A Simple Method for Modeling Change Over Time

Uri Korn

Ratemaking, Product, and Modeling Seminar
March 25 – 27, 2019

Changes

- How do you incorporate possible changes when loss rating large accounts?

- (Will discuss profitability studies and pricing models as well.)

Change?

- Talking about changes after adjusting for trend

Change?

- Talking about changes after adjusting for trend

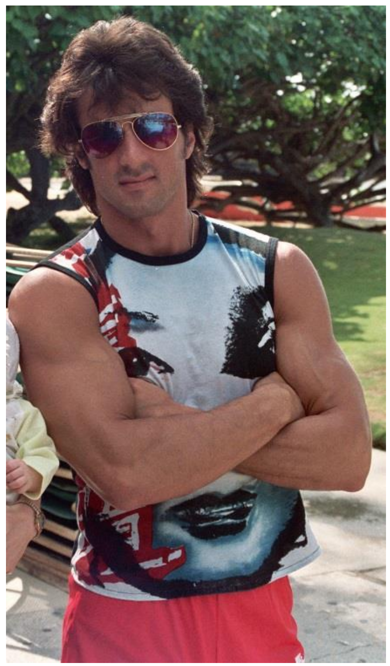
How do you handle changes: large account rating?

A) Ignore
B) By choosing the number of years to use
C) Ad hoc/examining the data
D) Exponential smoothing
E) Statistical model (e.g. ARIMA, etc.)

How do you handle changes: pricing models/profitability studies?

A) Ignore
B) By choosing the number of years to use
C) Ad hoc/examining the data
D) Adding year as a categorical variable in model
E) Exponential smoothing
F) Statistical model (e.g. ARIMA, etc.)

Because Things Change



[illegible]

Uri's Webpage!!!

Uri's Webpage!!!

Uri's Webpage!!!

Uri's Webpage!!!

Uri's Webpage!!!

Hobbies: Computers, Football

Uri's Webpage!!!

Hobbies: Computers, Football

Uri's Webpage!!!

Hobbies: Computers, Football

Pets: Fish (Axel)

Uri's Webpage!!!

Hobbies: Computers, Football

Pets: Fish (Axel)

Uri's Webpage!!!

Hobbies: Computers, Football

Pets: Fish (Axel)

Uri's Webpage!!!

Hobbies: Computers, Football

Pets: Fish (Axel)

Remember when webpages looked like this!?

What we need

- Powerful and simple

What we need

- Non-Gaussian errors
- Log link

What we need

- Works with "short" time series

What we need

- Works with big data

What we need

- Ability to handle credibility as well
  - To incorporate the basic limit exposure cost
  - Segmentations used in profitability studies
  - Relativities used in pricing models

What we need

- Robust
  - Handle volatile data
  - Small data changes shouldn’t result in big fitted value changes

What we need

- Suitable for presentation

What we need

- Intuitive and easy to explain

What we need

- Simple to implement – We got stuff to do

What we need

- Tastes great

What we need

- Low in sodium

What we need

- No trans fats

What we need

- Free shipping

What we need

- Great selection

What we need

- A supreme cardio-vascular workout!

Loss Development Methods

- Many different methods are available for loss development:
  - Bernheiter-Ferguson
  - Chain Ladder
  - Cape-Cod
  - Etc.

Loss Development Methods

- ~~Many different methods are available for loss development:~~
  - ~~Bernheiter-Ferguson~~
  - **Chain Ladder**
  - ~~Cape-Cod~~
  - ~~Etc.~~

- There is only one method!

Model Based Approach

- Inputs:
  - Target: Chain Ladder
  - Weights: Used Exposure (Exposure / LDF)

- If credibility/smoothing is applied to the changes, can result in BF or CL (or a credibility weighting of the two)

- Makes easier to analyze

Possible Solutions

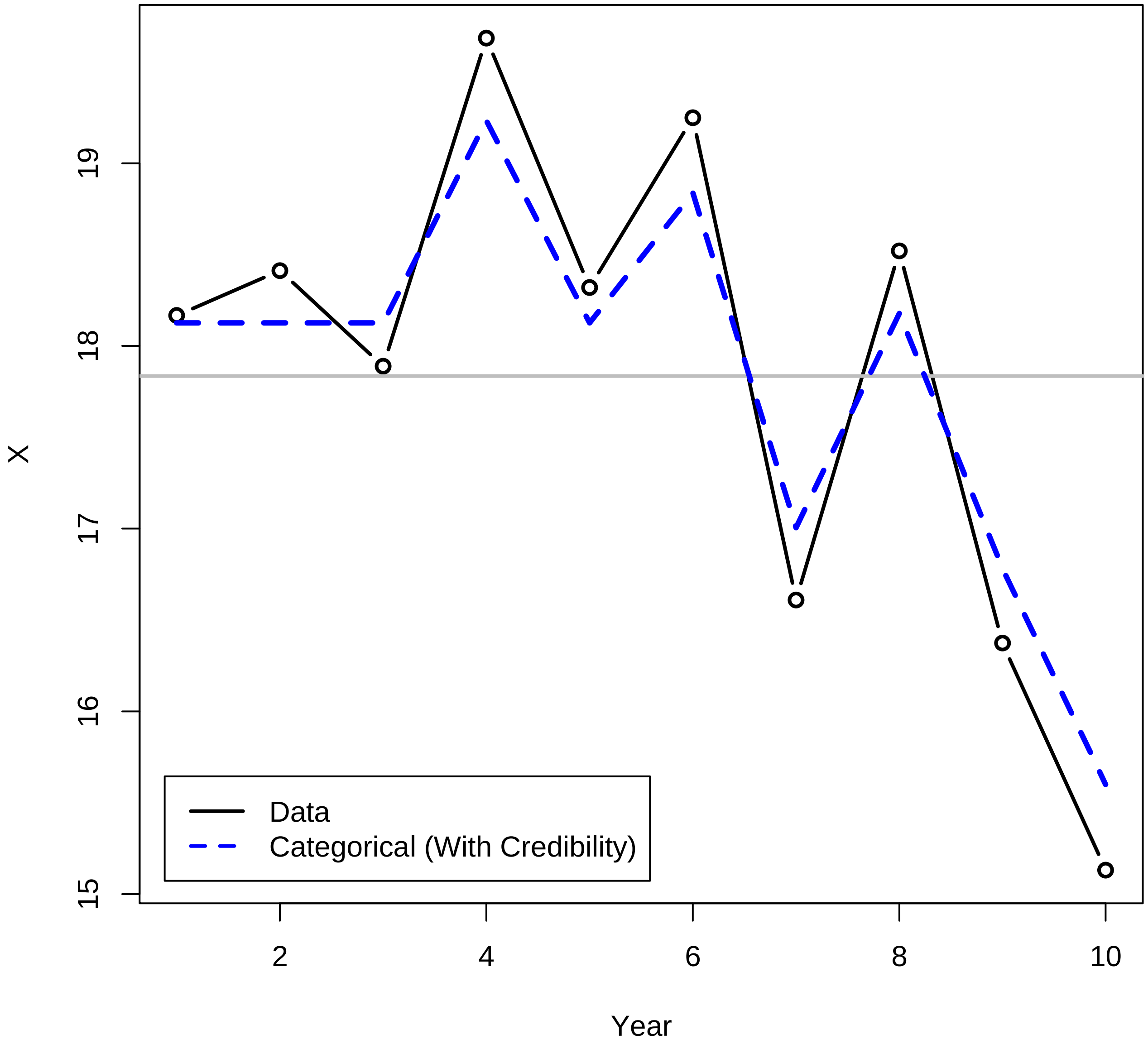
- Generalized Cape Cod (S. Gluck 1997)
  - Uses exponential smoothing to handle changes
  - Performs a BF method, selecting the a priori LR for each year using locally smoothed weights

  - Little guidance for selecting the smoothing parameter
  - Doesn't handle credibility weighting by segment

- This method separates out the development from the estimation of changes!

Possible Solutions

- Add year as a variable to the model (with credibility)



- Poor performance
- Non-intuitive

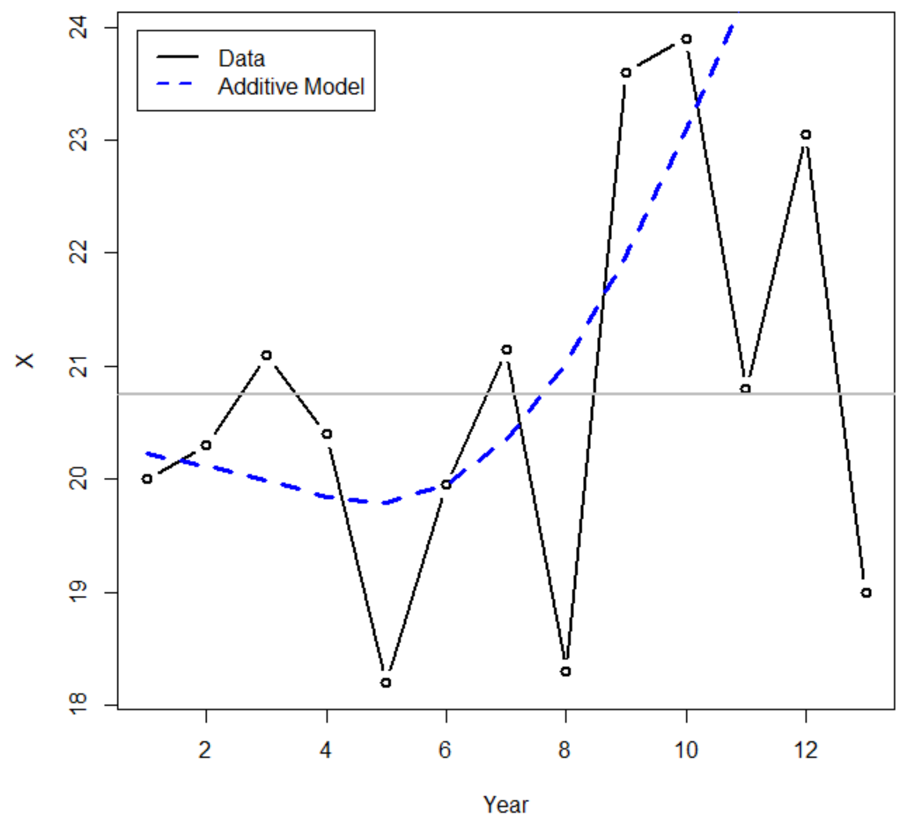
Possible Solutions

- ARIMAs
  - Robust time series method
  - Commonly used for forecasting

Possible Solutions

- ARIMAs
  - Robust time series method
  - Commonly used for forecasting
  - Behavior with ”short” time series?
  - Gaussian errors
  - Complex and non-intuitive
  - No credibility

Additive Models (Splines)



- Often show high trends at ends, that may not exist
- Very sensitive to data changes
-

State Space Models

- More simple and intuitive than ARIMAS
- Powerful, intuitive time series approach
- More modern time series approach
- No need to worry about stationarity, ACF plots, etc.

State Space Models!

The way of the future!

Trend (Drift) SSM

$$Y_t = X_t + e_t$$

$$X_t = X_{t-1} + u$$

- 'e' is an error term that is minimized
- 'u' is the trend/drift

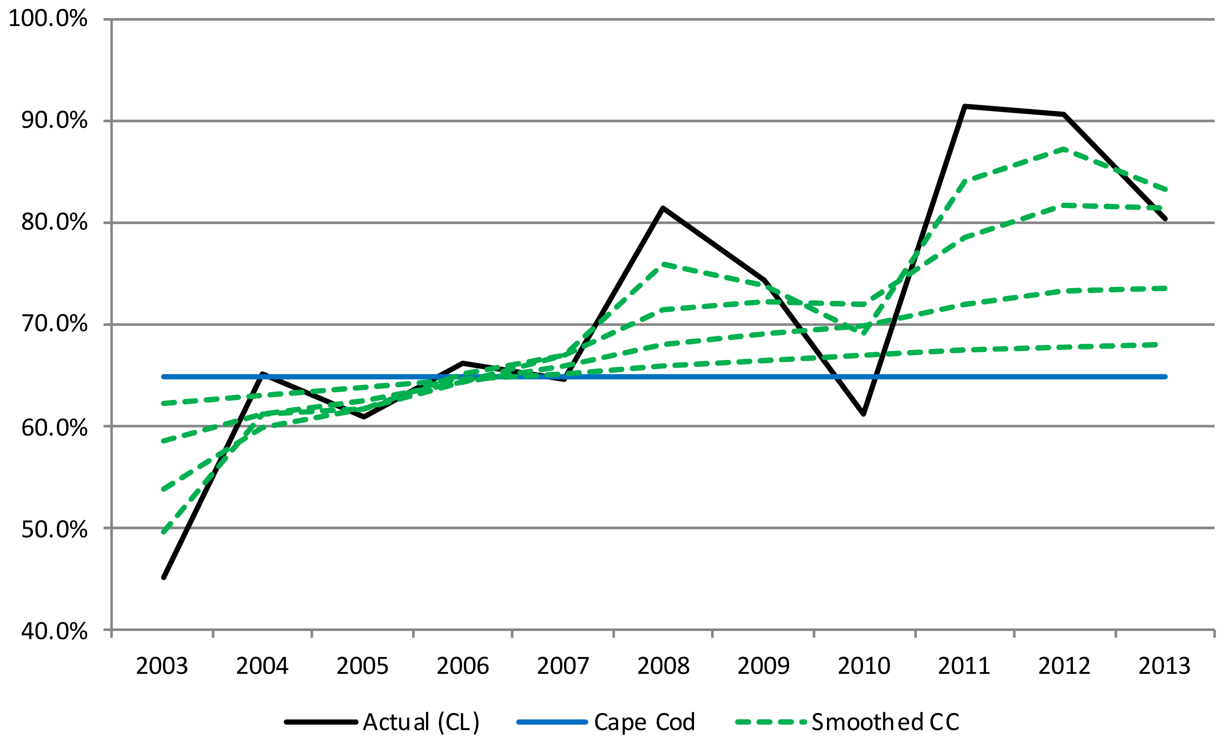
Random Walk SSM

$$Y_t = X_t + e_t$$

$$X_t = X_{t-1} + r_t$$

- ‘e’ and ‘r’ are error terms that are minimized
- Complement of credibility for each point is the <u>fitted</u> value of the previous point
-

Random Walk SSM



- Ratio of SD(e)/SD(r) determines the amount of smoothness/adapting to the data

SSM Fitting

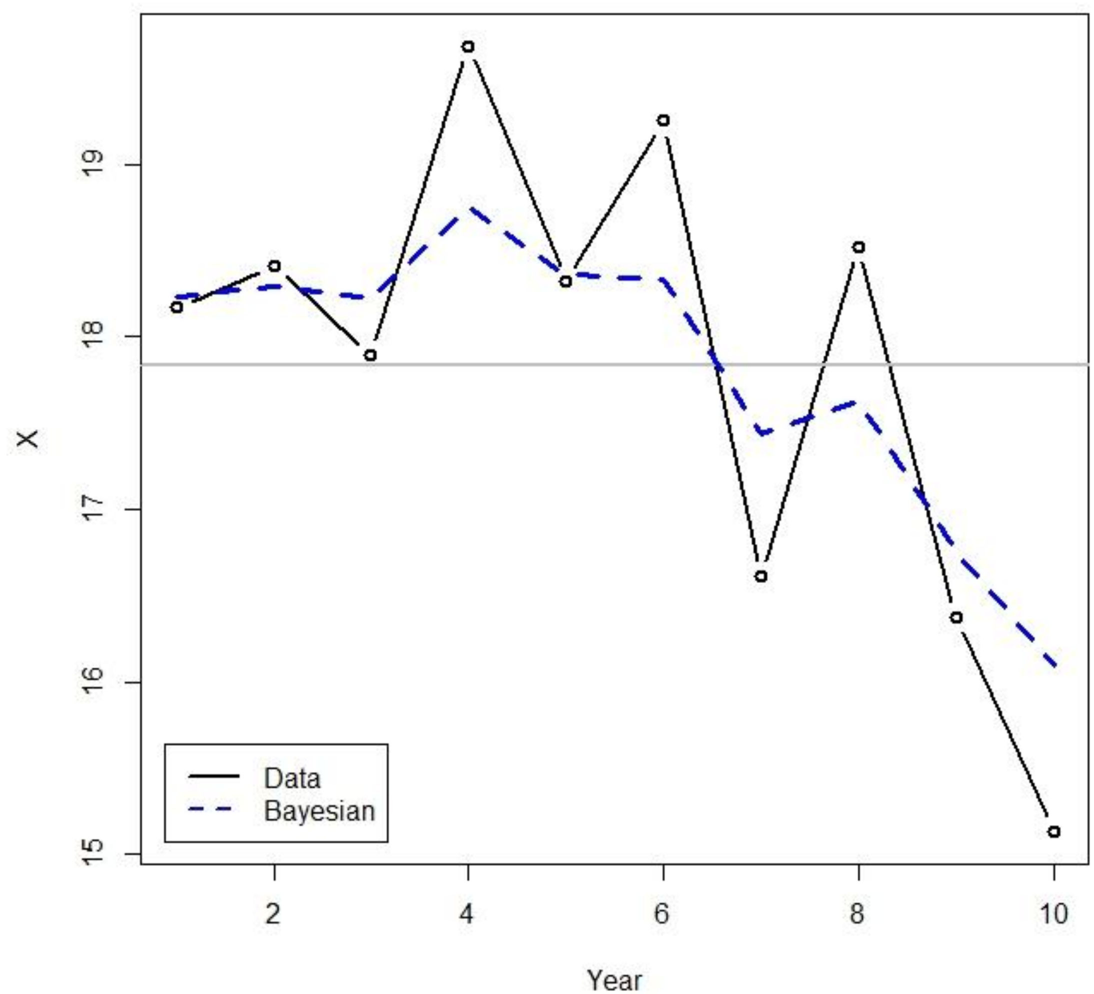
- Solving for the standard deviation of ‘e’ can be performed via maximum likelihood

- But how do we also solve for ’r’?
    - If we used maximum likelihood, the fit would adapt exactly to the data

SSM Fitting

- Bayesian
  - Uses MCMC simulations
  - Complex
  - Does not scale well
  - Not suitable for presentation

Bayesian Random Walk



SSM Fitting

- Kalman Filter
  - Uses look-ahead errors to solve for ‘r’
  - Complicated formulas
  - Gaussian errors
  - Not robust for ”short” time series

Penalized Regression

- Penalizes coefficients the more they deviate from zero – thus shrinking everything back towards the overall mean (similar to Mixed Models)

- K-fold cross validation used to determine penalty value

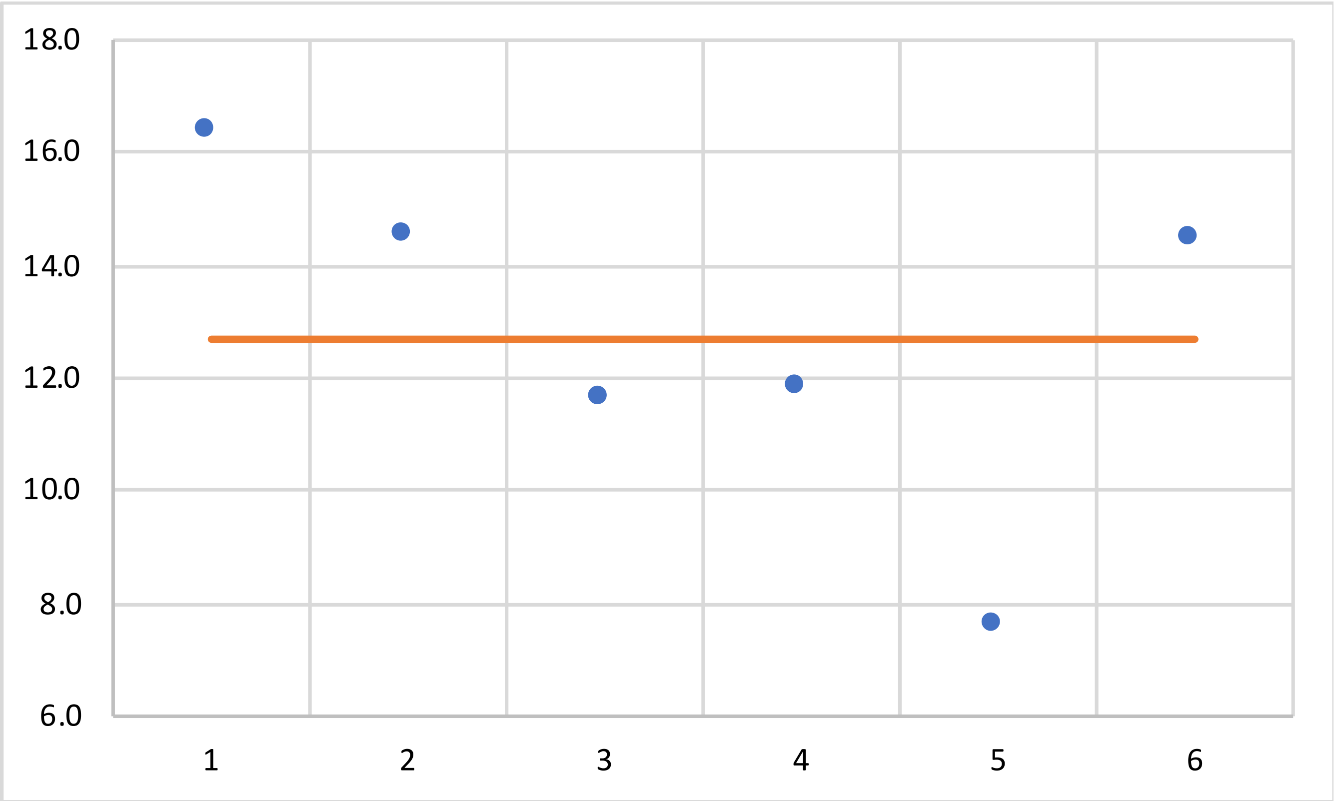
- Robust!

- Not normally used for time series

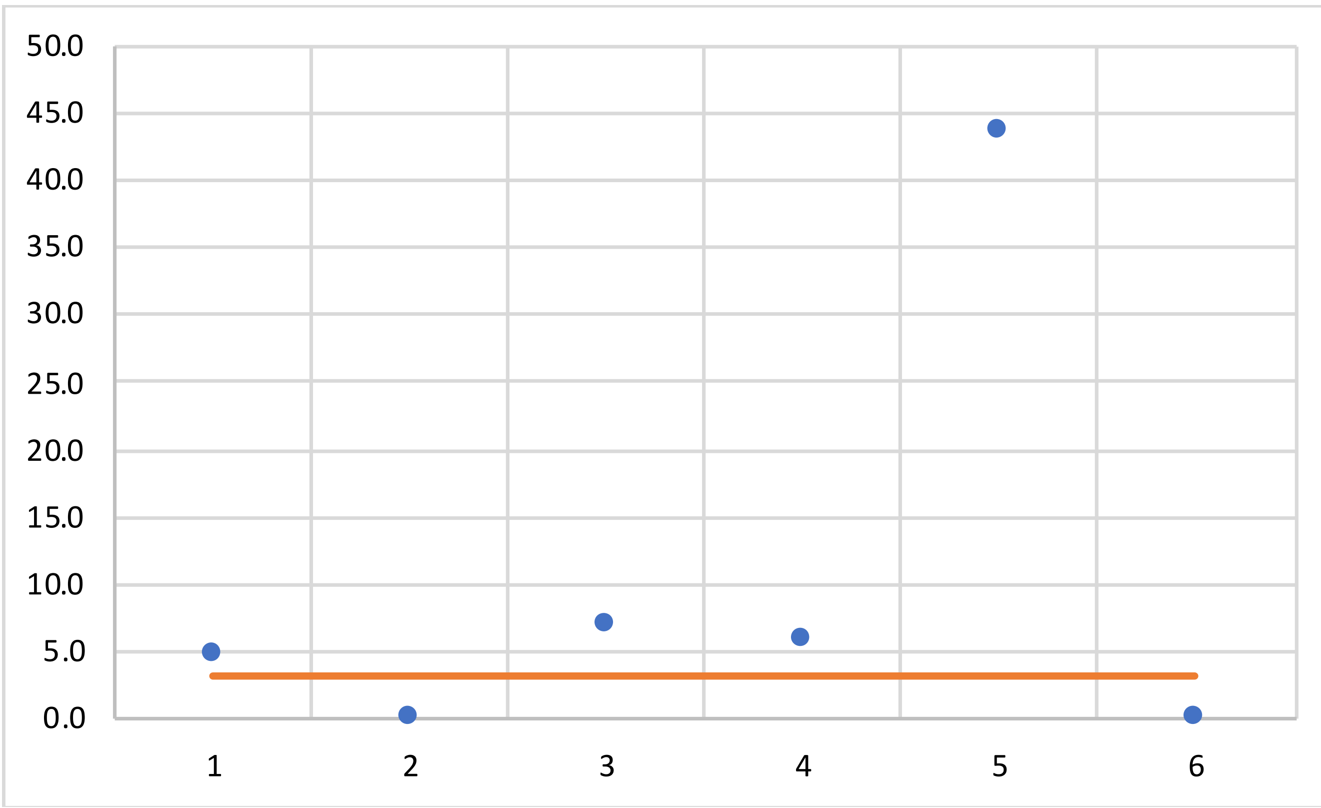
K-Fold Cross Validation

Run 1	Run 2	Run 3	Run 4	Run 5
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5

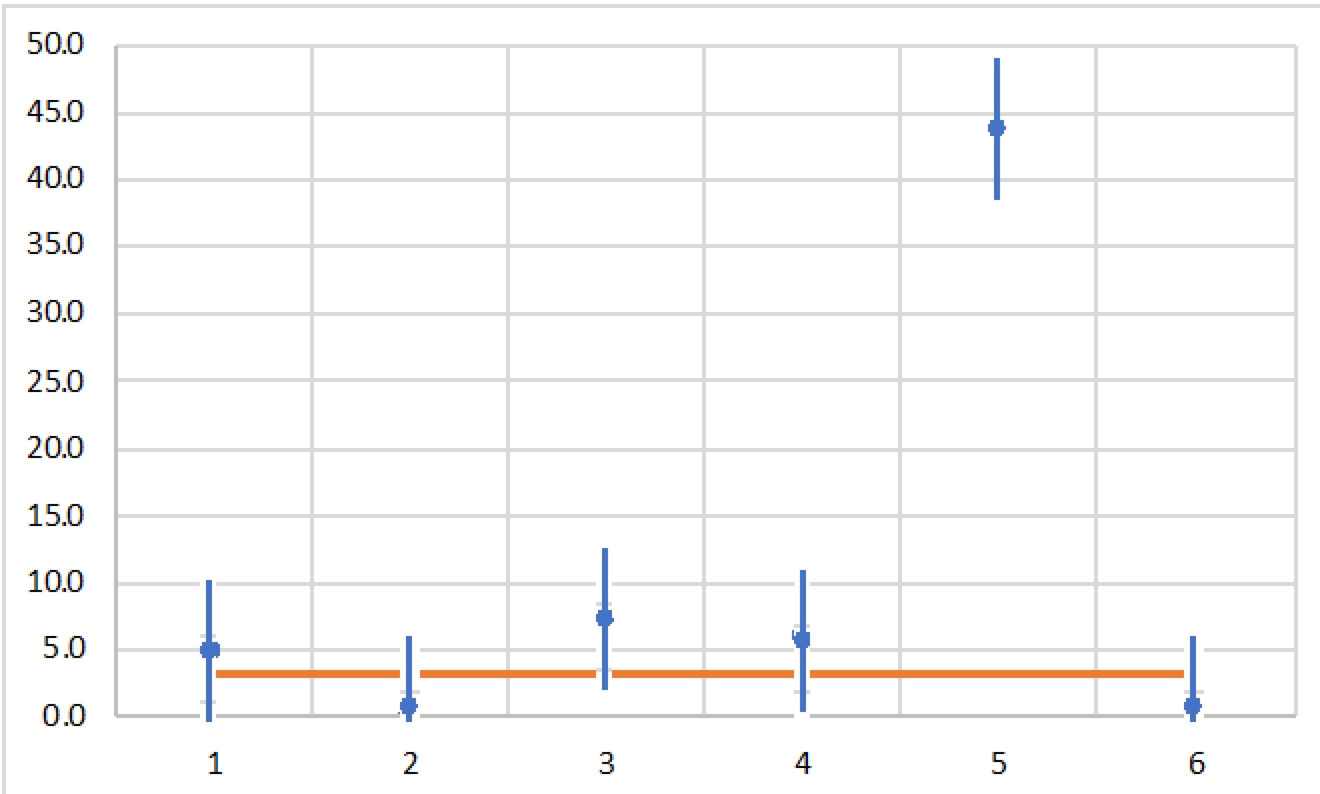
What is the mean?



What is the Variance?



What about now?



Intermission!

Proposed Approach

- Dummy encodings:

	2014	2015	2016
2013	0	0	0
2014	1	0	0
2015	0	1	0
2016	0	0	1

Proposed Approach

- Random walk dummy encodings:

	2014	2015	2016
2013	0	0	0
2014	1	0	0
2015	1	1	0
2016	1	1	1

- These equations will be equivalent to a random walk SSM (if some form of credibility is applied)
- (R allows the ability to change encodings – "contrasts")

Types of Penalized Regression

- Ridge
  - Penalty based on the square of the coefficient values
  - Equivalent to a normal prior (same as used in Mixed Models)

Types of Penalized Regression

- Ridge
  - Penalty based on the square of the coefficient values
  - Equivalent to a normal prior (same as used in Mixed Models)

- Lasso
  - Penalty based on the absolute value of the coefficient values
  - Also performs variable selection
  - Does not work well with correlated predictors

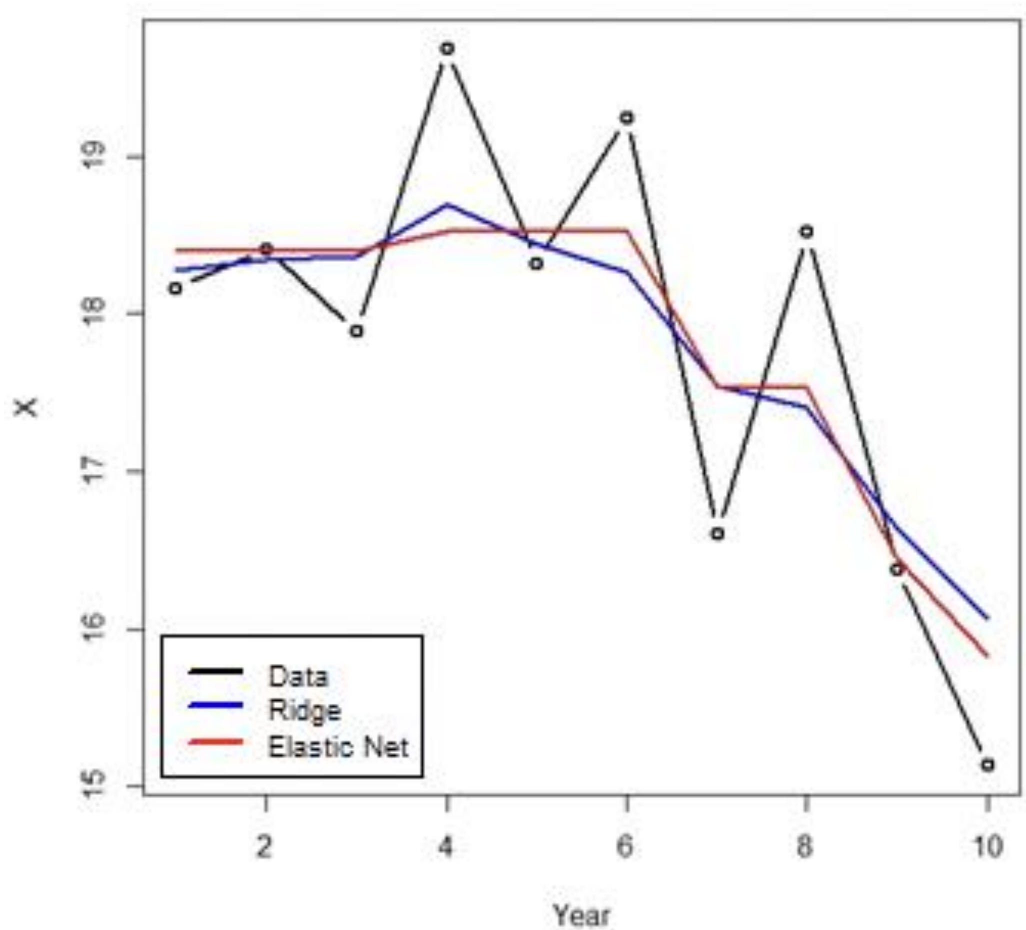
Types of Penalized Regression

- Ridge
  - Penalty based on the square of the coefficient values
  - Equivalent to a normal prior (same as used in Mixed Models)

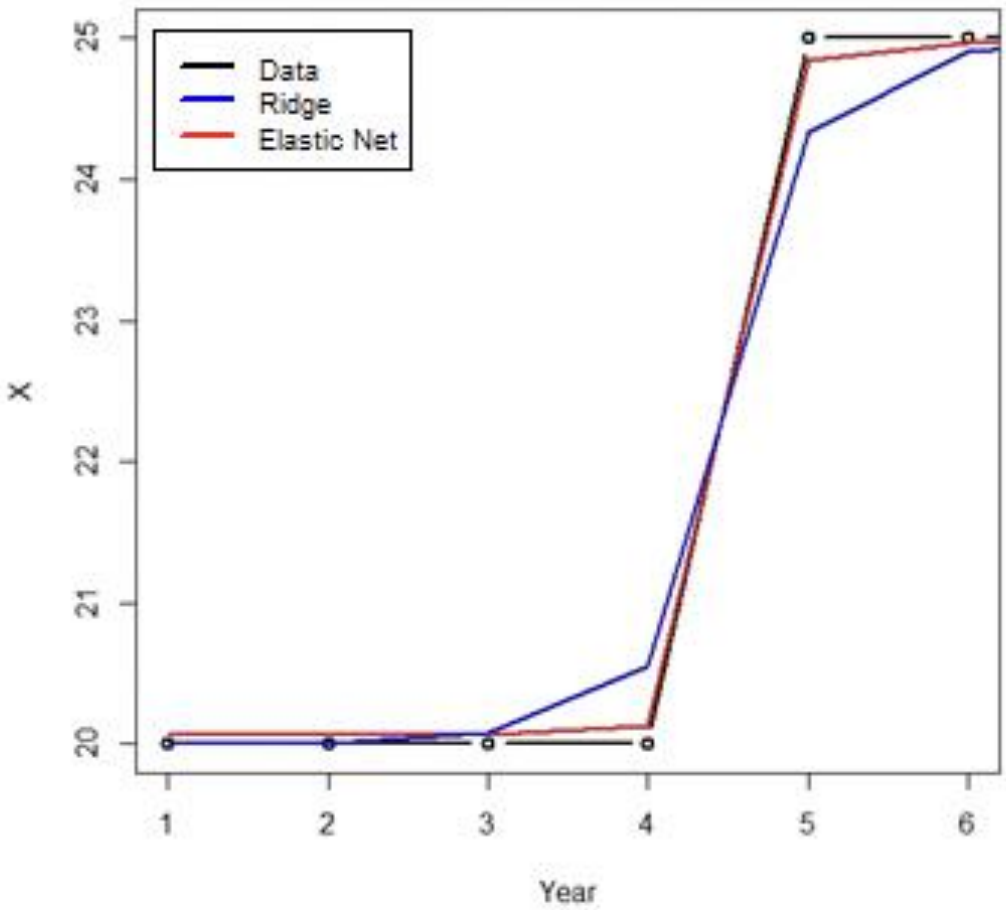
- Lasso
  - Penalty based on the absolute value of the coefficient values
  - Also performs variable selection
  - Does not work well with correlated predictors

- Elastic Net
  - Weighted average of ridge and lasso penalties
  - Best of both worlds – performs variable selection and works well with correlated predictors

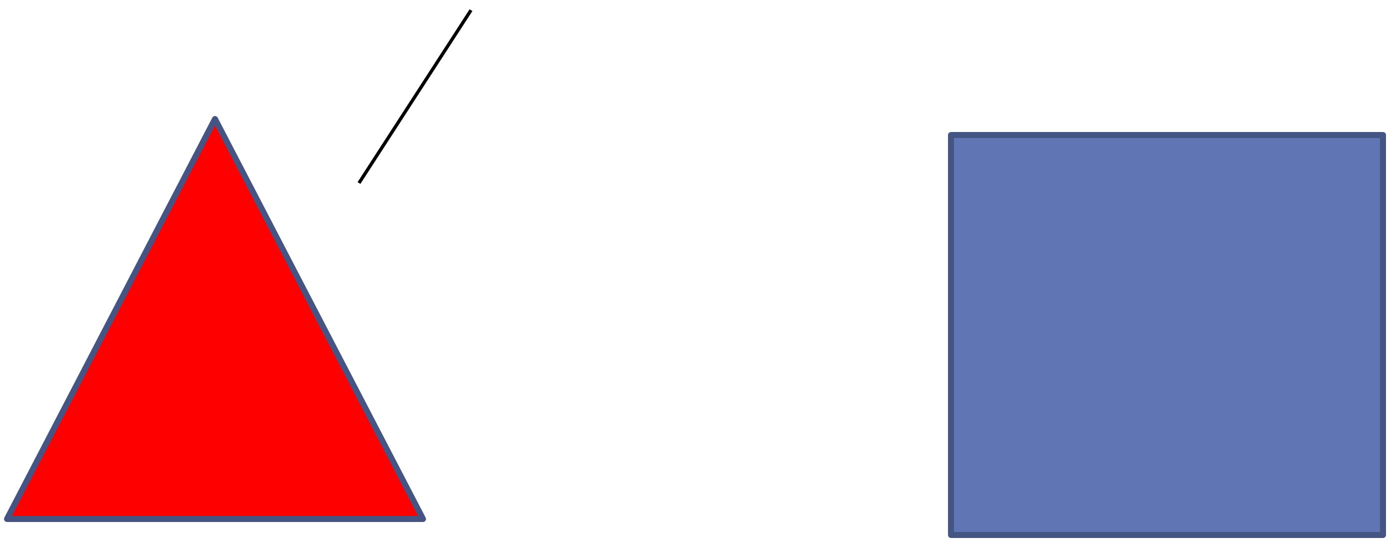
Types of Penalized Regression



Large Changes



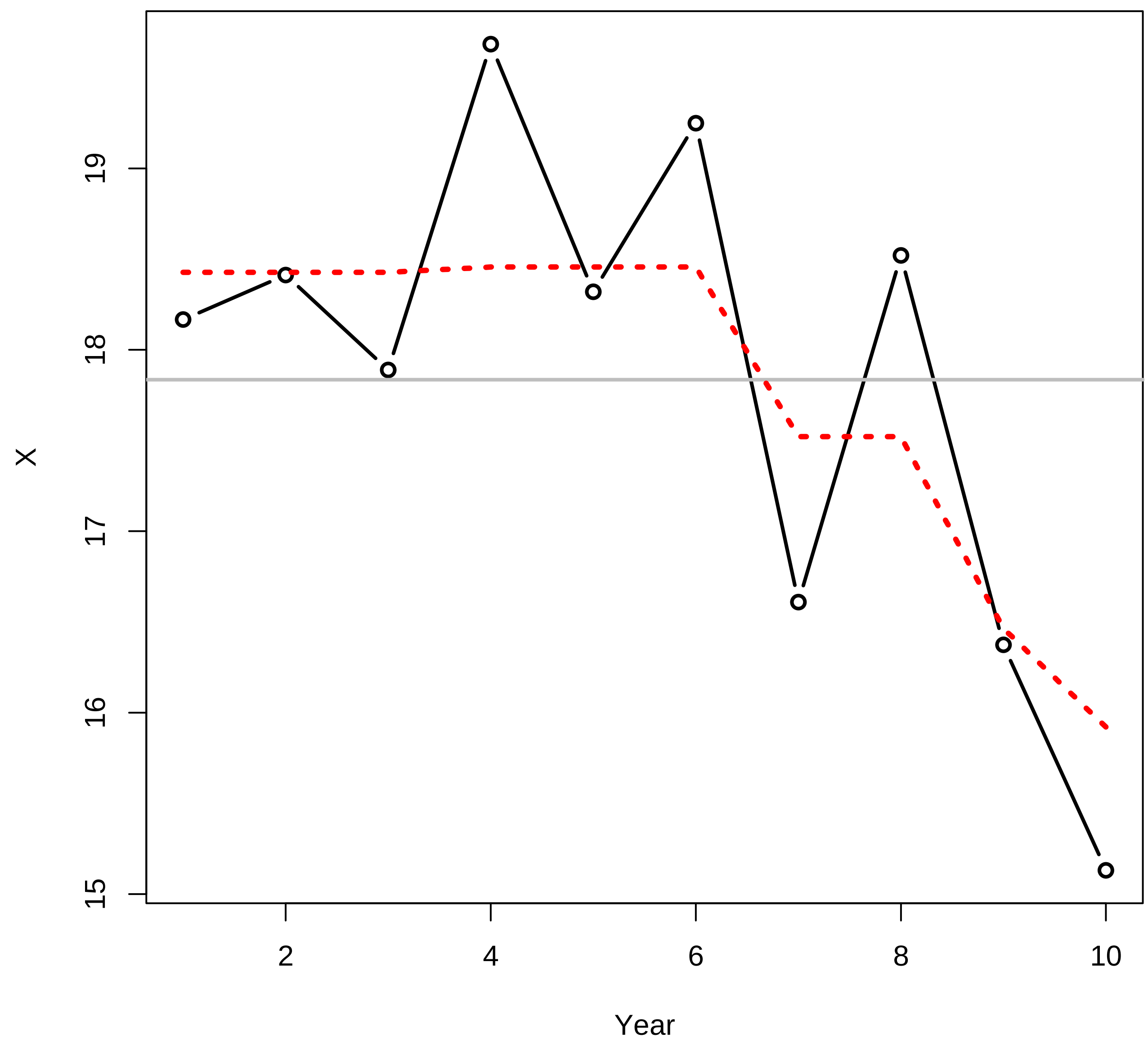
Come on, a little change
can be good.
Don’t be square.



Proposed Approach

- <u>Elastic Net Regression</u>
- **Response:** Chain ladder loss ratios/costs
- **Weights:** Used premiums/exposures

- Changes are determined scientifically and robustly
- Will give less weight to greener years
- Separates out loss development and estimation of changes
- Credibility Weighting between CL and BF



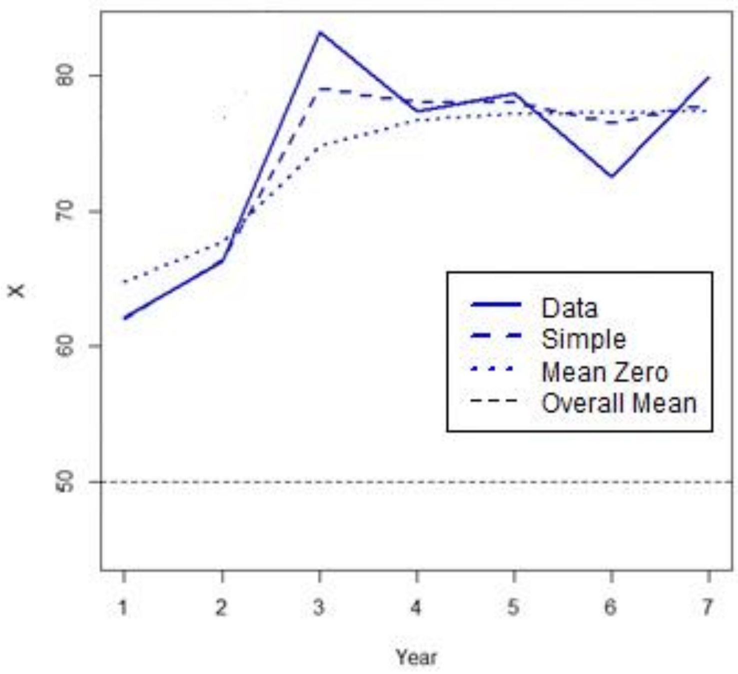
15
16
17
18
19
X
2
4
6
8
10
Year

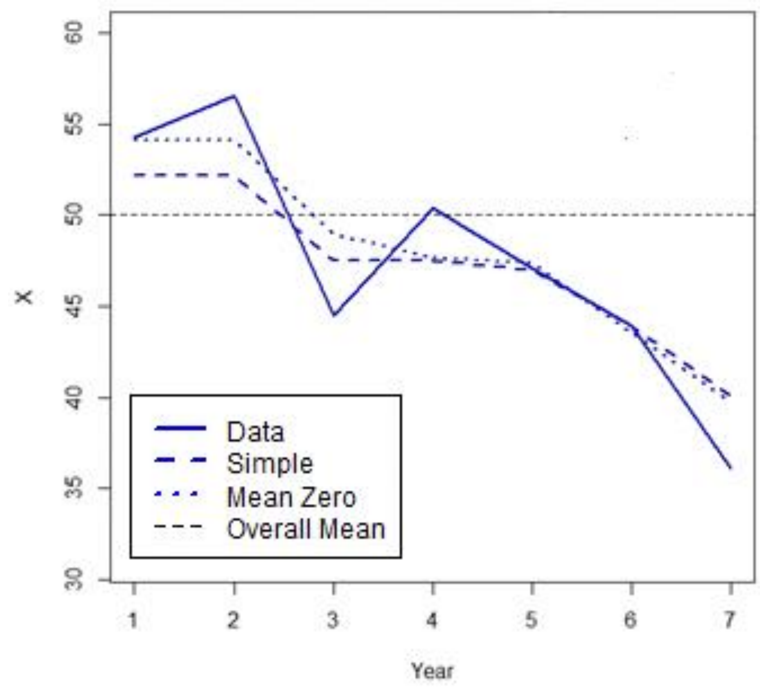
Multiple Segmentations

- Handle with interaction:

- **Formula:** Year + Segment + Segment:Year

Multiple Segmentations





- Solution is to set the mean of the encodings to zero
- This way the net effect of the random walk is zero, and so the segment coefficient represents the average values of the segments

Large Account Pricing

- **Formula:** Account + Account:Year (No intercept)
- **Offset:** Basic Limit Exposure Cost

- Determine credibility parameter(s) at the portfolio level

- Then fit the model to the account, using the selected credibility parameter

- Handles the basic limit credibility as well as changes!

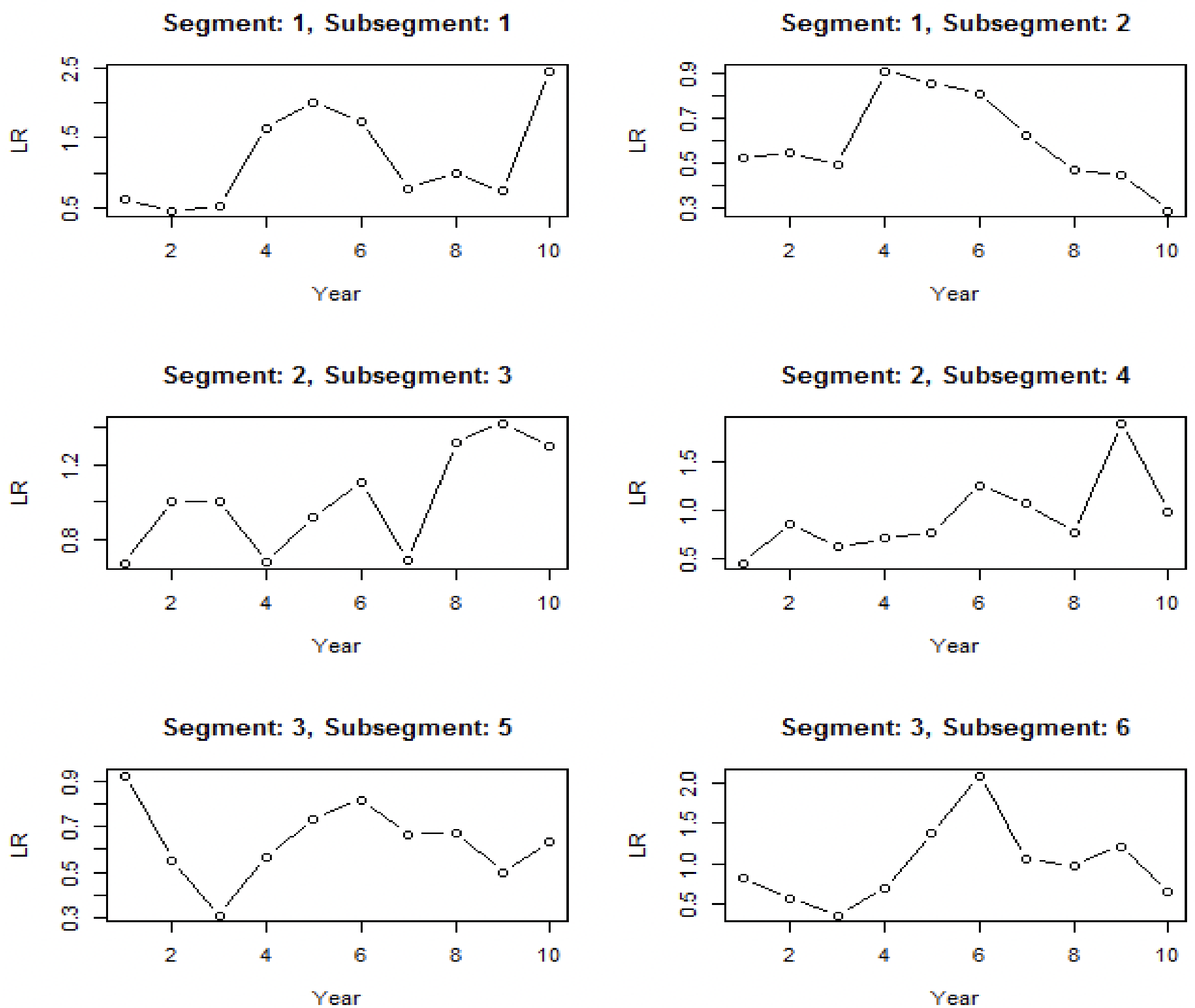
Minor Technical Note

- If using the R package glmnet:
  - Score = <u>Average</u> Loglik + $(Lambda \times \sum P^2)$
  - N x Score = Total Loglik + $(\mathrm{N} \times Lambda \times \sum P^2)$
  - When refitting on account data (or anything with a different number of rows), use:
    - Adjusted Lambda = Lambda x N / new N

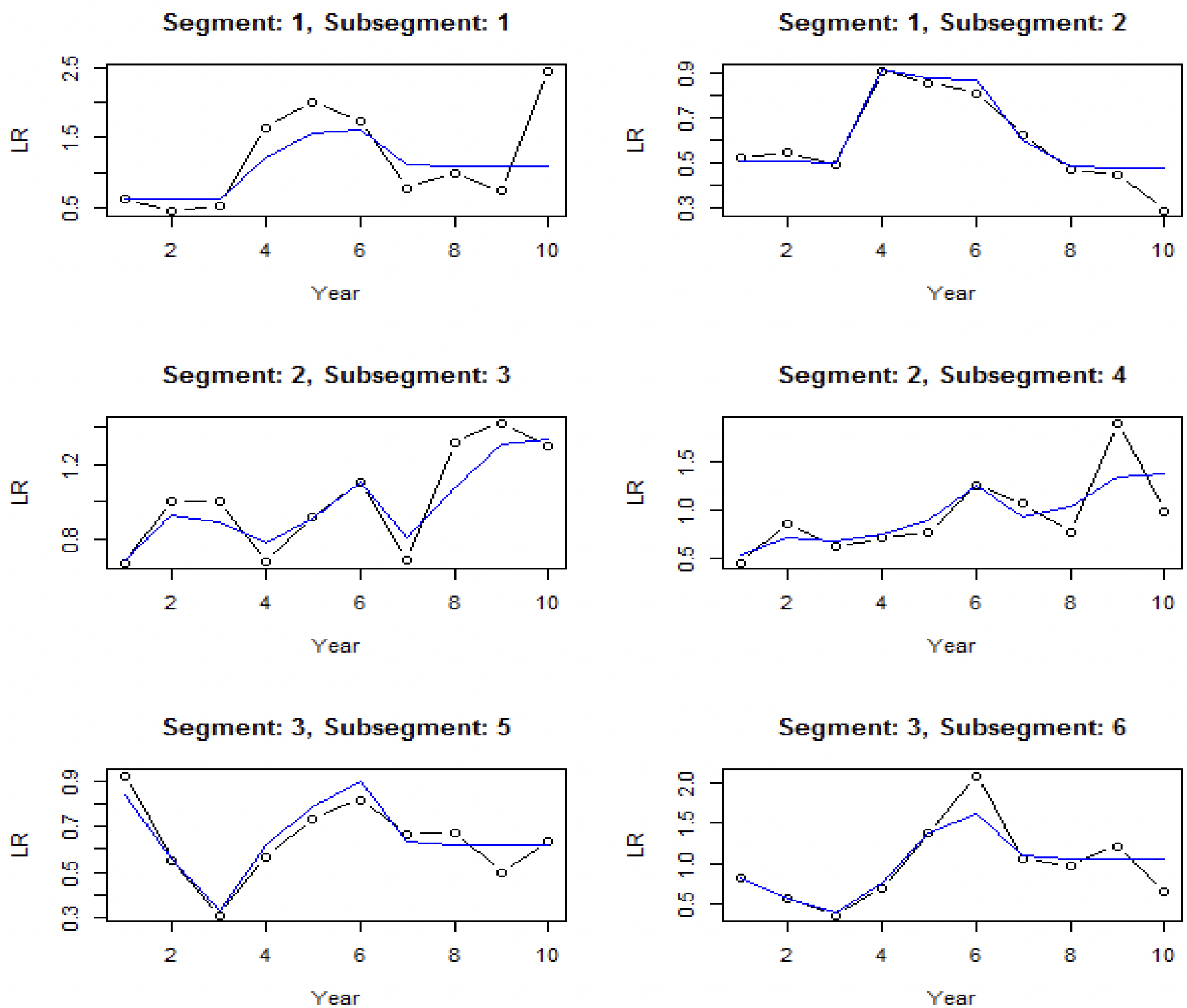
Profitability Studies/ Pricing Models

- Profitability studies
  - Cross validation allows it to work well with “short” time series
  - Credibility for segmentations
  - Segment + Year + Segment:Year
- Pricing Models
  - Regression framework with credibility
  - Scales well to very large datasets

Profitability Study Example



Profitability Study Example

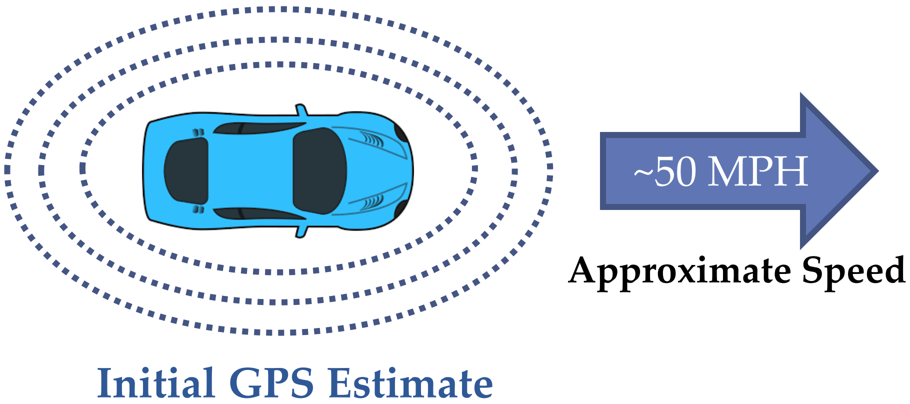


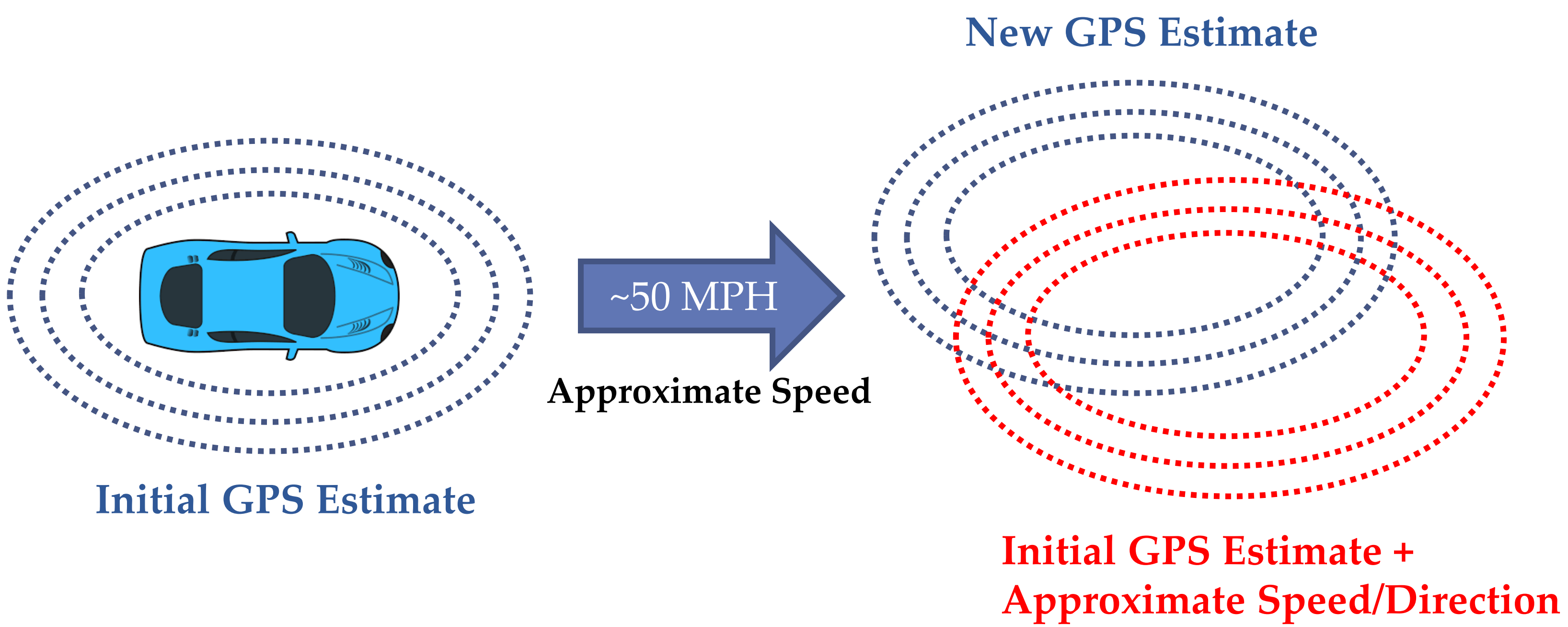
More Stuff!

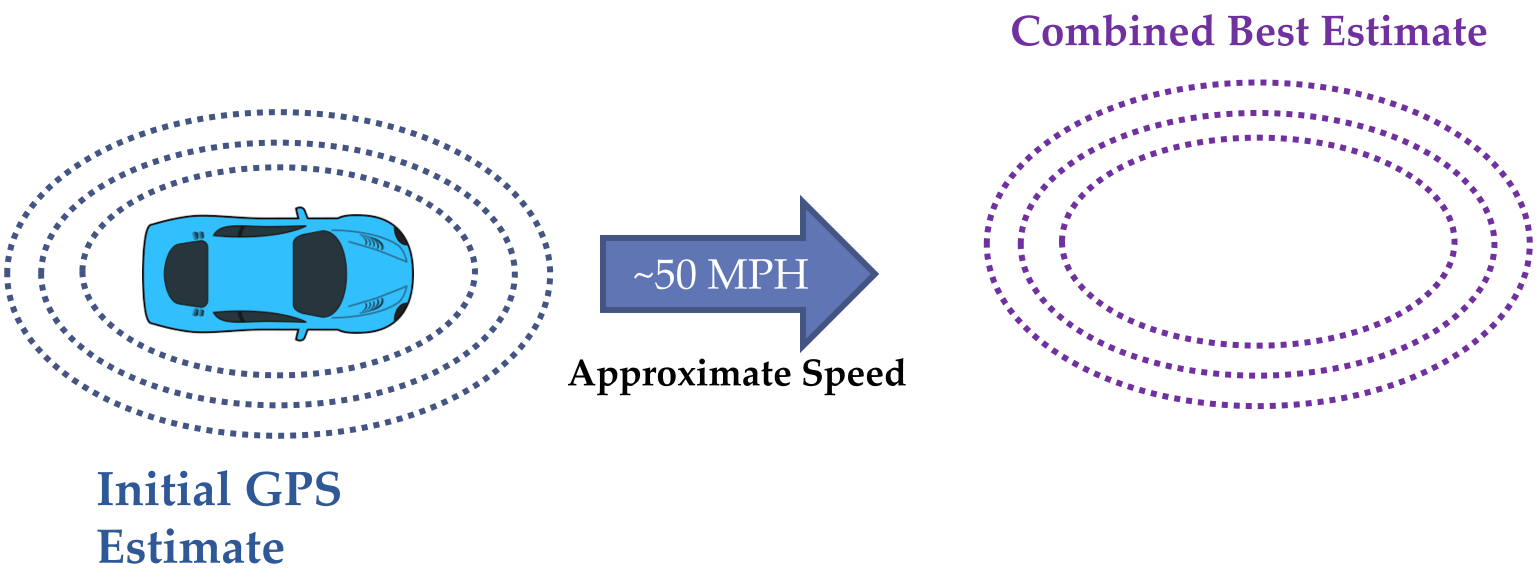
External Data

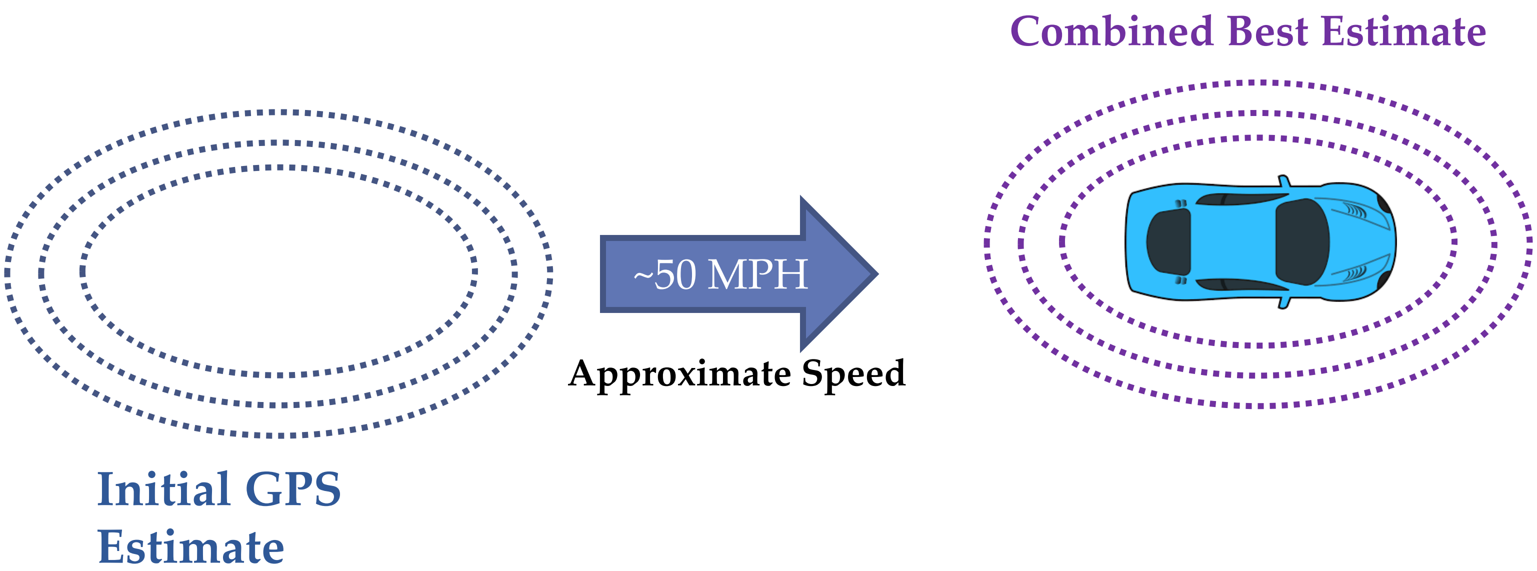
- Use the offset (and remove the intercept) – this will be used as the complement of credibility

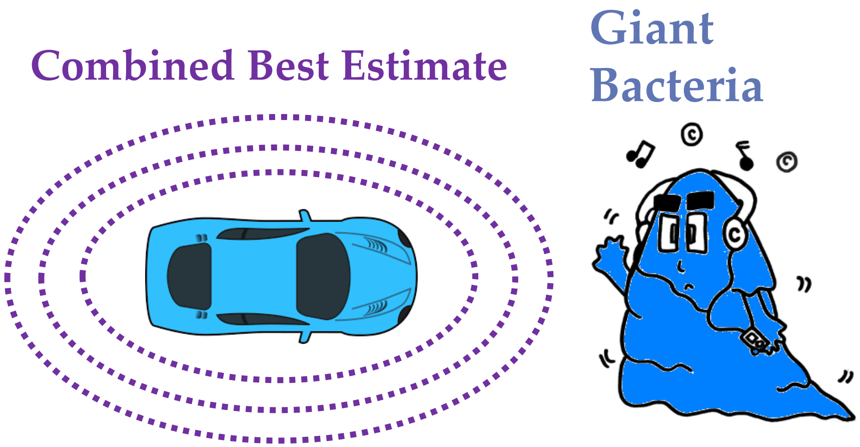
- If the external data only contains expected changes, keep the intercept
  - This will allow the overall level to change (without penalty)


~50 MPH
Approximate Speed
Initial GPS Estimate

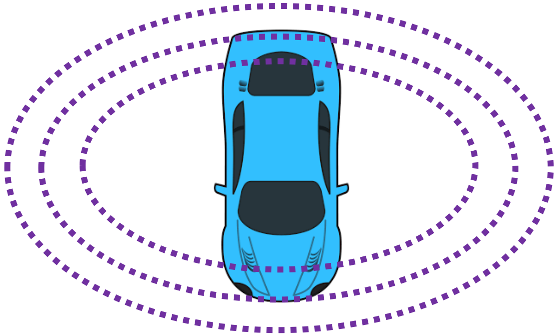

New GPS Estimate
~50 MPH
Approximate Speed
Initial GPS Estimate
Initial GPS Estimate +
Approximate Speed/Direction


Combined Best Estimate
~50 MPH
Approximate Speed
Initial GPS
Estimate

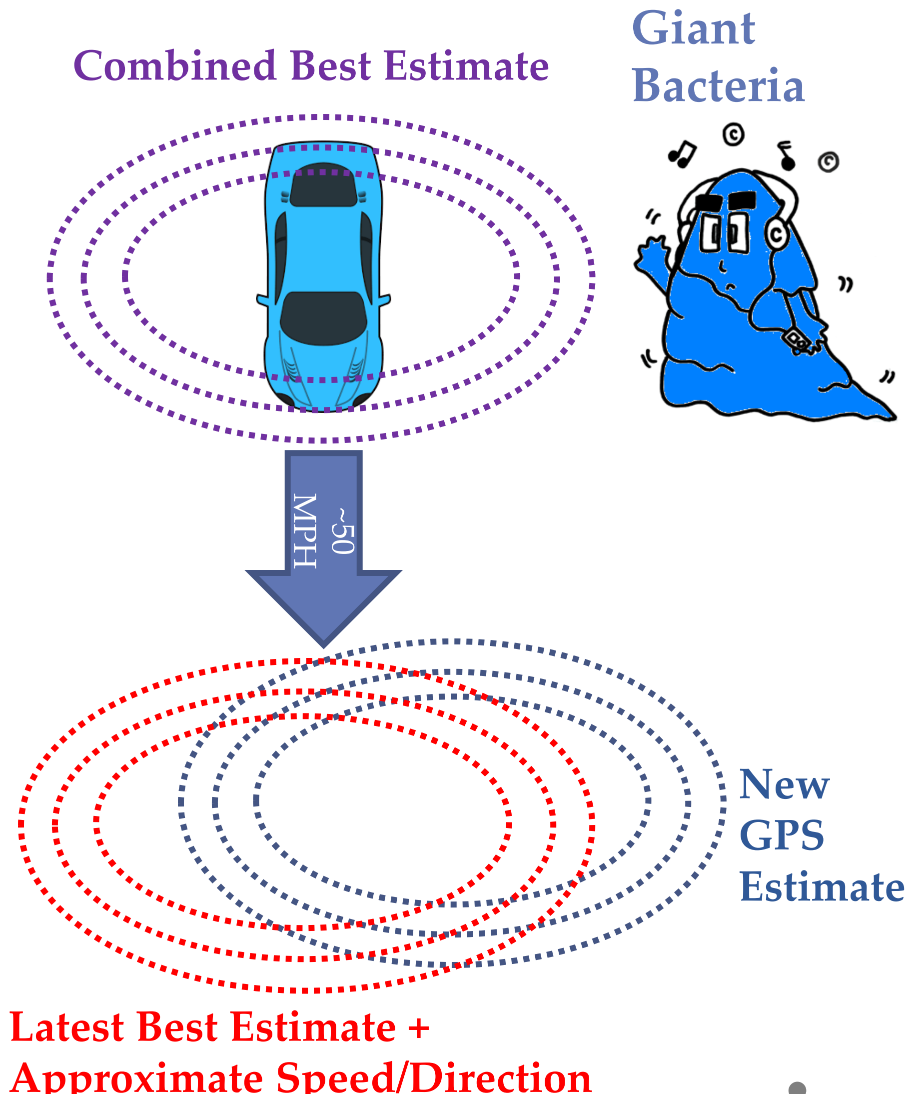

Combined Best Estimate
~50 MPH
Approximate Speed
Initial GPS
Estimate

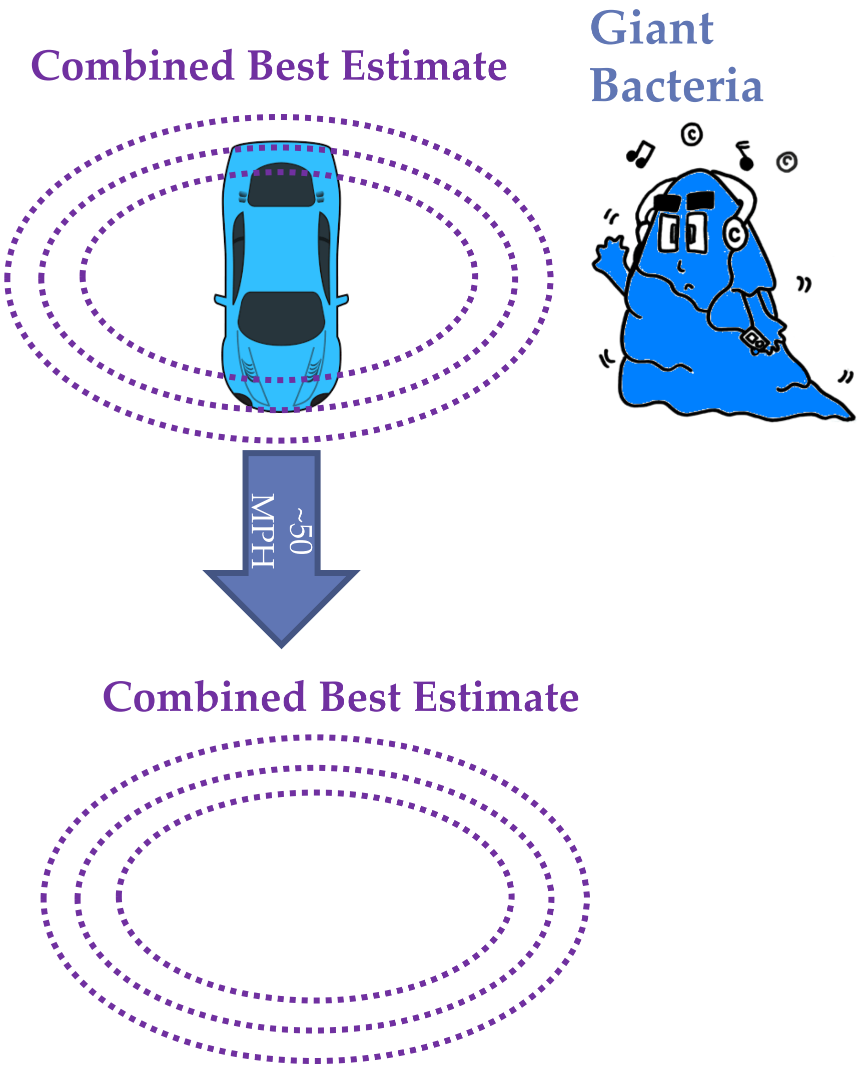

Combined Best Estimate
Giant Bacteria

Combined Best Estimate



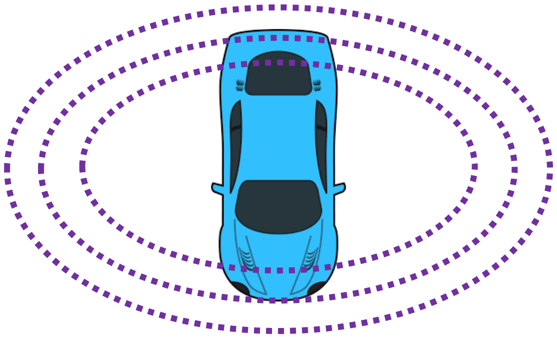
Giant Bacteria


Combined Best Estimate
Giant Bacteria
~50 MPH
New GPS Estimate
Latest Best Estimate + Approximate Speed/Direction


Combined Best Estimate
Giant
Bacteria
~50
MPH
Combined Best Estimate

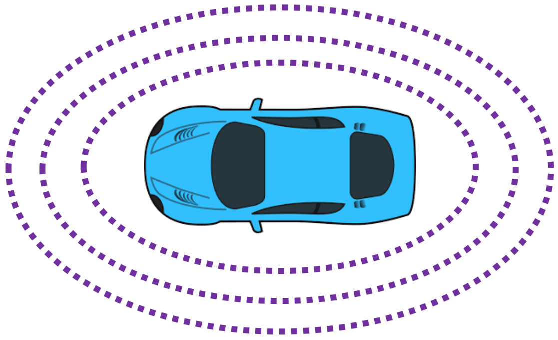
Giant Bacteria

Combined Best Estimate



Giant Bacteria

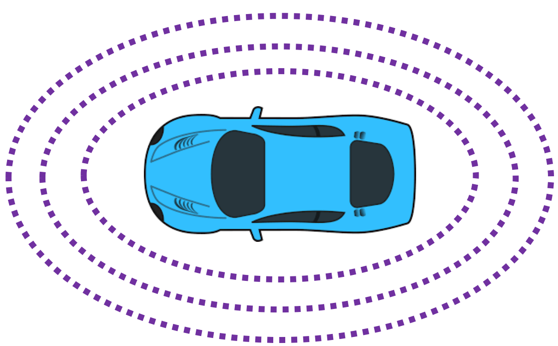
Combined Best Estimate



Giant Bacteria

Hacker

Combined Best Estimate



Giant Bacteria

Hacker

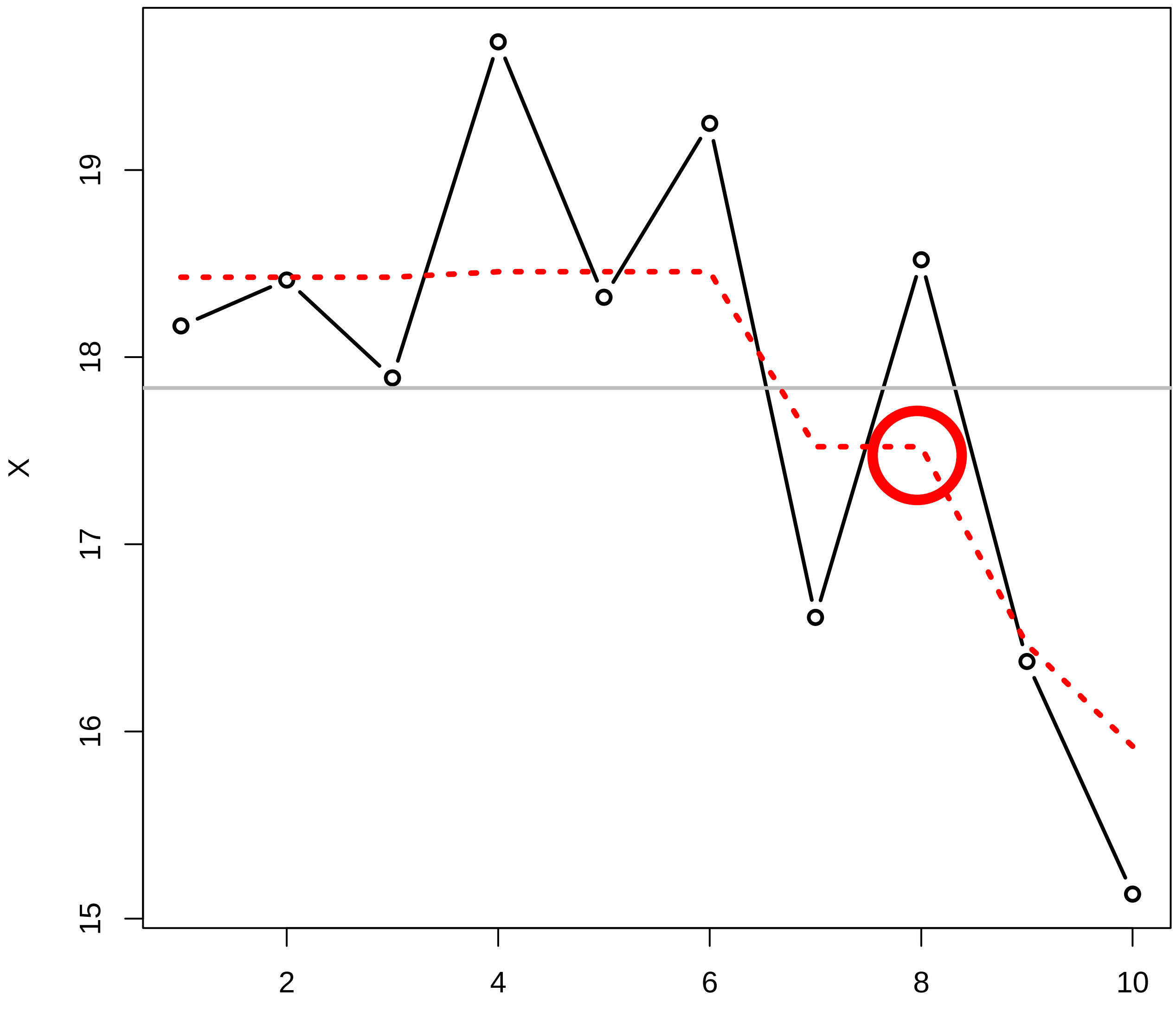
Random Walk with Drift

- The random walk assumes that the complement of credibility is the previous fitted value
- Adding a drift (or trend) parameter makes the complement of credibility the previous point plus the trend
- (See standardization in paper)

Changing Trend

	2014	2015	2016
2013	0	0	0
2014	1	0	0
2015	2	1	0
2016	3	2	1

- (But make columns sum to zero if used with interactions)



X
15
16
17
18
19
2
4
6
8
10
Year

Momentum/Mean Reversion (Extra Credit!)

$$Y_t = X_t + e_t$$

$$X_t = X_{t-1} + u_t$$

$$u_t = au_{t-1} + r_t$$

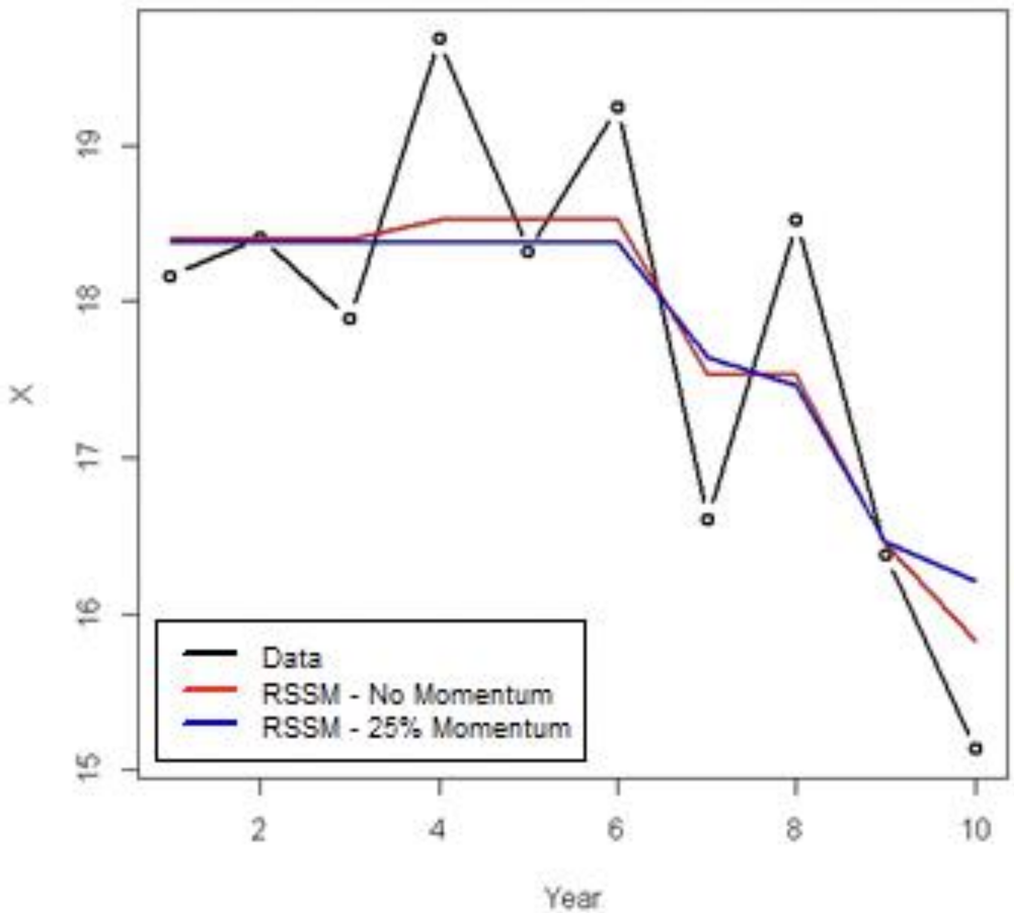
- If a = 1, changing trend model
- If a = 0, random walk model
-

Momentum/Mean Reversion

- Changing trend with 75% mean reversion
- Or random walk with 25% momentum

	2014	**2015**	**2016**
2013	0	0	0
2014	1	0	0
2015	$1 + 0.25$	1	0
2016	$1 + 0.25 + 0.25^2$	$1 + 0.25$	1

Momentum/Mean Reversion



Don't Ignore Change!

Conclusion

- Pricing/profitability forecasts require a robust, statistical method
- A random walk is a simple and intuitive way to forecast
- Using the proposed method, random walks can be incorporated into penalized regression models with credibility and forecast capabilities
- The random walk works well with the elastic net penalty, and its results are well suitable for interpretation
- For more details, see https://www.casact.org/pubs/forum/18spforumv2/06_Korn.pdf