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GLM II: Basic Modeling Strategy

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Overview

Basic GLM Specification

$$g(\mathbb{E}[y]) = \beta_0 + x_1\beta_1 + \cdots + x_k\beta_k + \text{offset}$$

1. The link function is g
2. The distribution of y is a member of the exponential family
3. The explanatory variables x_i may be continuous or discrete
4. The offset term can be used to adjust for exposure or to introduce known restrictions

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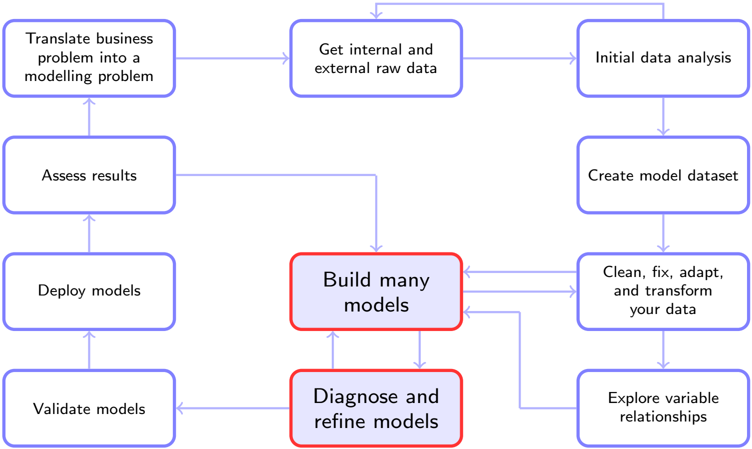
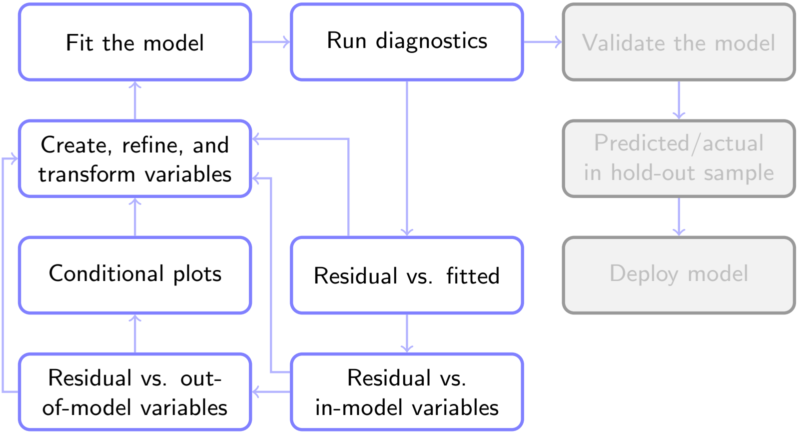
$$\mathbb{E}[y] = g^{-1}\left(\beta_0 + x_1\beta_1 + \cdots + x_k\beta_k + \text{offset}\right)$$

Overall Project Cycle



Model Building Cycle



Personal Auto Claims

The dataset contains $67,856$ policies taken out in 2004 or 2005. This is the `car.csv` dataset featured in the book by de Jong & Heller [3].

The available variables are:

1. Driver age
2. Gender
3. Garage location
4. Vehicle body
5. Vehicle age
6. Vehicle value (∞)
7. Exposure (∞)
8. Claim?
9. Number of claims
10. Total claim cost (∞)

(∞) denotes a continuous variable. All other variables are categorical or counts.

Variable Descriptions

Variable	Type	Comments
Driver Age	Cat	$1 =$ youngest, $2, \ldots, 6 =$ oldest
Gender	Cat	F = Female, M = Male
Garage Location	Cat	A, B, C, D, E, F
Vehicle Body	Cat	13 classes
Vehicle Age	Cat	1 to 4 = oldest
Vehicle Value	Cont	range: 0 to 34.56, in units of \$10K
Exposure	Cont	range: 0.003 to 0.999
Claim?	Cat	0 = no claim, 1 = claim
Number of Claims	Count	$0, 1, 2, 3, 4$
Total Claim Cost	Cont	range: \$0 to $\$55,922$

Exploratory Analysis

- Tabular summaries
- Univariate exploration (along with exposure)
- Bivariate relationships
- Correlations

Preparing to Stay Honest

Take precautions to make sure that the results achieved are actually worth having. To this end split your data into three sets:

1. *Build*: used to create many models
2. *Test*: used to check intermediate models
3. *Validate*: used only once to check your final model

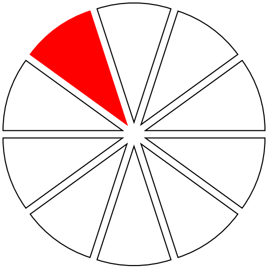
One rule of thumb: $(50\%, 25\%, 25\%)$.

Set	Records
Build	33,928
Test	16,964
Validate	16,964
Total	67,856

Preparing to Stay Honest

What if you don't have a large dataset that would allow you to split it in three segments (Build, Test, Validate)?

Use Cross-Validation!



Summary Statistics for Build Dataset

Continuous Variables

	total claim cost	exposure	veh.value
Min. :	0.0	0.003	0.000
1st Qu.:	0.0	0.219	1.010
Median :	0.0	0.446	1.500
Mean :	143.4	0.469	1.777
3rd Qu.:	0.0	0.709	2.150
Max. :	55920.0	0.999	34.560

Vehicle value is in units of $10,000.

Summary Statistics for Build Dataset

Categorical Variables (record counts)

```
   veh.body    veh.age    area
 SEDAN:11149   1: 6017    A: 8216
 HBACK: 9372   2: 8332    B: 6603
 STNWG: 8114   3:10126    C:10344
 UTE  : 2351   4: 9453    D: 4035
 TRUCK:  886              E: 2971
 HDTOP:  770              F: 1759
 COUPE:  396
 PANVN:  378
 MIBUS:  373
 MCARA:   60
 CONVT:   37
 BUS  :   27
 RDSTR:   15
```

Summary Statistics for Build Dataset

Categorical Variables (record counts)

age.cat	gender	claim?	claim count
1:2852	F:19264	No :31599	0:31599
2:6501	M:14664	Yes: 2329	1: 2185
3:7971			2: 133
4:8086			3: 10
5:5290			4: 1
6:3228			

Summary Statistics for Build Dataset

Categorical Variables (record counts)

```
                                    claim
age.cat   gender     claim?         count
 1:2852  F:19264  No :31599       0:31599
 2:6501  M:14664  Yes: 2329       1: 2185
 3:7971                           2:  133
 4:8086                           3:   10
 5:5290                           4:    1
 6:3228
```

What is the claim frequency?

A naive GLM model for Claim Counts

```
Call: glm(formula = num.claims ~ 1,
           family = poisson(link = "log"),
            data = car[b.idx, ])

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.61397    0.02006  -130.3   <2e-16 ***

    Null deviance: 13437 on 33927 degrees of freedom
Residual deviance: 13437 on 33927 degrees of freedom
```

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$$e^{-2.61397} = 0.0732$$

How to adjust for Exposure?

For a frequency model with a log-link we have

$$\log\left(\frac{\mathbb{E}[\text{counts}]}{\text{exposure}}\right) = \text{linear predictor}$$

$$\log\left(\mathbb{E}[\text{counts}]\right) = \text{linear predictor} + \underbrace{\log\left(\text{exposure}\right)}_{\text{offset term}}$$

A simple GLM model for Claim Counts

```
Call: glm(formula = num.claims ~ 1,
           family = poisson(link = "log"),
             data = car[b.idx, ],
           offset = log(exposure))

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.85591    0.02006  -92.52   <2e-16 ***

    Null deviance: 12864 on 33927 degrees of freedom
Residual deviance: 12864 on 33927 degrees of freedom
```

$$e^{-1.85591} = 0.1563 = \frac{2485}{15897.84}$$

Continue with Paul's presentation

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