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GLM II: Basic Modeling Strategy

Ernesto Schirmacher

Liberty Mutual Insurance

Casualty Actuarial Society
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Overview

Quick Review of GLMs

Project Cycle

Modeling Cycle

Personal Auto Claims Example

Exploratory Analysis

Build, Test, Validate

Exposure Adjustments

Basic GLM Specification

$$g(\mathbb{E}[y]) = \beta_0 + x_1\beta_1 + \cdots + x_k\beta_k + \text{offset}$$

1. The link function is g
2. The distribution of y is a member of the exponential family
3. The explanatory variables x_i may be continuous or discrete
4. The offset term can be used to adjust for exposure or to introduce known restrictions

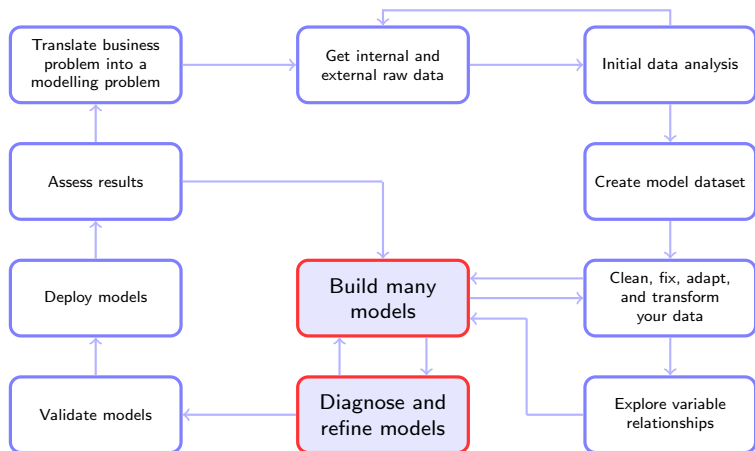
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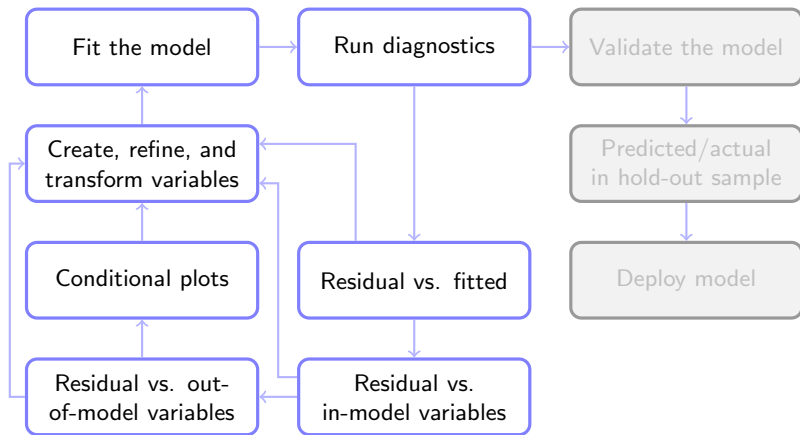
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$$\mathbb{E}[y] = g^{-1}(\beta_0 + x_1\beta_1 + \cdots + x_k\beta_k + \text{offset})$$

Overall Project Cycle



Model Building Cycle



Personal Auto Claims

The dataset contains 67,856 policies taken out in 2004 or 2005. This is the `car.csv` dataset featured in the book by de Jong & Heller [3].

The available variables are:

1. Driver age
2. Gender
3. Garage location
4. Vehicle body
5. Vehicle age
6. Vehicle value (∞)
7. Exposure (∞)
8. Claim?
9. Number of claims
10. Total claim cost (∞)

(∞) denotes a continuous variable. All other variables are categorical or counts.

Variable Descriptions

Variable	Type	Comments
Driver Age	Cat	1 = youngest, 2, . . . , 6 = oldest
Gender	Cat	F = Female, M = Male
Garage Location	Cat	A, B, C, D, E, F
Vehicle Body	Cat	13 classes
Vehicle Age	Cat	1 to 4 = oldest
Vehicle Value	Cont	range: 0 to 34.56, in units of \$10K
Exposure	Cont	range: 0.003 to 0.999
Claim?	Cat	0 = no claim, 1 = claim
Number of Claims	Count	0, 1, 2, 3, 4
Total Claim Cost	Cont	range: \$0 to \$55,922

Exploratory Analysis

- ▶ Tabular summaries
- ▶ Univariate exploration (along with exposure)
- ▶ Bivariate relationships
- ▶ Correlations

Preparing to Stay Honest

Take precautions to make sure that the results achieved are actually worth having. To this end split your data into three sets:

1. *Build*: used to create many models
2. *Test*: used to check intermediate models
3. *Validate*: used only once to check your final model

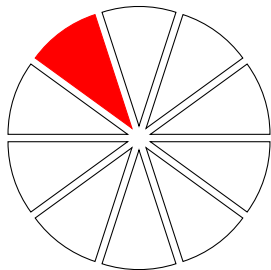
One rule of thumb: (50%, 25%, 25%).

Set	Records
<i>Build</i>	33,928
<i>Test</i>	16,964
<i>Validate</i>	16,964
Total	67,856

Preparing to Stay Honest

What if you don't have a large dataset that would allow you to split it in three segments (Build, Test, Validate)?

Use Cross-Validation!



Summary Statistics for Build Dataset

Continuous Variables

	total	claim	cost	exposure	veh.value
Min.	:	0.0	0.003	0.000	
1st Qu.:		0.0	0.219	1.010	
Median	:	0.0	0.446	1.500	
Mean	:	143.4	0.469	1.777	
3rd Qu.:		0.0	0.709	2.150	
Max.	:	55920.0	0.999	34.560	

Vehicle value is in units of \$10,000.

Summary Statistics for Build Dataset

Categorical Variables (record counts)

veh.body	veh.age	area
SEDAN:11149	1: 6017	A: 8216
HBACK: 9372	2: 8332	B: 6603
STNWG: 8114	3:10126	C:10344
UTE : 2351	4: 9453	D: 4035
TRUCK: 886		E: 2971
HDTOP: 770		F: 1759
COUPE: 396		
PANVN: 378		
MIBUS: 373		
MCARA: 60		
CONVT: 37		
BUS : 27		
RDSTR: 15		

Summary Statistics for Build Dataset

Categorical Variables (record counts)

			claim
age.cat	gender	claim?	count
1:2852	F:19264	No :31599	0:31599
2:6501	M:14664	Yes: 2329	1: 2185
3:7971			2: 133
4:8086			3: 10
5:5290			4: 1
6:3228			

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What is the claim frequency?

A naive GLM model for Claim Counts

```
Call: glm(formula = num.claims ~ 1,
          family = poisson(link = "log"),
          data = car[b.idx, ])
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-2.61397	0.02006	-130.3	<2e-16 ***

Null deviance: 13437 on 33927 degrees of freedom

Residual deviance: 13437 on 33927 degrees of freedom

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$$e^{-2.61397} = 0.0732$$

How to adjust for Exposure?

For a frequency model with a log-link we have

$$\log \left(\frac{\mathbb{E}[\text{counts}]}{\text{exposure}} \right) = \text{linear predictor}$$

$$\log (\mathbb{E}[\text{counts}]) = \text{linear predictor} + \underbrace{\log (\text{exposure})}_{\text{offset term}}$$

A simple GLM model for Claim Counts

```
Call: glm(formula = num.claims ~ 1,
          family = poisson(link = "log"),
          data = car[b.idx, ],
          offset = log(exposure))
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-1.85591	0.02006	-92.52	<2e-16 ***




Null deviance: 12864 on 33927 degrees of freedom

Residual deviance: 12864 on 33927 degrees of freedom





$$e^{-1.85591} = 0.1563 = \frac{2485}{15897.84}$$

Continue with Pong's presentation

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