusted, Trended, & Developed)
X
ILF

More volatile, but more relevant

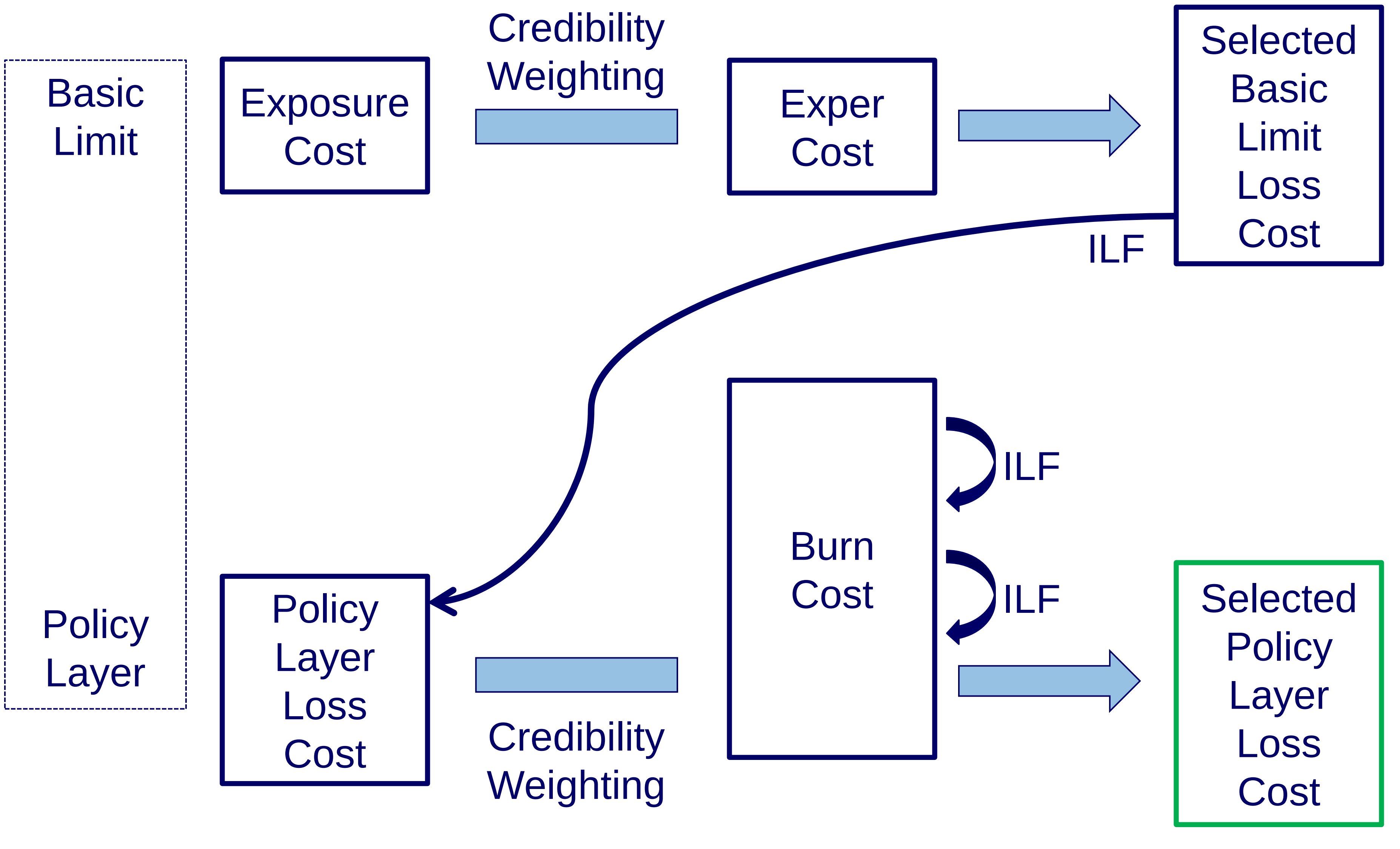
Burn Cost
(Exposure Adjusted, Trended, & Developed)

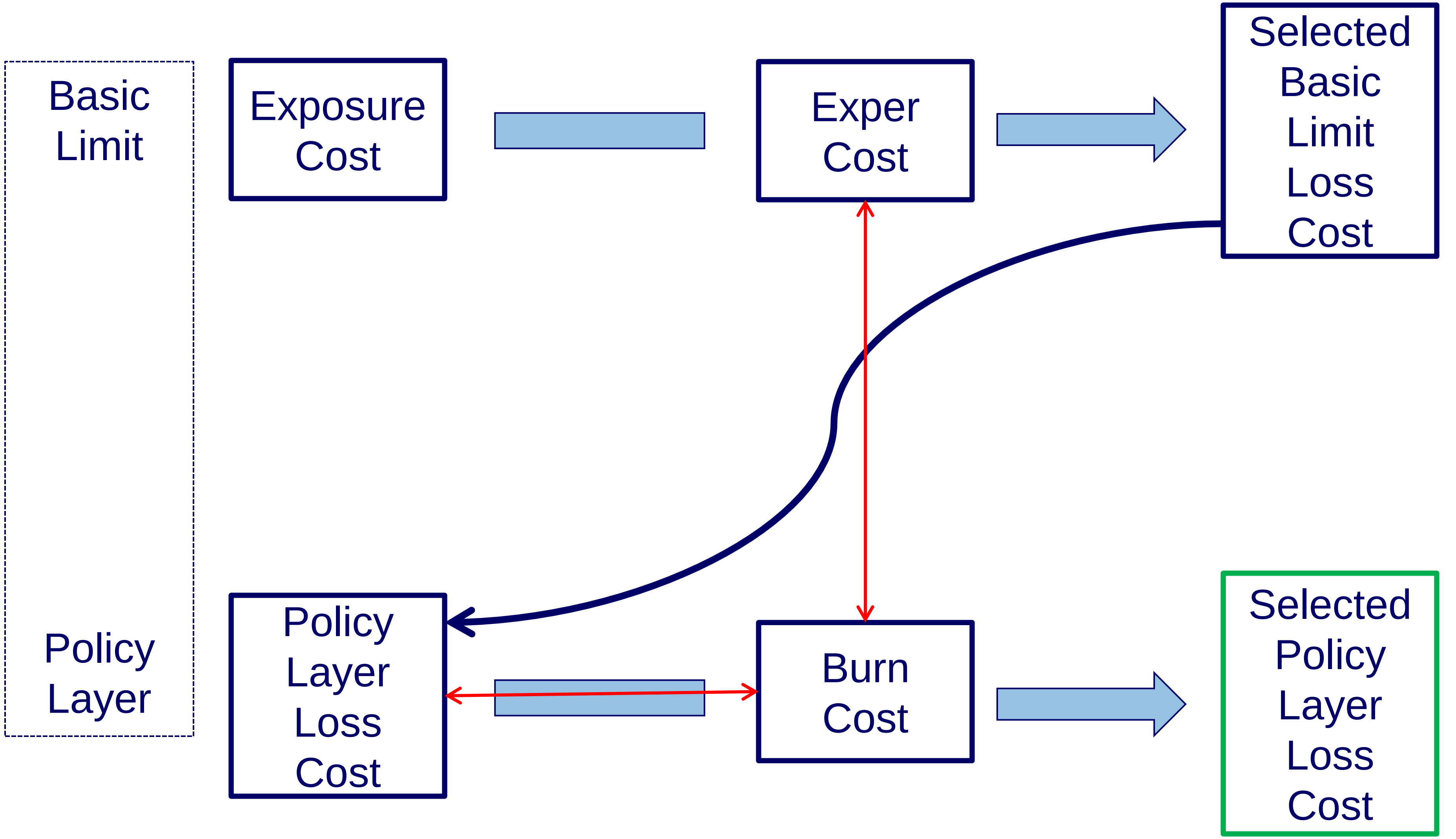
Most volatile but most relevant

Traditional Approach

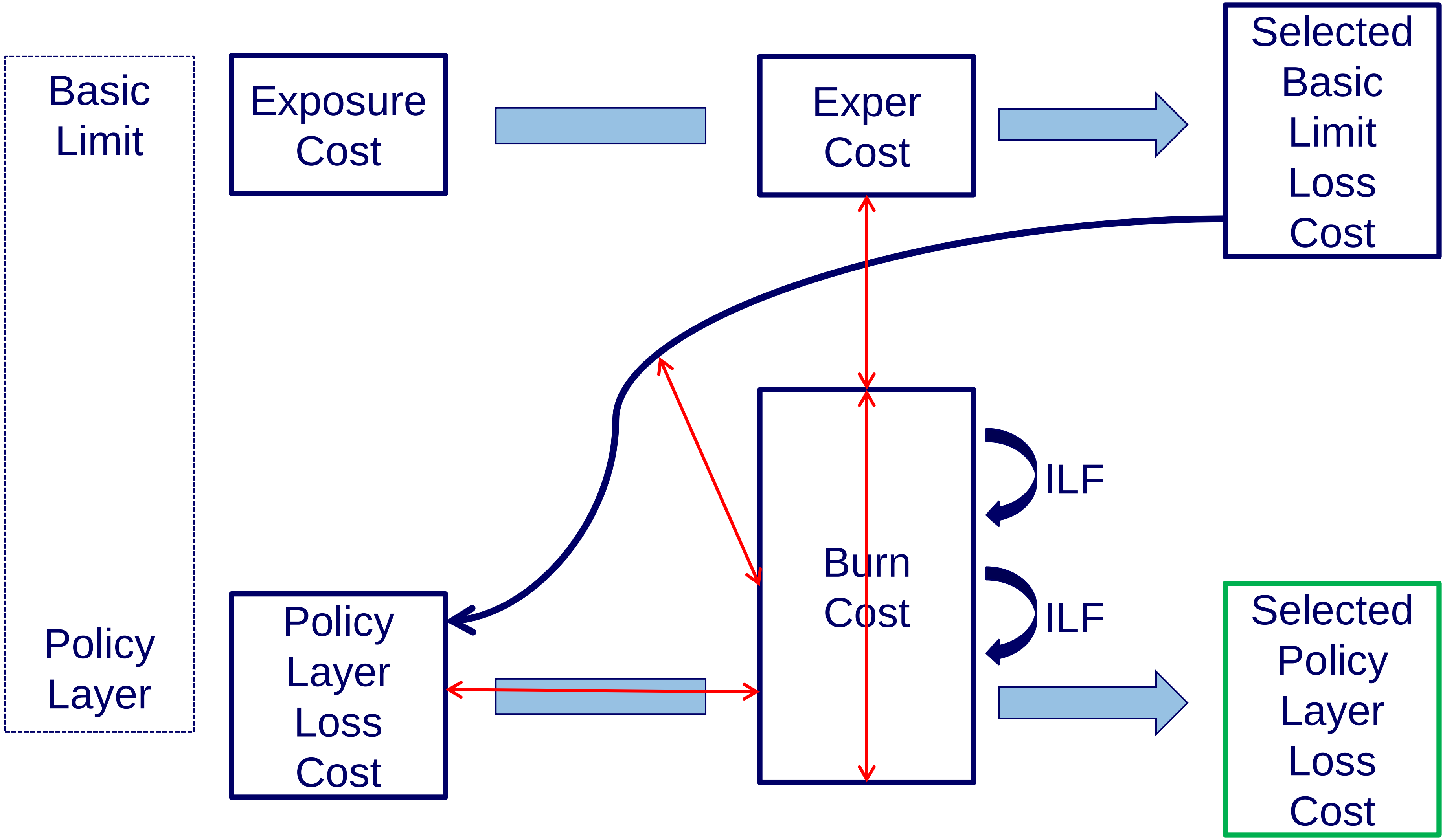


Traditional Approach 2



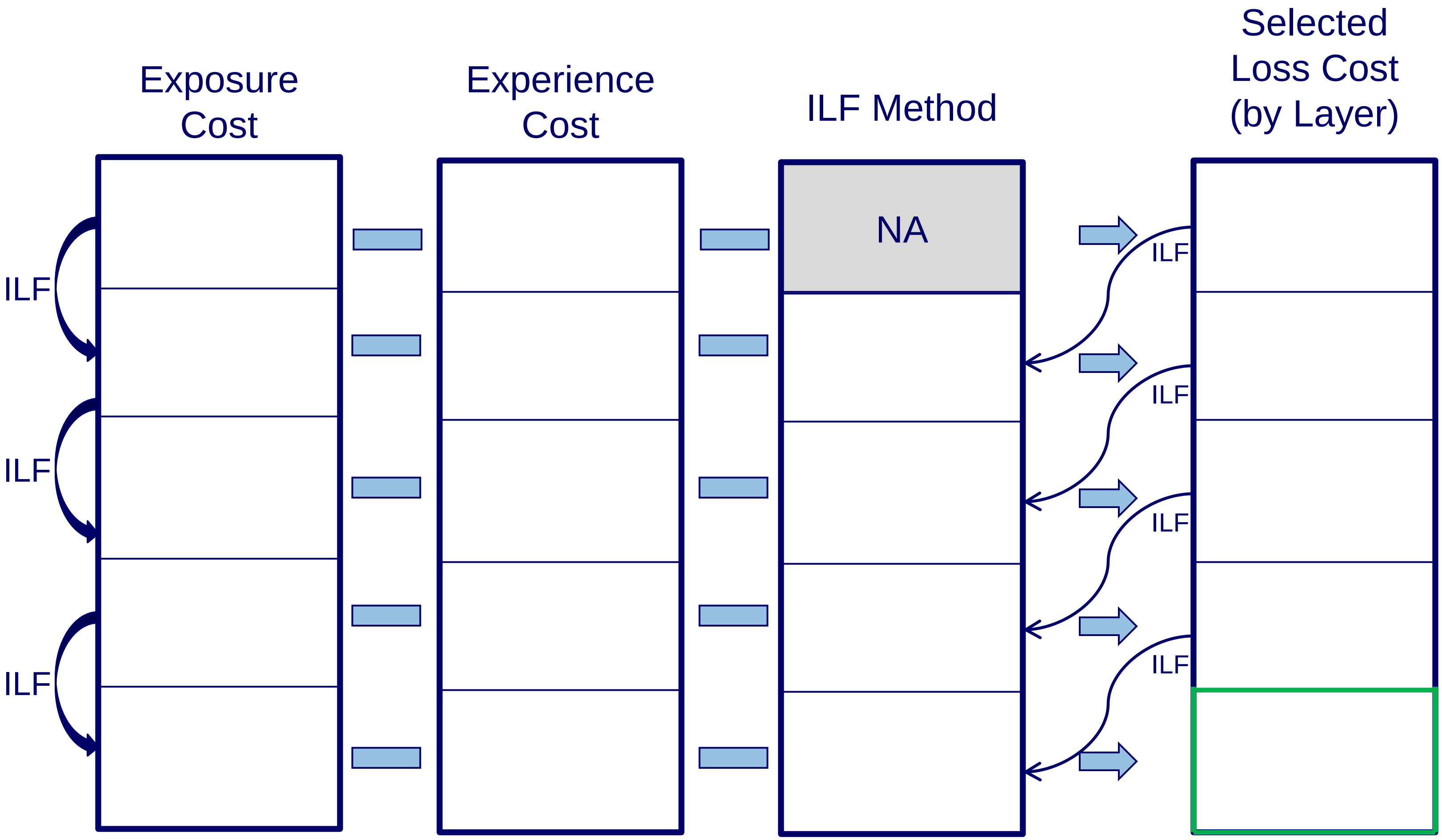


- Calculating the credibilities requires the calculation of the variance of the exposure cost, loss cost, burn cost, increased limits factor, as well as all covariances (red lines)



- Even more correlations

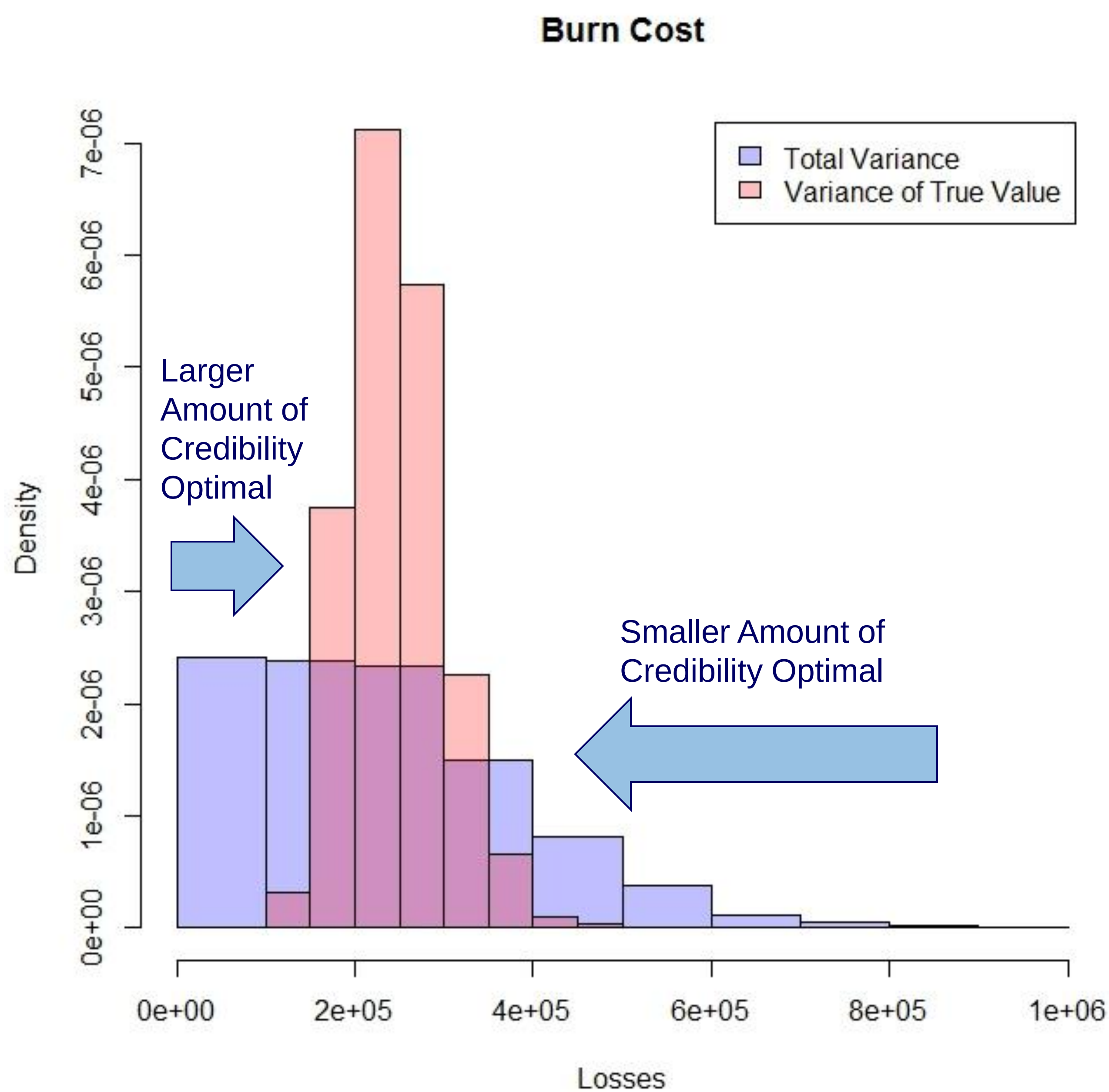
Comprehensive Traditional Approach – Clark (2011)



- Uses all available data in a comprehensive framework
- Credibility for each layer is based on taking the sums of the rows (or columns) of the inverse of the covariance matrix
- Formulas for calculating all relevant variances and covariances

Issues With The Traditional Approaches

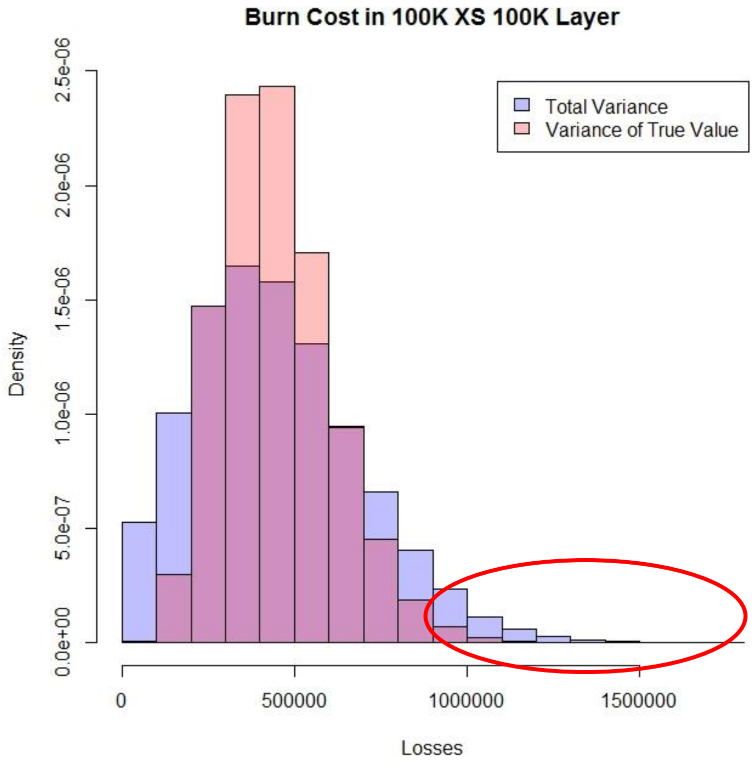
- Requires development and estimation of the experience cost for each layer used
- Requires estimating the variances and covariances of all components, which can be difficult. Clark helps, but obtaining everything needed is still difficult.
- The Burn Cost distribution is right skewed. Linear credibility methods (i.e., that use Z) do not work well with this data (Venter 2003).


Burn Cost
Total Variance
Variance of True Value
Larger Amount of Credibility Optimal
Smaller Amount of Credibility Optimal
Density
0e+00
1e-06
2e-06
3e-06
4e-06
5e-06
6e-06
7e-06
0e+00
2e+05
4e+05
6e+05
8e+05
1e+06
Losses

Simulation – Predicting the 100K xs 100K Layer

25 claims, large loss threshold of 100K, 5000 simulations

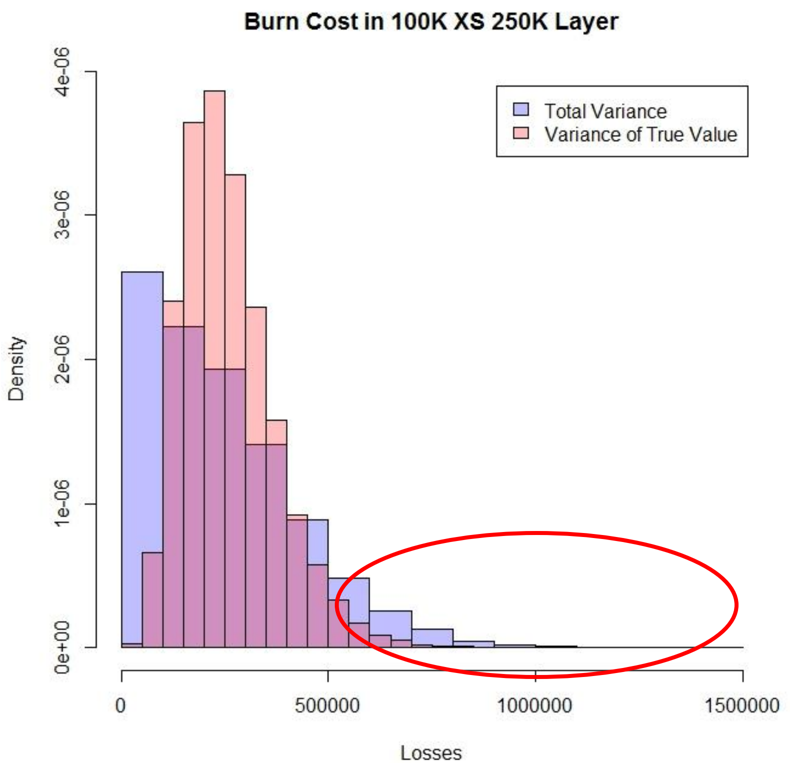
RMSE Percentage			
1) Portfolio	2) Burn Cost	3) Credibility Weighted Burn Cost	Cred Burn Cost vs Port (3 vs 1)
35.6%	39.8%	26.5%	-25.8%



Simulation – Predicting the 100K xs 250K Layer

25 claims, large loss threshold of 100K, 5000 simulations

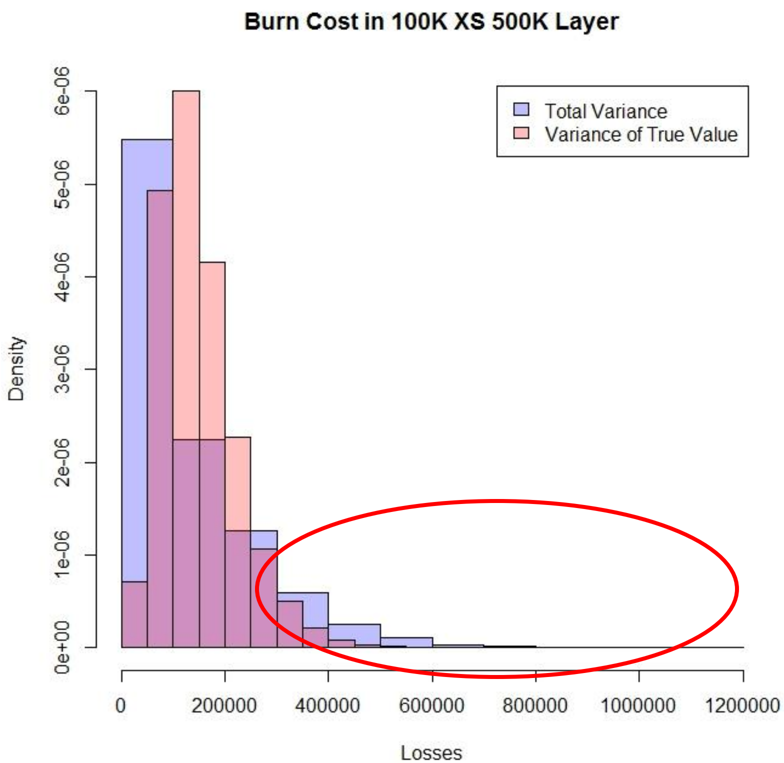
RMSE Percentage			
1) Portfolio	2) Burn Cost	3) Credibility Weighted Burn Cost	Cred Burn Cost vs Port (3 vs 1)
43.2%	56.6%	34.5%	-20.2%



Simulation – Predicting the 100K xs 500K Layer

25 claims, large loss threshold of 100K, 5000 simulations

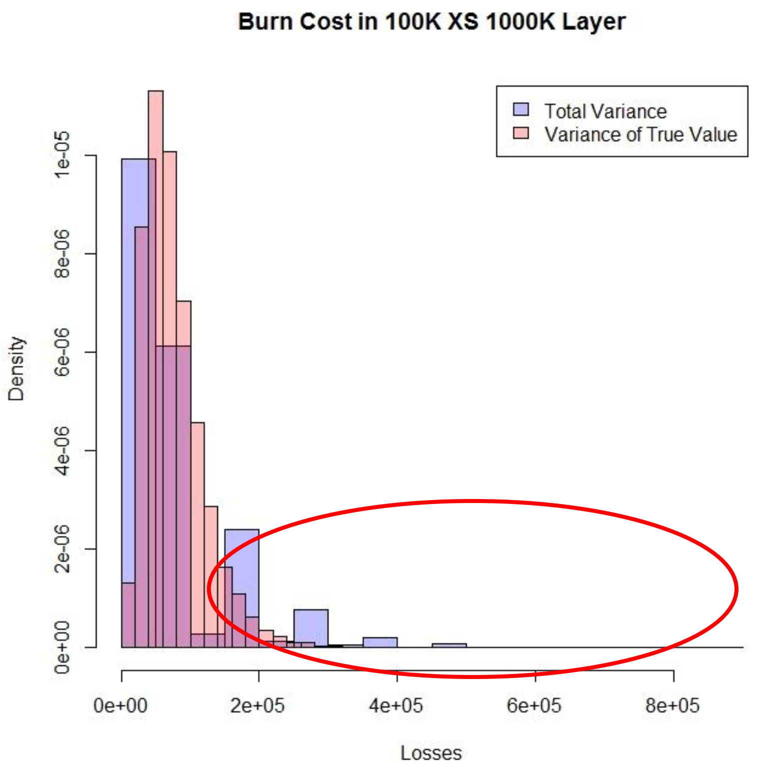
RMSE Percentage			
1) Portfolio	2) Burn Cost	3) Credibility Weighted Burn Cost	Cred Burn Cost vs Port (3 vs 1)
50.2%	77.9%	42.5%	-15.4%



Simulation – Predicting the 100K xs 1M Layer

25 claims, large loss threshold of 100K, 5000 simulations

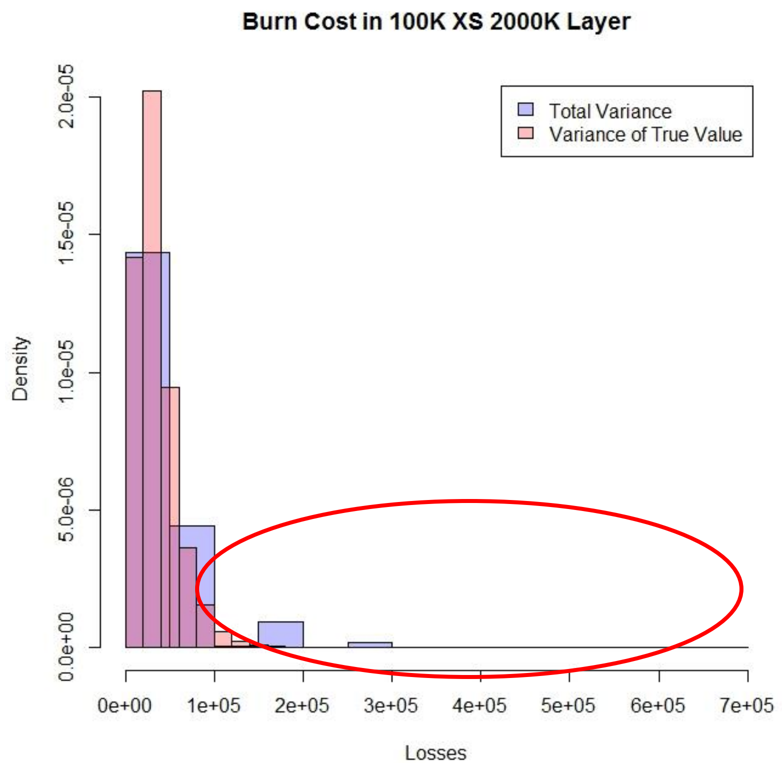
RMSE Percentage			
1) Portfolio	**2) Burn Cost**	**3) Credibility Weighted Burn Cost**	**Cred Burn Cost vs Port (3 vs 1)**
58.3%	111.3%	52.0%	-10.8%



Simulation – Predicting the 100K xs 2M Layer

25 claims, large loss threshold of 100K, 5000 simulations

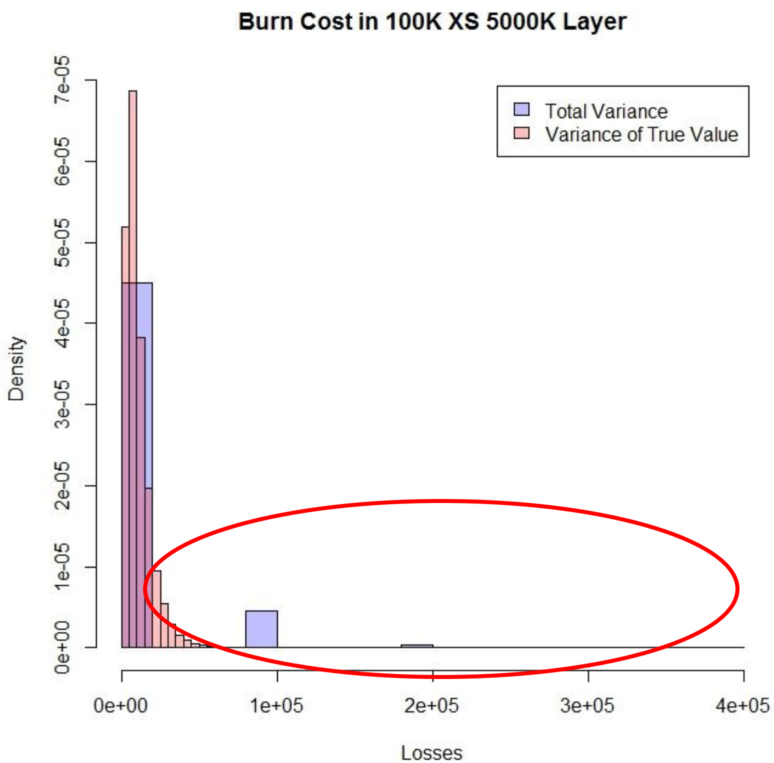
RMSE Percentage			
1) Portfolio	2) Burn Cost	3) Credibility Weighted Burn Cost	Cred Burn Cost vs Port (3 vs 1)
67.3%	169.8%	62.3%	-7.3%



Simulation – Predicting the 100K xs 5M Layer

25 claims, large loss threshold of 100K, 5000 simulations

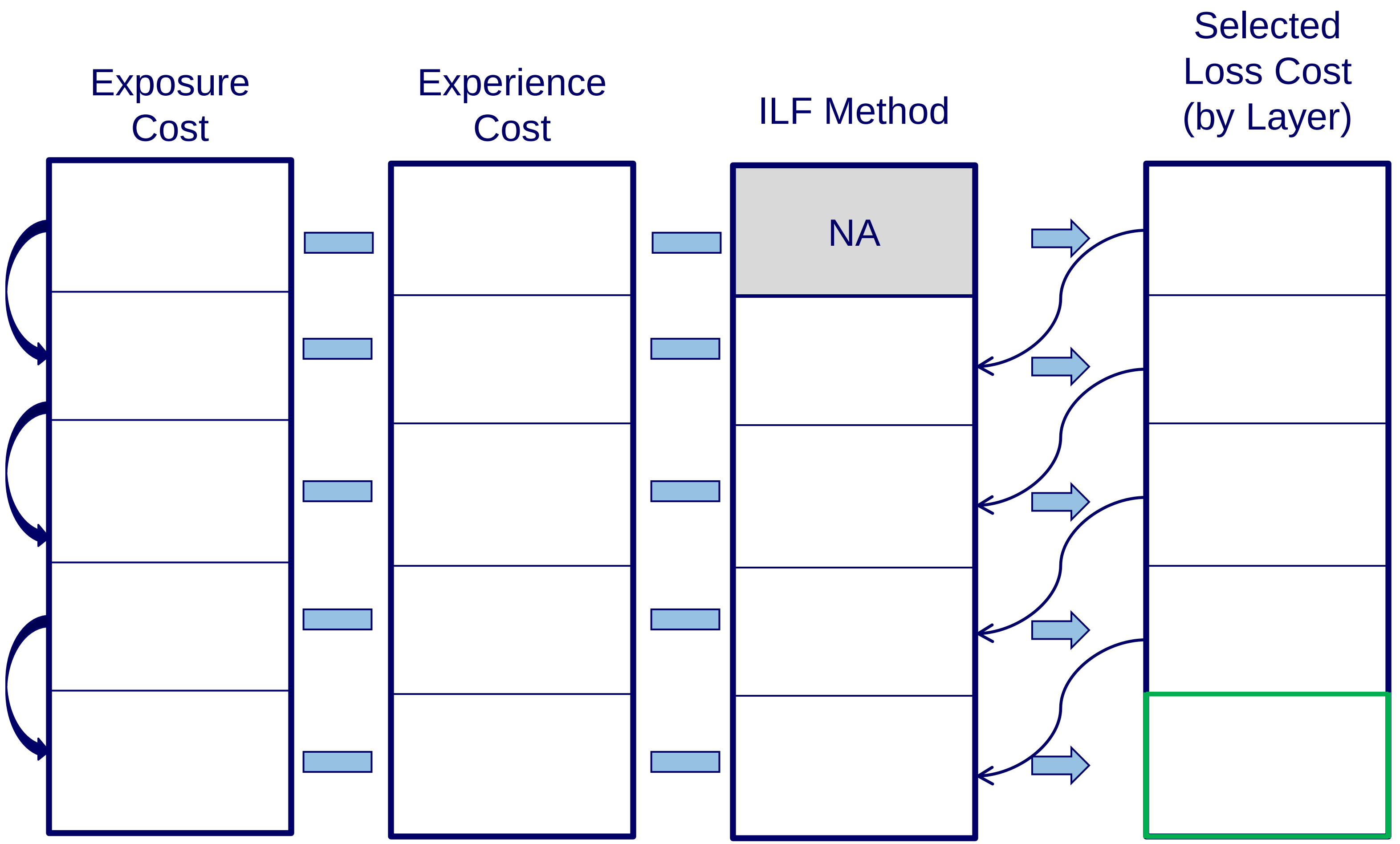
RMSE Percentage			
1) Portfolio	2) Burn Cost	3) Credibility Weighted Burn Cost	Cred Burn Cost vs Port (3 vs 1)
80.8%	313.2%	77.5%	-4.0%



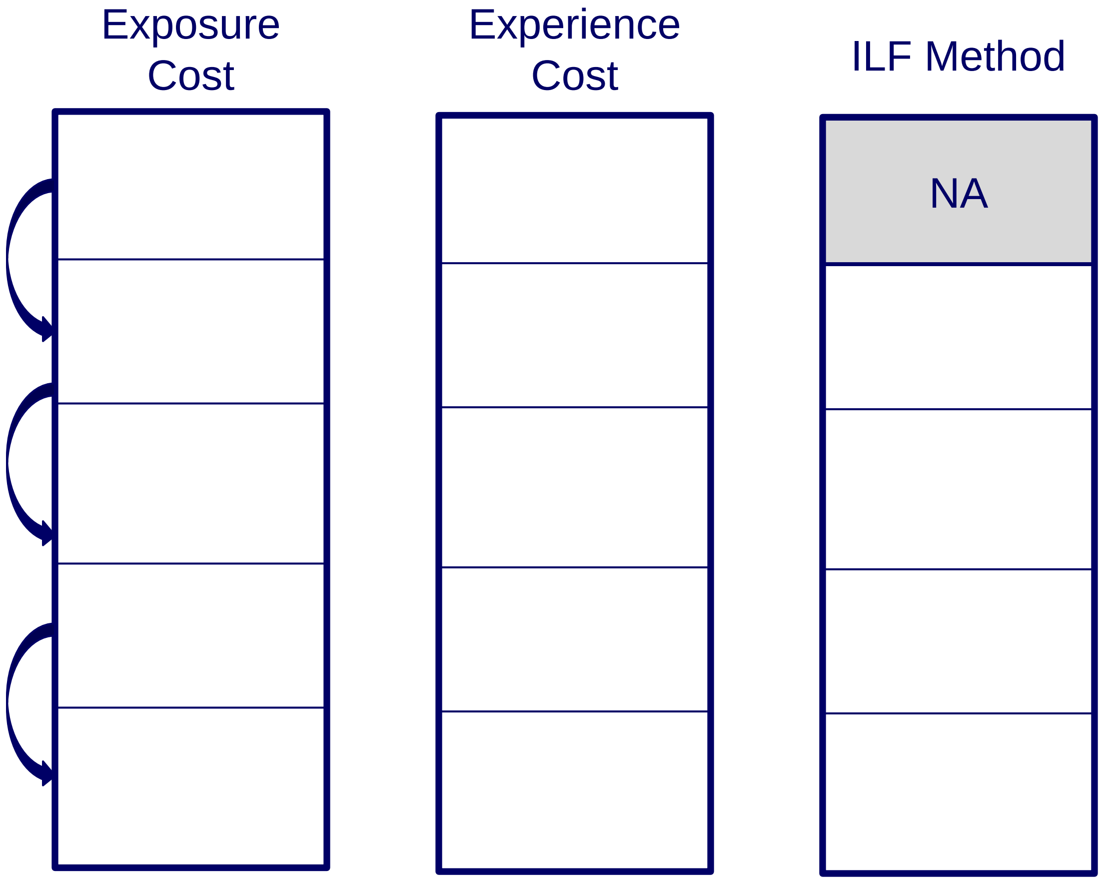
Simulation Summary

Layer	1) Portfolio	2) Burn Cost	3) Credibility Weighted Burn Cost	Credibility for #3	Cred Burn Cost vs Port (3 vs 1)
100K xs 100K	35.6%	39.8%	26.5%	44.5%	-25.8%
100K xs 250K	43.2%	56.6%	34.5%	36.7%	-20.2%
100K xs 500K	50.2%	77.9%	42.5%	29.1%	-15.4%
100K xs 1M	58.3%	111.3%	52.0%	21.1%	-10.8%
100K xs 2M	67.3%	169.8%	62.3%	13.8%	-7.3%
100K xs 5M	80.8%	313.2%	77.5%	7.0%	-4.0%

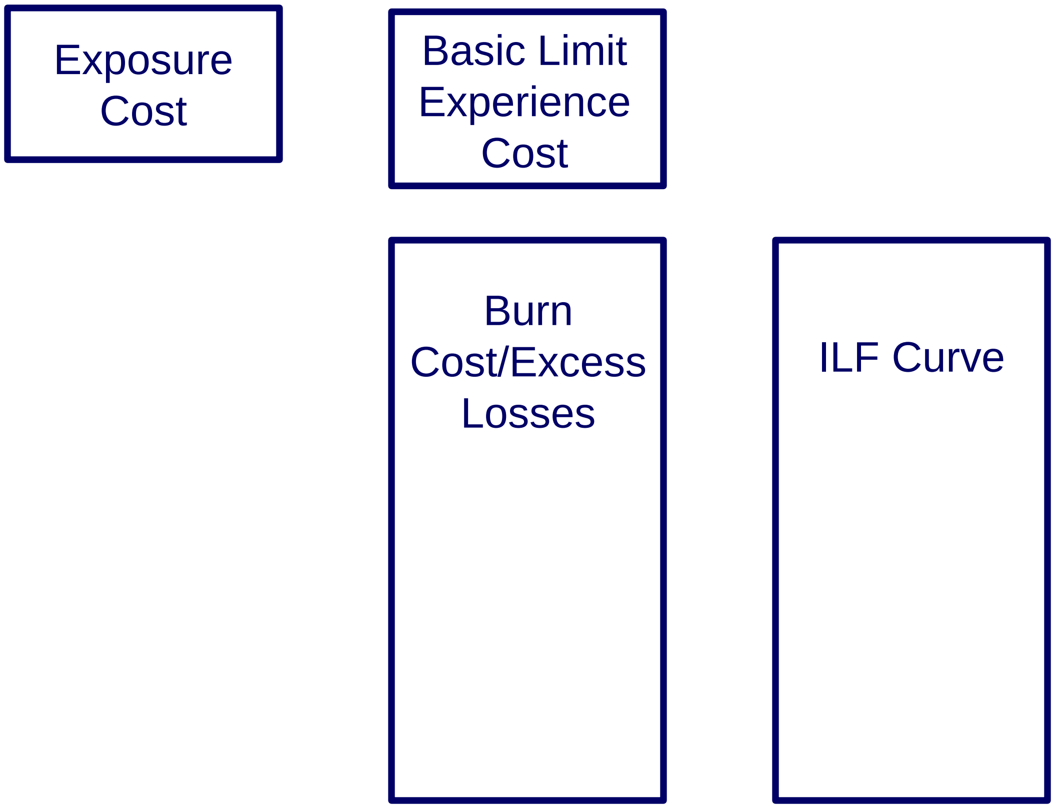
Taking a Step Back – What Data is Available?



Taking a Step Back – What Data is Available?

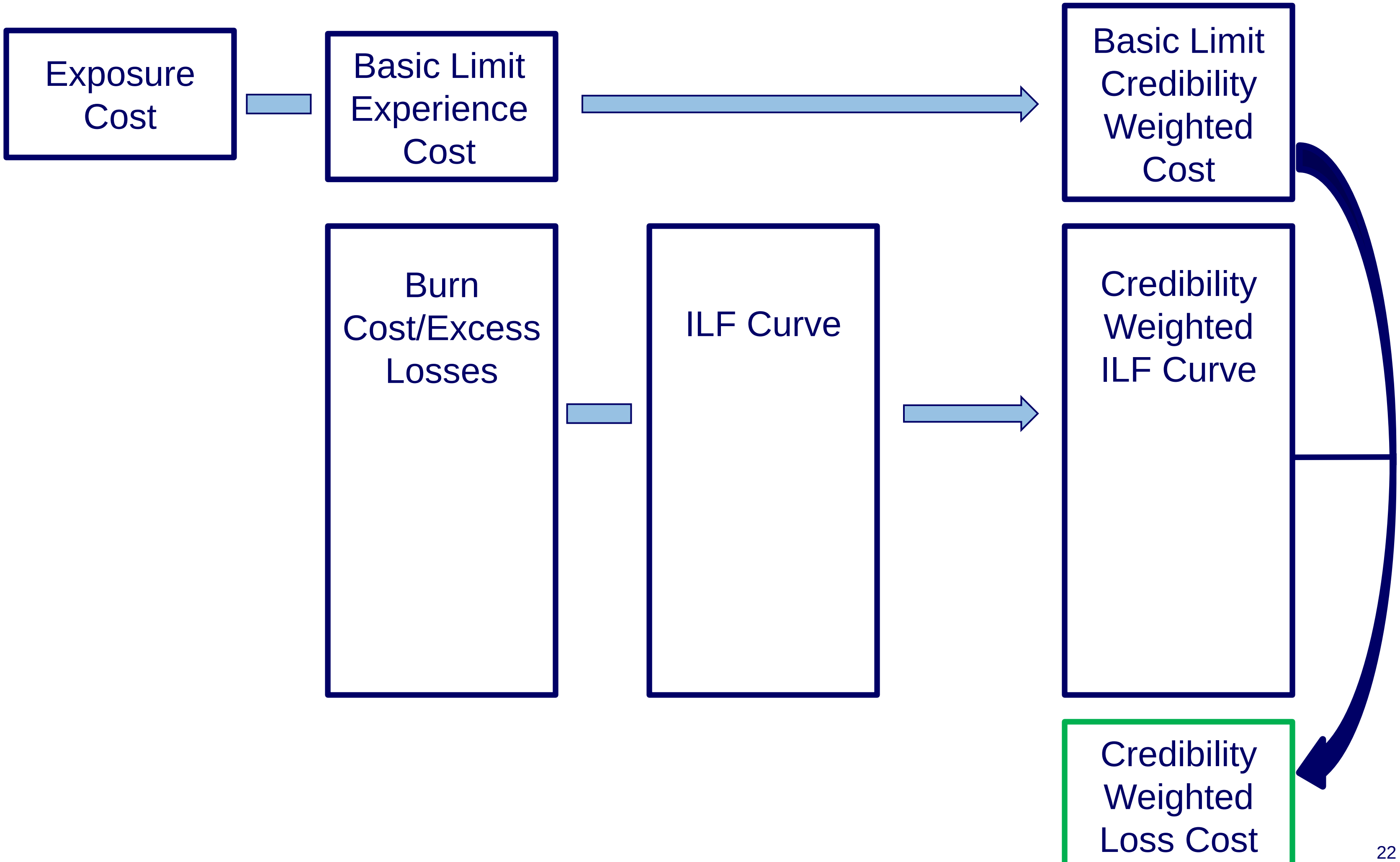


Taking a Step Back – What Data is Available?



Taking a Step Back – What Data is Available?

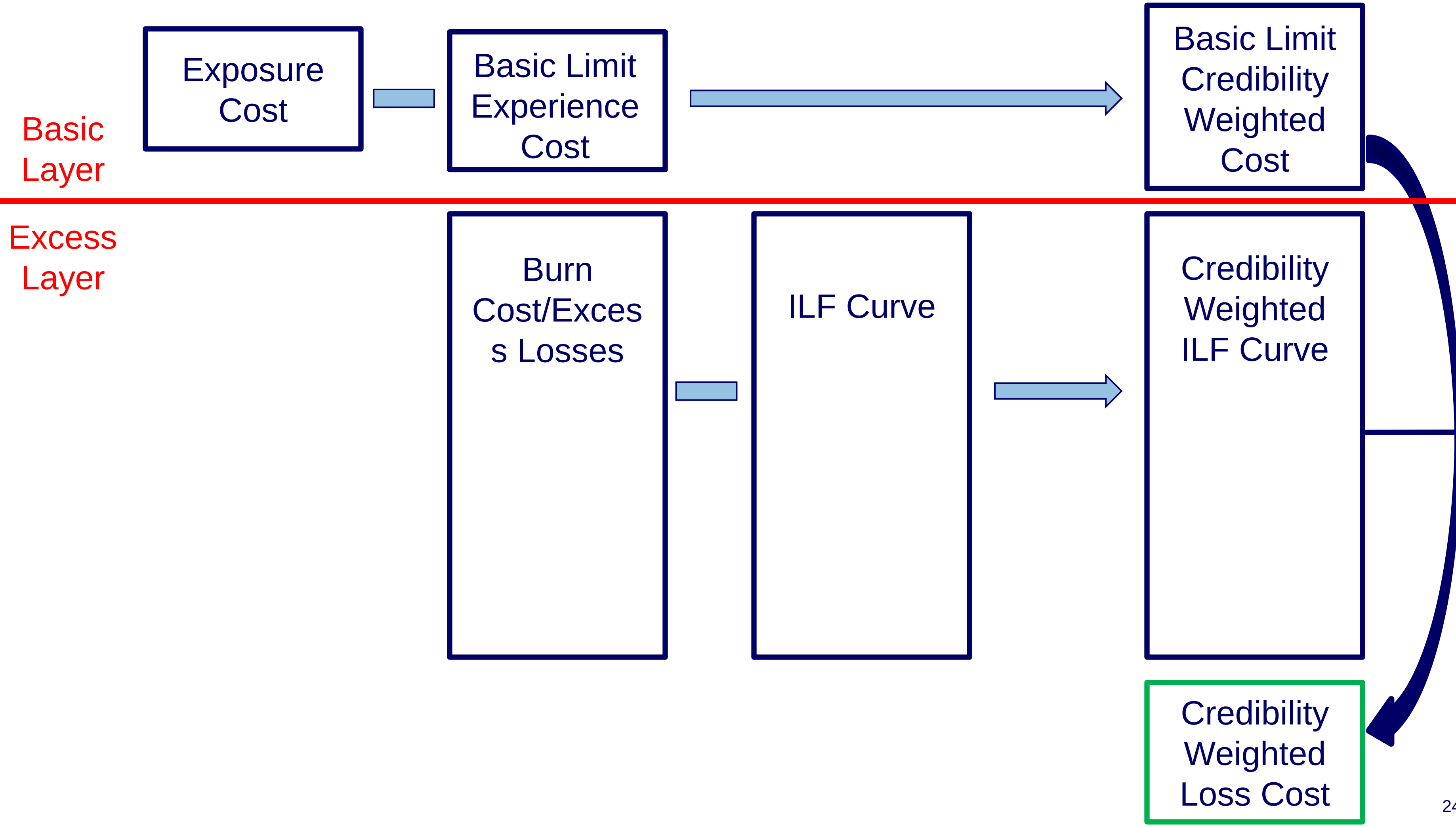
This type of approach utilizes all of the available data as well



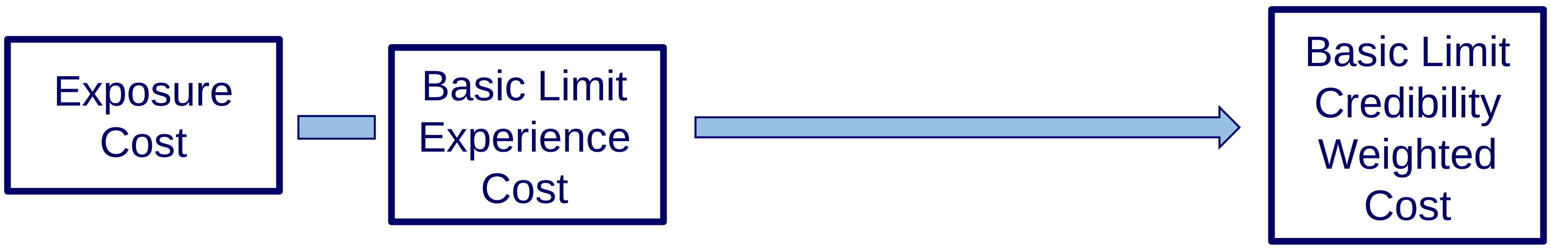
Proposed Approach

- Does not require trending & developing of every layer
- Does not require excess LDFs or trend factors
- Does not utilize burn costs which are highly volatile and right skewed
- Does not require the estimation of covariances
- Utilizes claim count information

We will address the Basic Layer first, and then the Excess Layer



Credibility Weighting the Basic Layer



- Use Buhlmann-Straub credibility
- The Within and Between Variance of the account's experience can be calculated using a sampling of actual accounts
- Can calculate on frequency/severity separately or on the combined aggregate losses

Problems with Buhlmann-Straub for Individual Account Pricing

1. Each item has a different a prior loss cost, but the formulas assume they are the same

2. If accounts' losses are capped at different amounts, their expected variances will differ, and the assigned credibilities should differ as well

3. Additional information is available in the ILF curve to help estimate some of the expected values and variances

Issue #1: Each item has a different a prior loss cost

- Modify the variance components in the formulas (the squared differences) to take into account the relationship between the variances and the expected values
- The common assumption is that the variance of frequency is proportional to the mean
- Modified Buhlmann-Straub Frequency Formulas:

$$\widehat{EPV} = \frac{\sum_{g=1}^{G} \sum_{n=1}^{N_g} e_{gn} \left(f_{gn} - \bar{f}_g \right)^2 / \bar{F}_g}{\sum_{g=1}^{G} \left(N_g - 1 \right)}$$

$$\widehat{VHM} = \frac{\sum_{g=1}^{G} e_g \left(\bar{f}_g - \bar{F}_g \right)^2 / \bar{F}_g - (G - 1)\,\widehat{EPV}}{e - \frac{\sum_{g=1}^{G} e_g^2}{e}}$$

$$k = \frac{\widehat{EPV}}{\widehat{VHM}} \qquad Z = \frac{e}{e + k}$$

Item # 1

- For severity, the common assumption is that the variance is proportional to the mean squared.
- Initial modified formulas for severity:

$$\widehat{EPV} = \frac{\sum_{g=1}^{G} \sum_{n=1}^{N_g} c_{gn} \left(s_{gn} - \bar{s}_g \right)^2 / \bar{S}_g^2}{\sum_{g=1}^{G} \left(N_g - 1 \right)}$$

$$\widehat{VHM} = \frac{\sum_{g=1}^{G} c_g \left(\bar{s}_g - \bar{S}_g \right)^2 / \bar{S}_g - (G - 1)\, \widehat{EPV}}{c - \frac{\sum_{g=1}^{G} c_g^2}{c}}$$

- But, these formulas do not consider the information in the severity distribution or take into account different loss caps (Items #2 & 3)

Issues #2, 3: Different capping values, additional info in ILFs

- Instead of the actual data, use the ILF curve/severity distribution to calculate the Expected Process Variance (EPV) (Refer to the paper for the derivation and final formulas)
- This will be more stable and reliable than using the account's actual experience

$$\widehat{EPV}_{g,cap} = \frac{LEV2(cap) - LEV(cap)^2}{\bar{S}^2}$$

$$k = \frac{\widehat{EPV}_{g,cap}}{\widehat{VHM}}$$

$$Z = \frac{c}{c + k}$$

- The higher the cap, the higher the EPV, and the lower the credibility

Aggregate Data

- For aggregate data (frequency & severity combined), the assumption is that the variance is proportional to the mean taken to some power between 1 and 2. (A common assumption is to use 1.67.)
- Formulas for aggregate data: (refer to the paper for derivation)

$$\widehat{EPV}_{g,cap} = \frac{[LEV2(cap) - LEV(cap)^2] \times \bar{F} + \widehat{EPV}_f \times \bar{F} \times \bar{S}^2}{\bar{L}^P}$$

$$\widehat{VHM} = \frac{\sum_{g=1}^{G} e\left[(\bar{l}_g - \bar{L}_g)^2 / \bar{L}_g{}^P - \frac{(G-1)\,\widehat{EPV}_{g,cap}}{G\, e_g}\right]}{e - \frac{\sum_{g=1}^{G} e_g^2}{e}}$$

Excess Frequency

- Sometimes, only excess frequency above a certain threshold is used to modify the lost cost
- The modified Buhlmann-Straub formulas can be derived by using the following relationship for the variance-to-mean ratio (where p is the chance of exceeding the threshold):

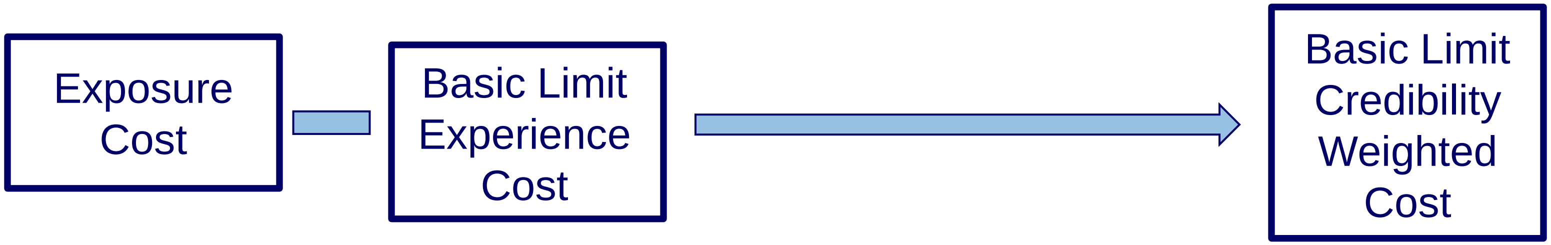
$$\text{XS VTM} = (\text{GU VTM} - 1) \times p + 1$$

- Refer to the paper for the final formulas
- Once the EPV and VHM are computed, *k* can be calculated as follows (where EPV and VHM are the ground-up values):

$$\mathrm{k} = \frac{(EPV - 1) \times p + 1}{VHM \times p}$$

- Higher thresholds will have a lower *p* value, which will result in higher *k* values and lower credibilities, as expected

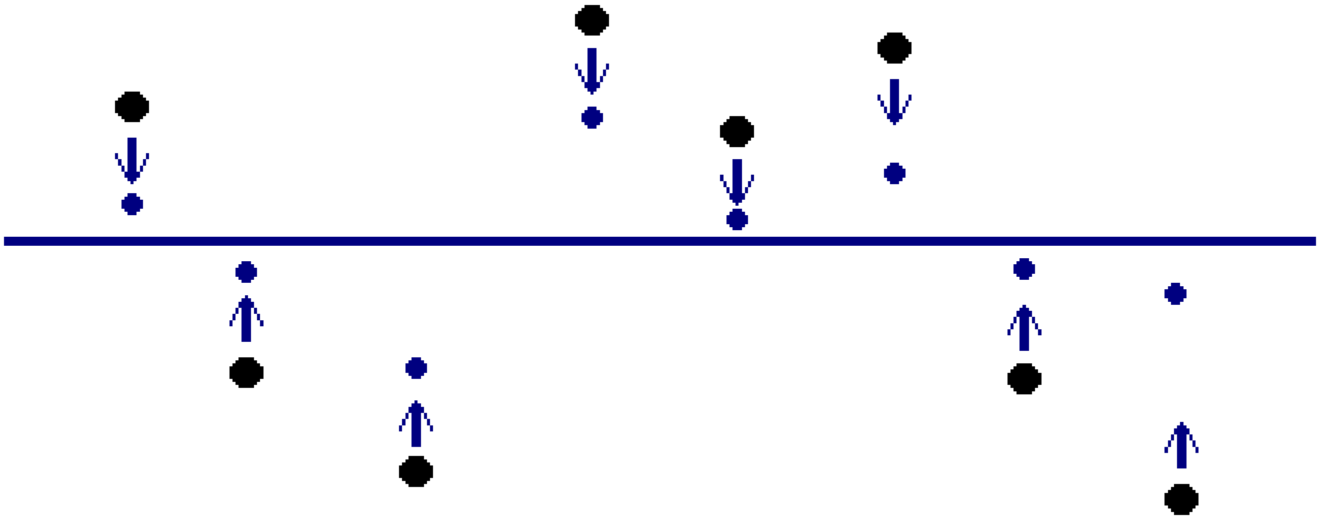
Credibility Weighting the Basic Layer



- For all of the final modified formulas, refer to the paper

Accounting for Development

- When calculating the within variance (EPV), do <u>not</u> use a Bornhuetter-Ferguson method since it pushes each year towards the mean and thus artificially lowers the volatility inherent in the experience
- (Don't confuse the prediction with the data)



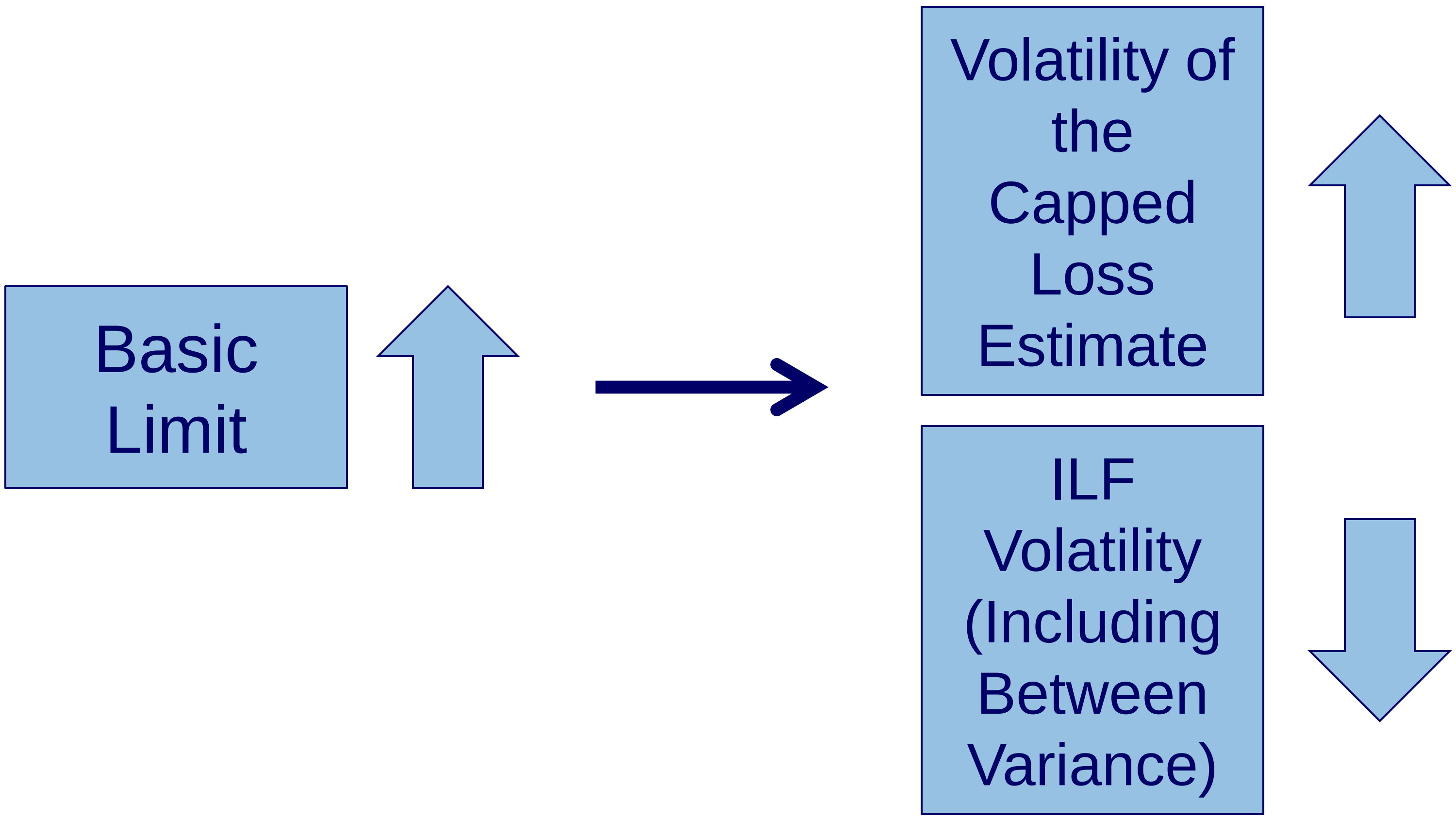
Accounting for Development

- Instead, use a Cape-Cod-like approach
    - Chain ladder estimates per year
    - “Used” Exposures = Exposures / LDF – takes into account that more recent years have higher volatility
- This allows for a more direct analysis of the experience

- For aggregate losses, use the LDF
- For claim counts, use the CCDF (claim count development factor)
- For severity, use LDF / CCDF or derive factors directly
    - Multiply the average severities by this factor
    - Use the actual claim counts as the exposures (do not apply a factor)

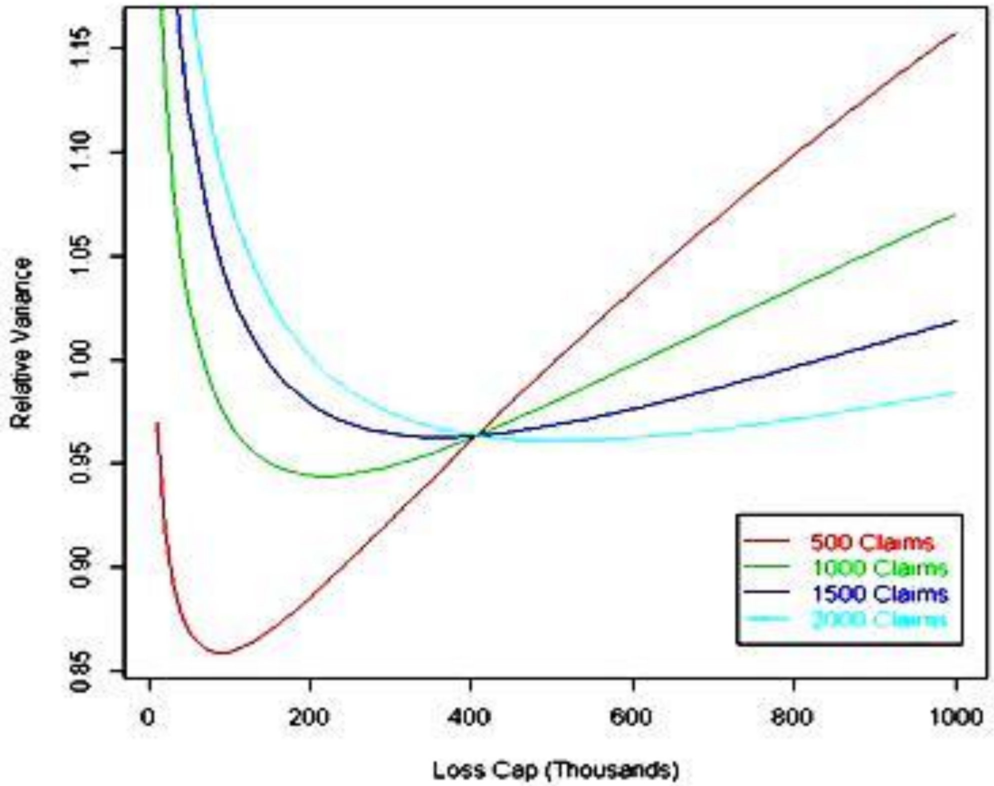
Intermission

A slight tangent: Calculating the optimal basic limit

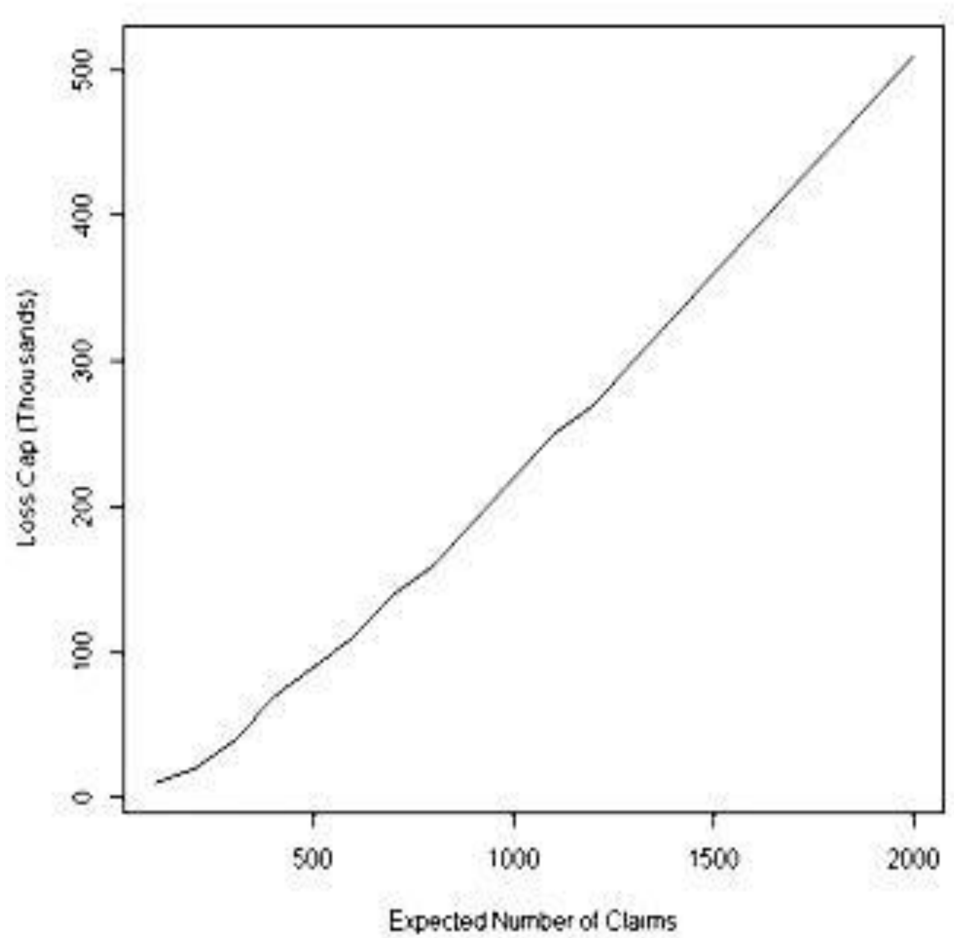


Calculating the optimal basic limit

- Based on what we have covered so far (along with a bit soon to come), we can now quantify this!

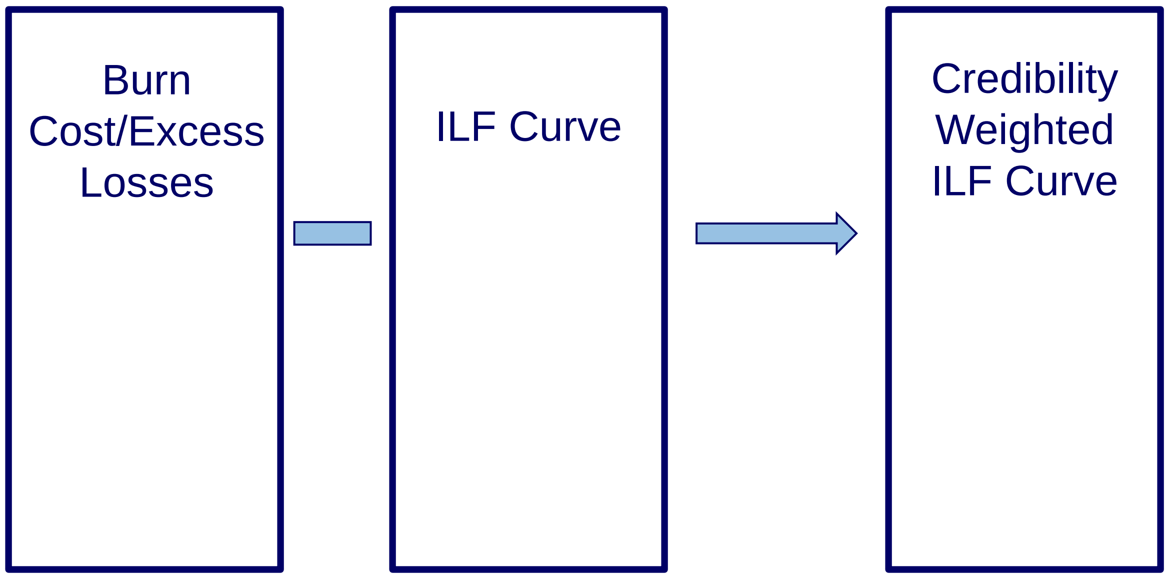


Summary of results for different sized accounts



Refer to the paper for the final formulas

Excess Layer



What is the best way to fit the losses?

- Note that data is often only available in aggregate form
- Often only a relatively small number of claims are available

- Test Methods:
  - Maximum Likelihood (with modified formula for aggregate losses)
  - Minimize Layer CSP Errors Squared
  - Minimize Layer CSP Error Percentages Squared
  - Layer CSPs - Binomial Maximum Likelihood
  - Minimize Layer LEV Error Percentages Squared
  - Minimize claim counts in layer Chi-Square statistic

Simulation Results

- A simulation was performed with 25 ground up claims and a large loss threshold of 200K
- Assumed only aggregate data was available
- For each iteration, use each of the below methods to fit a (lognormal) severity distribution to the data
- Using the fitted severity distributions, calculate the average severity in the 10M xs 10M layer
- The results show that MLE is the best, even with thin, aggregate data

Method	Bias	RMSE (Thousands)
MLE	4.7%	194
CSP Error Squared	16.5%	239
CSP Error Percent Squared	13.5%	243
CSP Binomial	8.9%	209
LEV Error Percent Squared	55.2%	282
Counts Chi-Square	41.4%	256

Increased Limit Factor

- What data should be considered when constructing the credibility weighted ILF curve?

Capped Losses X ILF(Policy Layer)

= Account Claim Count x LEV-Account (Loss Cap) X LEV-Port(Policy Layer) ÷ LEV-Port(Loss Cap)

= Account Claim Count x LEV-Port (Policy Layer) X LEV-Account(Loss Cap) ÷ LEV-Port(Loss Cap)

Increased Limit Factor

- So, applying an ILF is the same as multiplying the claim count by the portfolio estimated average severity in the policy layer, multiplied by an experience factor related to the actual severity in the basic limit

- Using an ILF already considers the basic limit losses (which have already been credibility weighted in the first part)

- When constructing a credibility weighted ILF, only consider the excess losses, otherwise they will be double counted

Likelihood formula for aggregate data

- The claims below the large loss threshold are left censored, since we know the number but not the amounts

$$\sum_{x=Claims\,>\,LLT} PDF(x) + n \times CDF(LLT)$$

Where:

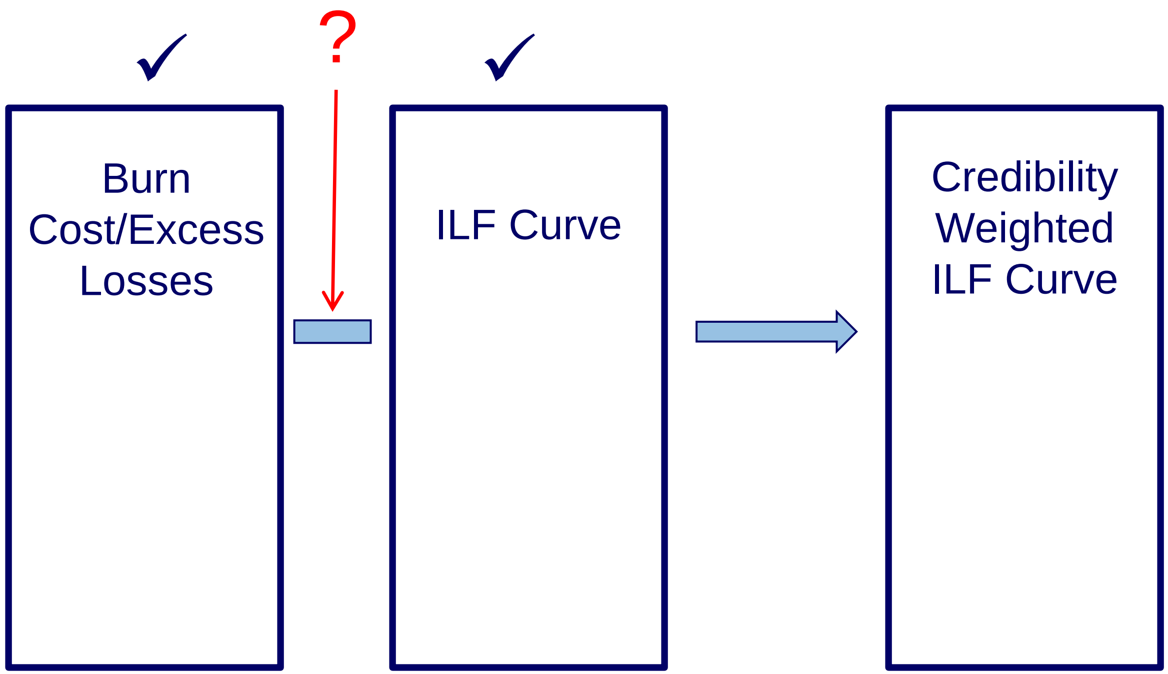
n is the number of claims that do not exceed the large loss threshold

PDF is the <u>logarithm</u> of the probability density function

CDF is the <u>logarithm</u> of the cumulative density function

LLT is the large loss threshold

What credibility method should we use for the excess losses?



Bayesian Credibility

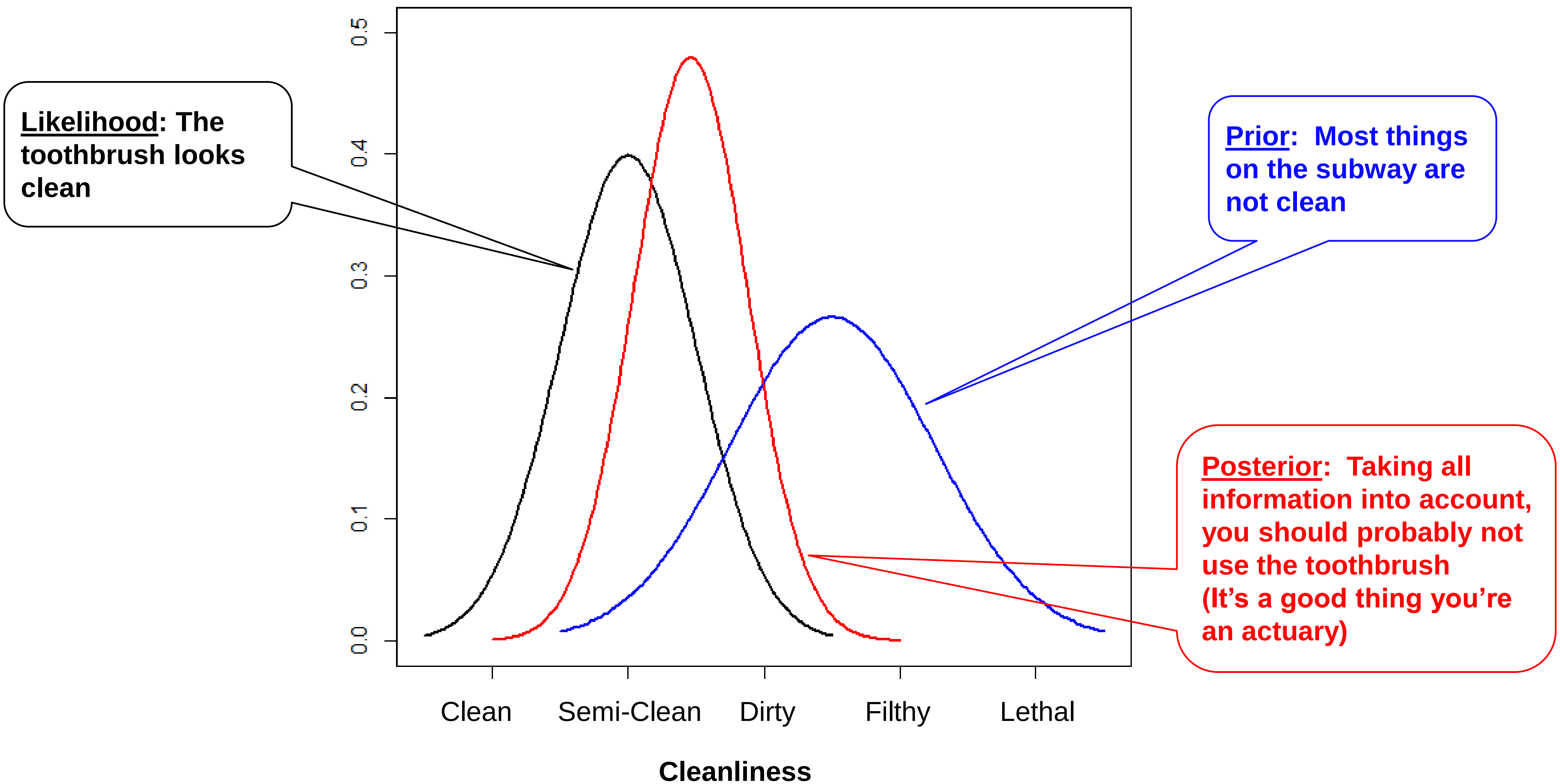
f(Params | Data) ~ f(Data | Params) x f(Params)

Posterior ~ Likelihood x Prior

- Optimal credibility method
- Is exact
- Can handle right skewed data

Bayesian Credibility Example

- You find a toothbrush on the subway!
- It looks semi-clean!
- Should you use it?



Bayesian Credibility for a Severity Distribution

- Performs credibility weighting on the parameters of the severity distribution simultaneously while fitting the distribution

- This is done by adding another component to the log-likelihood which pushes each parameter closer to the mean

Likelihood

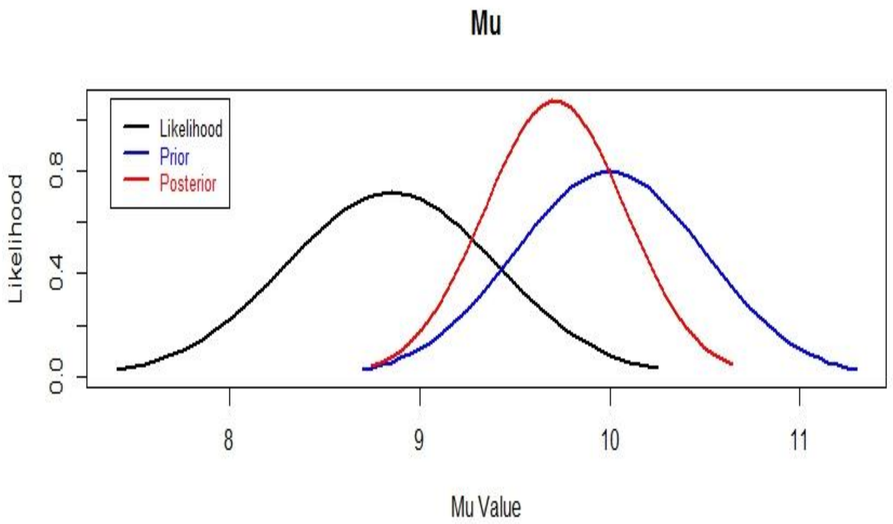
$$\sum PDF(x, p1, p2) +$$

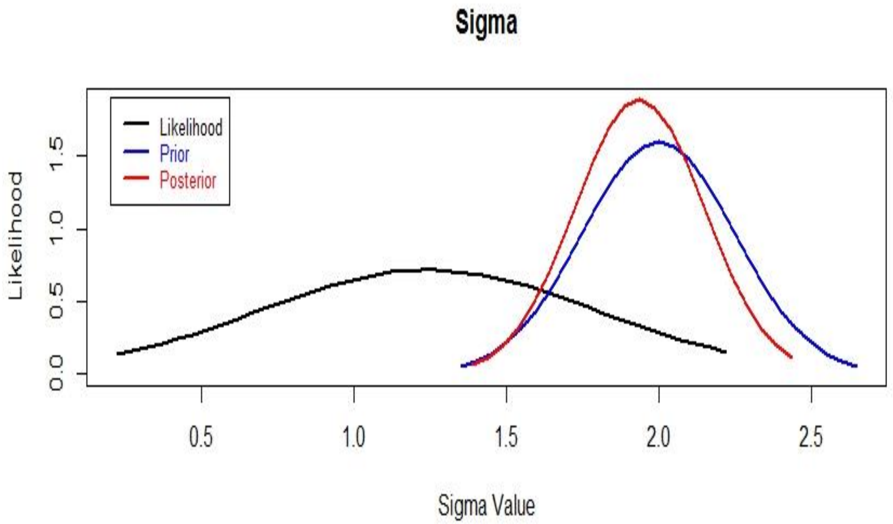
$$Norm(p1, Porfolio\ p1, Between\ Var1) + Norm(p2, Porfolio\ p2, Between\ Var2)$$

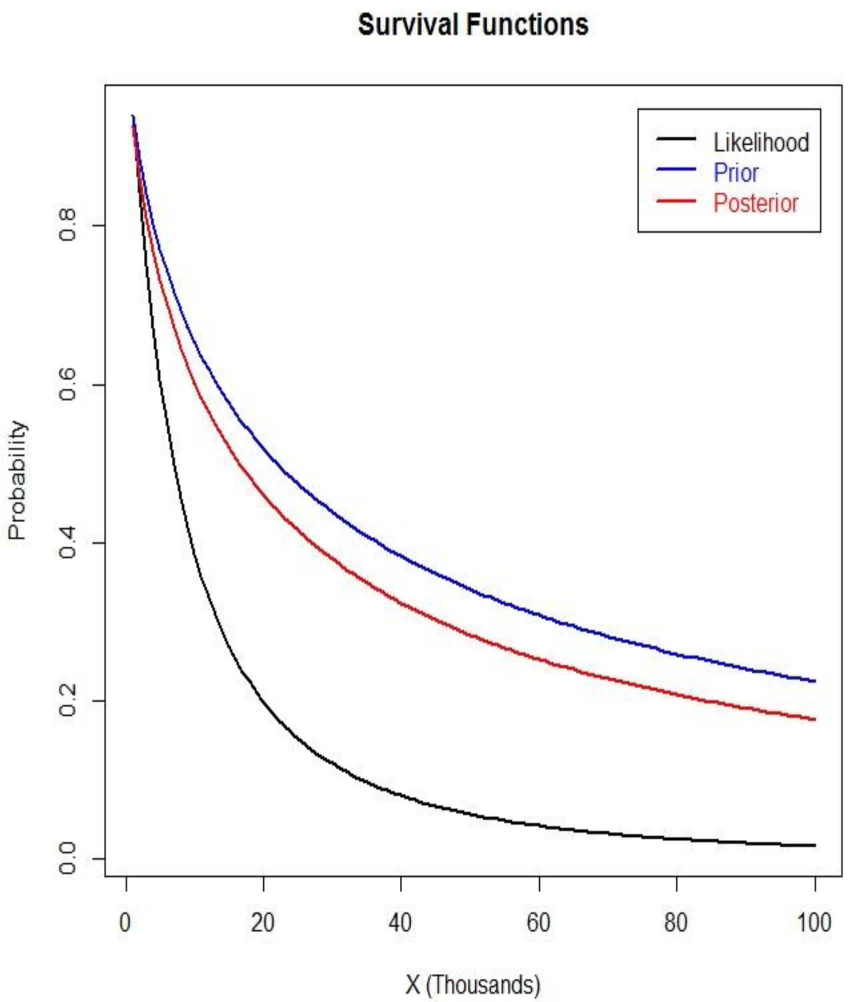
Prior

Where:
PDF is the <u>logarithm</u> of the probability density function
Norm is the <u>logarithm</u> of the normal probability density function

Bayesian Credibility for Distributions



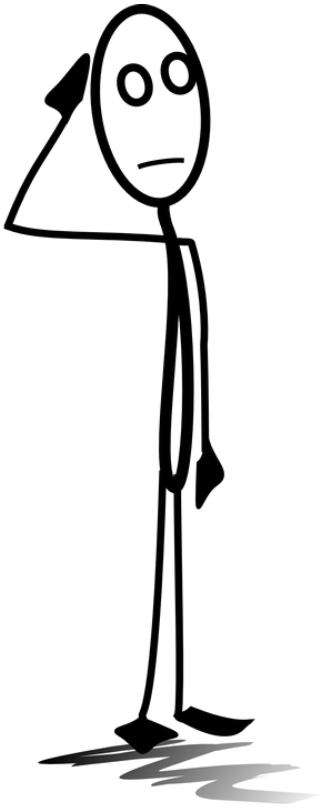




Bayesian Credibility

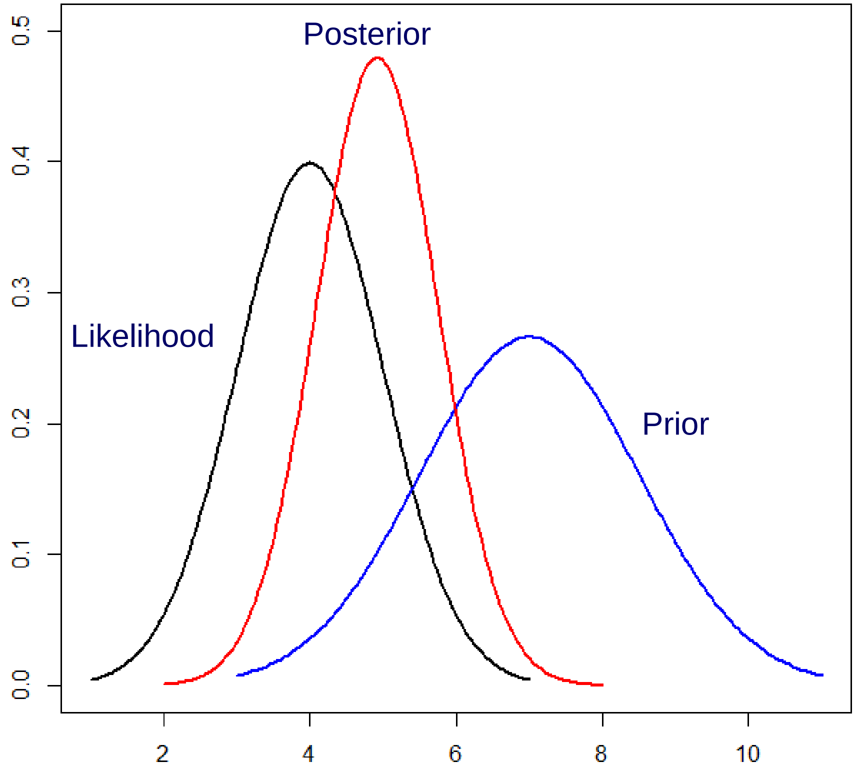
But this requires specialized software to run??

Bayesian Credibility



Or does it??

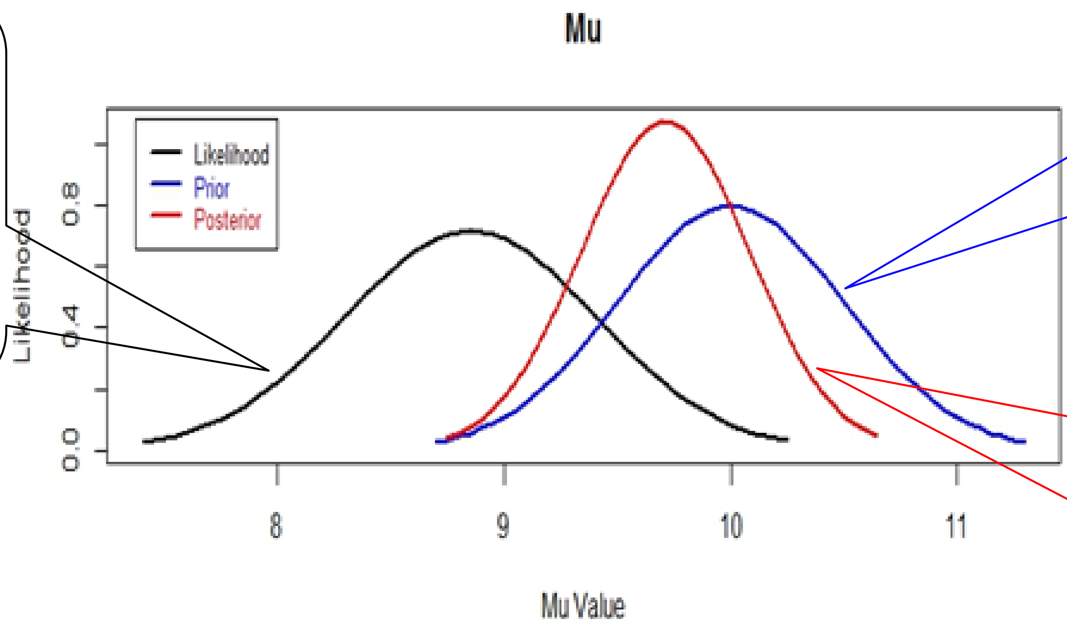
Normal Conjugate



- If both the likelihood and the prior are normally distributed, the posterior is normally distributed as well

Implementing Bayesian Credibility via MLE

MLE parameters are approximately normally distributed (asymptotically)



Normal prior on the parameters (the common assumption)

This is a conjugate prior and the posterior is normally distributed as well

- Since the result is normal, the mode equals the mean
- MLE, which returns the mode, also returns the mean in this case
- The result will match that returned from using specialized Bayesian software!

Implementing Bayesian Credibility via MLE

- Revised likelihood formula – MLE with additional prior/credibility component

Likelihood

$$\sum_{x=Claims>LLT} PDF(x,p1,p2) + n \times CDF(LLT,p1,p2) +$$

$$Norm(p1, Porfolio\ p1, Between\ Var1) + Norm(p2, Porfolio\ p2, Between\ Var2)$$

Prior

Where:

n is the number of claims that do not exceed the large loss threshold

PDF is the logarithm of the probability density function

CDF is the logarithm of the cumulative density function

Norm is the logarithm of the normal probability density function

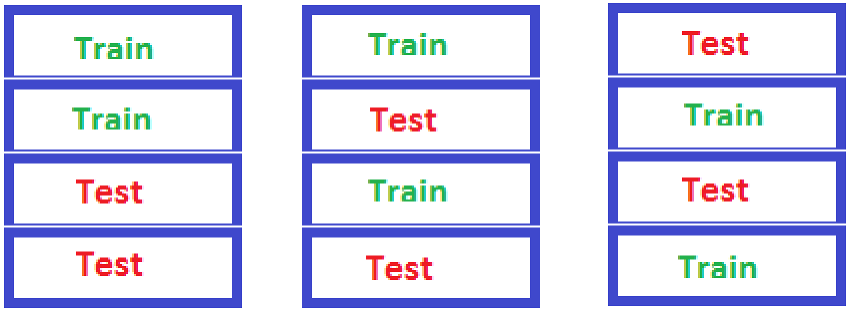
LLT is the large loss threshold

- (If a maximization routine cannot be run, a grid search can be used instead)

Calculating the Parameter Between Variance

- Options:
  - Buhlmann-Straub formulas <u>cannot</u> be used for this
  - Use specialized Bayesian software (JAGS, Stan)
  - Cross Validation (can even be implemented in Excel)

Cross Validation



- Test different possible values for the between variances
- For each, fit the model on a fraction of the data
- Using the fitted parameters, calculate the log-likelihood (without the prior) on the remainder of the data
- Repeat using different training/testing sets
- Plotting the average log-likelihoods from the test data should yield a smooth curve – otherwise more iterations may be needed
- Select the variances with the maximum cross validated likelihood
- Important: For this to work, the same train and test data should be used on each of the tested variance values

Severity Development

- Since larger losses take longer to settle on average, severity development needs to be taken into account when considering the excess losses
- Some simple options:
  - Assume development affects all layers the same – derive and apply severity development factors to the losses and to the large loss threshold
  - Use the same method that was used to apply severity development to the ILF curve – credibility weight the losses against the undeveloped portfolio parameters, and then apply the adjustment to the result
  - Many others…

Distributions with more than 2 parameters

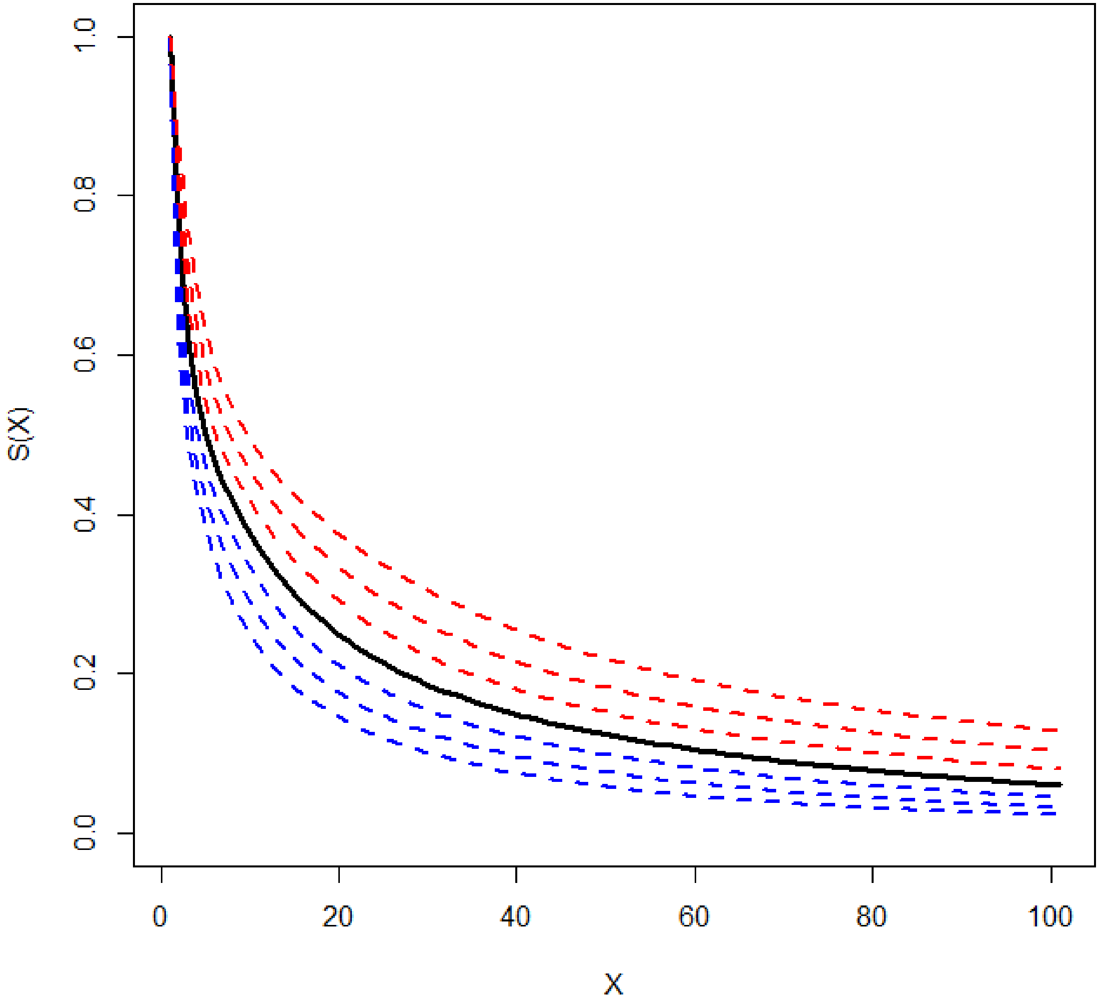
- Add two additional “adjustment” parameters that modify the severity distribution
- For a mixed distribution, for example:
  - Have one adjustment parameter apply a scale adjustment
  - Have the other shift the weights back and forth, which will affect the tail

$$\theta_i' = \theta_i \times exp(Adj1)$$

$$R_i = W_i \times exp(i \times Adj2)$$

$$W_i' = R_i / \sum R$$

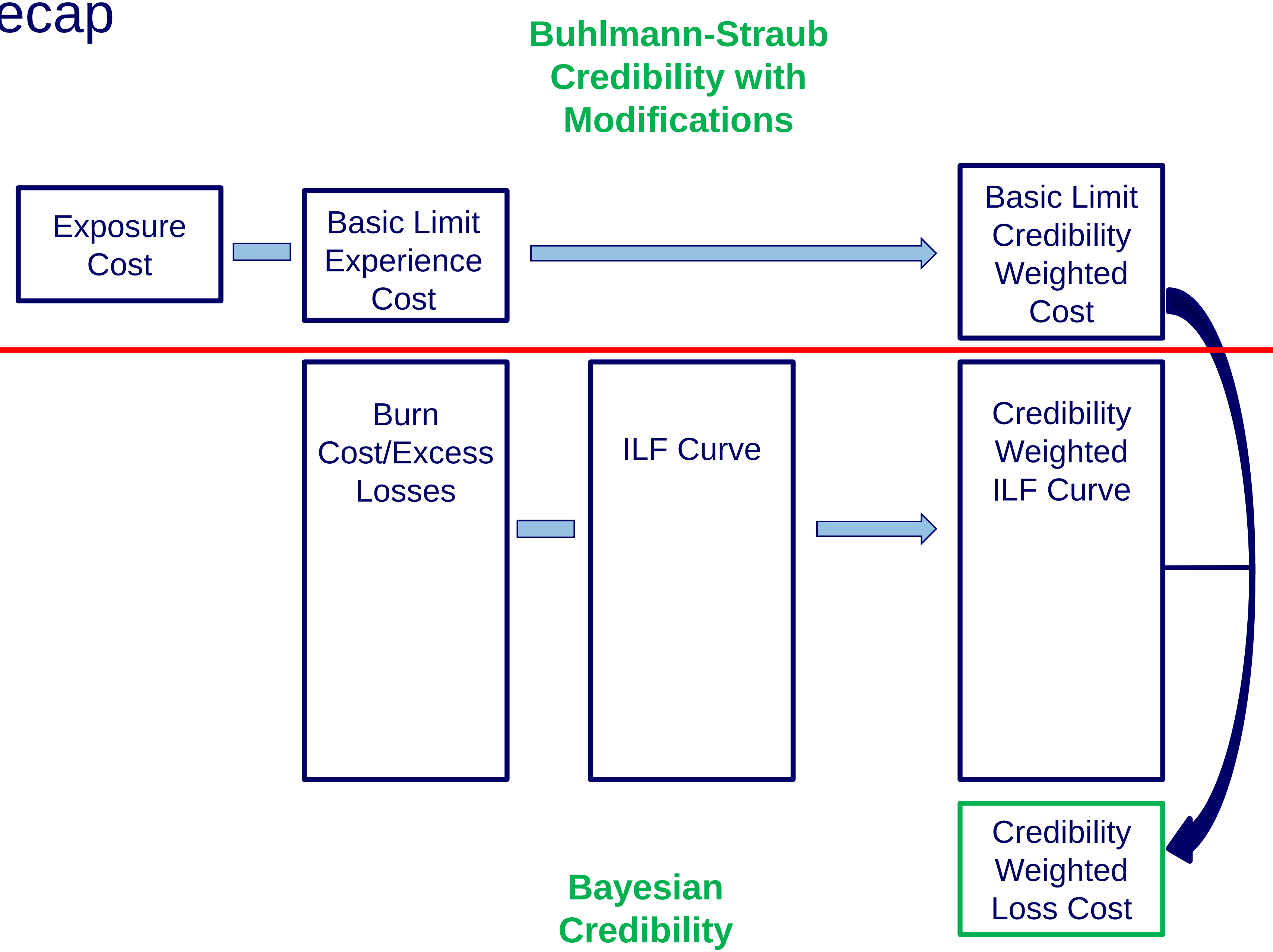
Scale adjustments



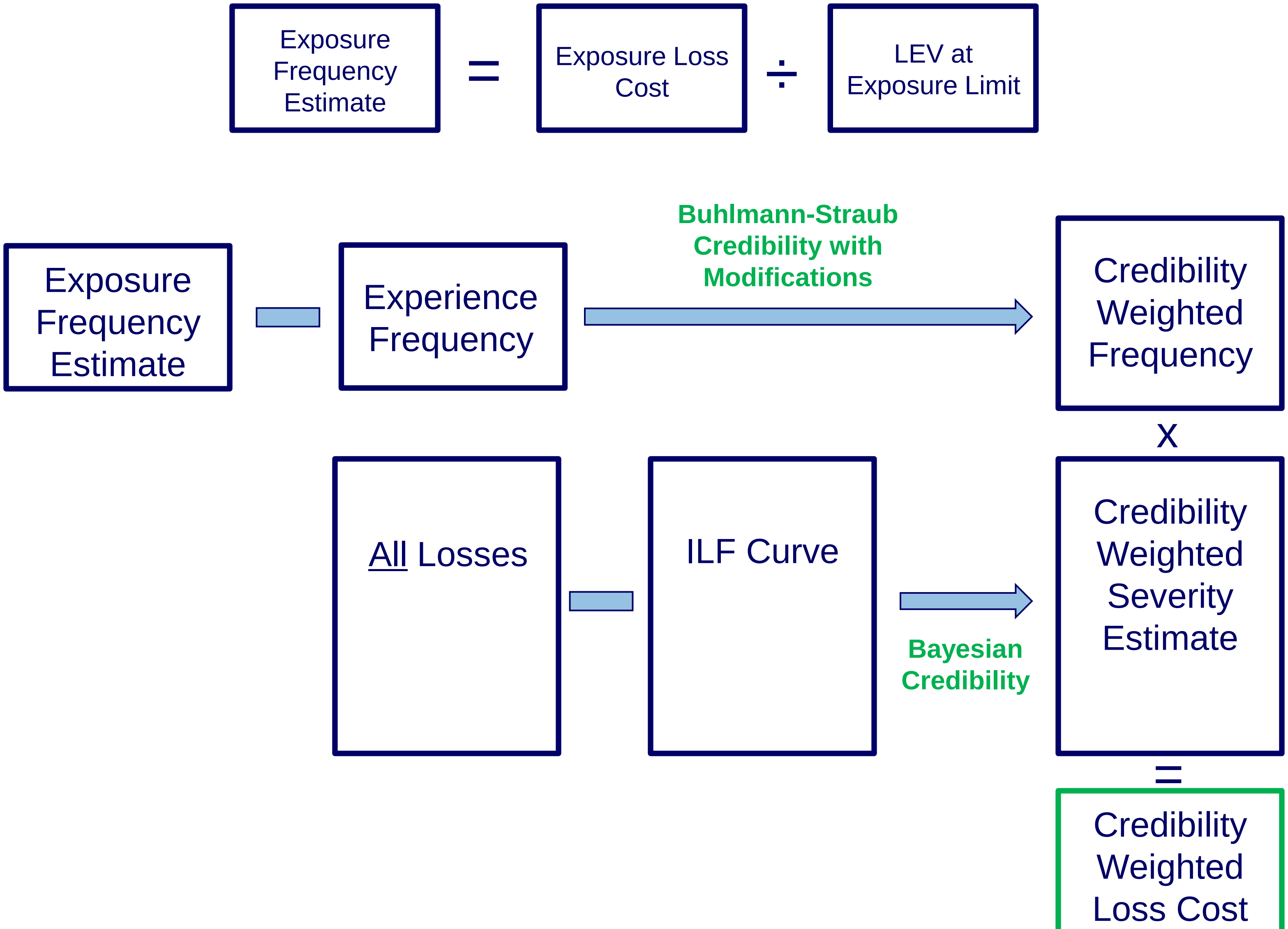
Other Issues

- Risk load: Remove from the ILF curve before credibility weighting and add back in afterwards
- Zero or legal only claims: If these are in the data, develop factors by year to remove
- Separate primary and excess distributions: Refer to the paper
- Log of zero rounding errors: Set a minimum value above zero before taking the logarithm

Recap


Buhlmann-Straub Credibility with Modifications
Exposure Cost
Basic Limit Experience Cost
Basic Limit Credibility Weighted Cost
Burn Cost/Excess Losses
ILF Curve
Credibility Weighted ILF Curve
Credibility Weighted Loss Cost
Bayesian Credibility

Going a step further – A Frequency/Severity Approach



This method does not require selection of a basic limit!

Revised Likelihood Formula to Incorporate All of the Data

$$\sum_{x=Claims > LLT} PDF(x, p1, p2) + n \times CDF(LLT, p1, p2) +$$

$$\boxed{Norm(Average\ Capped\ Severity, \mu, \sigma^2) +}$$

$$Norm(p1, Portfolio\ p1, Between\ Var1) + Norm(p2, Portfolio\ p2, Between\ Var2)$$

Where:

$$\mu = LEV(Basic\ Limit, p1, p2)$$

$$\sigma^2 = [\,LEV2(Basic\ Limit, p1, p2) - LEV(Basic\ Limit, p1, p2)^2\,] / m$$

Average Capped Severity is the average severity at the large loss threshold/basic limit calculated from the account's losses

n is the number of claims that do not exceed the large loss threshold

m is the total number of claims

PDF is the logarithm of the probability density function

CDF is the logarithm of the cumulative density function

Norm is the logarithm of the normal probability density function

LLT is the large loss threshold

LEV and *LEV2* and the limited expected value first and second moments respectively

Revisiting the Simulation

Layer	1) Portfolio	2) Burn Cost	3) Credibility Weighted Burn Cost	Credibility for #3	4) MLE – Full Credibility	5) Credibility Weighted MLE	Cred Burn Cost vs Port (3 vs 1)	Cred MLE vs Port (5 vs 1)
100K xs 100K	35.6%	39.8%	26.5%	44.5%	36.0%	24.9%	-25.8%	-30.3%
100K xs 250K	43.2%	56.6%	34.5%	36.7%	48.6%	30.3%	-20.2%	-30.0%
100K xs 500K	50.2%	77.9%	42.5%	29.1%	63.1%	35.3%	-15.4%	-29.6%
100K xs 1M	58.3%	111.3%	52.0%	21.1%	84.4%	41.3%	-10.8%	-29.2%
100K xs 2M	67.3%	169.8%	62.3%	13.8%	118.7%	48.1%	-7.3%	-28.5%
100K xs 5M	80.8%	313.2%	77.5%	7.0%	206.3%	58.6%	-4.0%	-27.4%

Sharp Decline

Very Stable

Excess Layer Simulations: Lognormal, 2M xs 2M

Method	Bias: LEV Method	RMSE: LEV Method (Millions)	Bias: ILF Method	RMSE: ILF Method (Millions)	RMSE LEV Method - Relative to Portfolio ILF Method	RMSE ILF Method - Relative to Portfolio ILF Method
Portfolio	0.0%	3.05	**0.0%**	**2.06**	+48.2%	**0.0%**
Credibility - Aggregate, Including Capped Sum	**-0.3%**	**1.43**	2.8%	1.49	**-30.5%**	-27.7%
Credibility - Aggregate, NOT Including Capped Sum	2.1%	1.46	**3.6%**	**1.47**	-29.2%	**-28.7%**

- 25 group up claims
- Large loss threshold of 200K
- See paper for more details

Excess Layer Simulations: Mixed Exponential, 10M xs 10M

Method	Bias: LEV Method	RMSE: LEV Method (Millions)	Bias: ILF Method	RMSE: ILF Method (Millions)	RMSE LEV Method - Relative to Portfolio ILF Method	RMSE ILF Method - Relative to Portfolio ILF Method
Portfolio	0.0%	3.53	**-0.8%**	**2.68**	+31.8%	**0.0%**
Credibility - Aggregate, Including Capped Sum	**1.5%**	**2.06**	4.6%	2.17	**-23.0%**	-19.1%
Credibility - Aggregate, NOT Including Capped Sum	3.8%	2.08	**5.2%**	**2.11**	-22.3%	**-21.2%**

- 25 group up claims
- Large loss threshold of 200K
- See paper for more details

Extreme Value Theory for High Up Layers

Extreme Value Theory for High Up Layers

- For high up layers, smaller losses may be less relevant

- For cases when the policy layer is well above the account's experience, how relevant is the account's claim experience for modifying the severity distribution? Wouldn't this be an extrapolation?

  - Note, this is less of an issue when credibility weighting with the portfolio severity distribution (assuming this distribution includes losses approaching the policy layer)

  - Would really only be an issue if full credibility was given to the account's losses

Extreme Value Theory for High Up Layers

- Leverage Extreme Value Theory for estimating the policy layer severity and for determining which losses are relevant

- Based on the Peak Over Threshold method of Extreme Value Theory, excess severity potential can be estimated by fitting a Generalized Pareto Distribution (GPD) to the loss data above a certain threshold

- The GPD is usually a good fit to tail data

- The GPD can be extrapolated, unlike most other distributions which often yields questionable results
  - (Note that the Single-Parameter Pareto is a subclass of the GPD)

- In deciding what claims to include, there is a tradeoff between goodness of fit (higher threshold) vs including more data (lower threshold)
  - Looking at goodness of fit graphs can help decide which data to include (other methods as well)

Extreme Value Theory for High Up Layers

- Fit a GPD to the account's excess losses (ignoring those underneath)
  - Don't use the likelihood formulas above – just use the PDF. The GPD already assumes that it is excess of the threshold

- Is often a good fit to an account's higher losses (even if it is too low for the portfolio)

- Any distribution can be used for the portfolio

- To enable the use of any distribution for the portfolio as well as different thresholds across accounts, the GPD parameters can be reparameterized

$$p1 = log(\, f(t_1) \,/\, s(t_1)\,)$$

$$p2 = log(\, f(t_1) \,/\, s(t_1) \;-\; f(t_2) \,/\, s(t_2)\,)$$

$$loglik = \sum_i GPD(\, x_i, \alpha, \beta, threshold\,) + N(\, p1, h\,\alpha v1\,) + N(\, p2, h\,\alpha v2\,)$$

Where GPD() and N() are the logarithms of the PDFs

- Refer to the paper for further details and explanation of this method

Conclusion

- Method to incorporate all relevant information in statistically robust way
- Avoids use of burn costs
- Eliminates the need to develop each layer separately
- Does not require an arbitrary selection of a basic limit