# Utility-Theoretic Underwriting

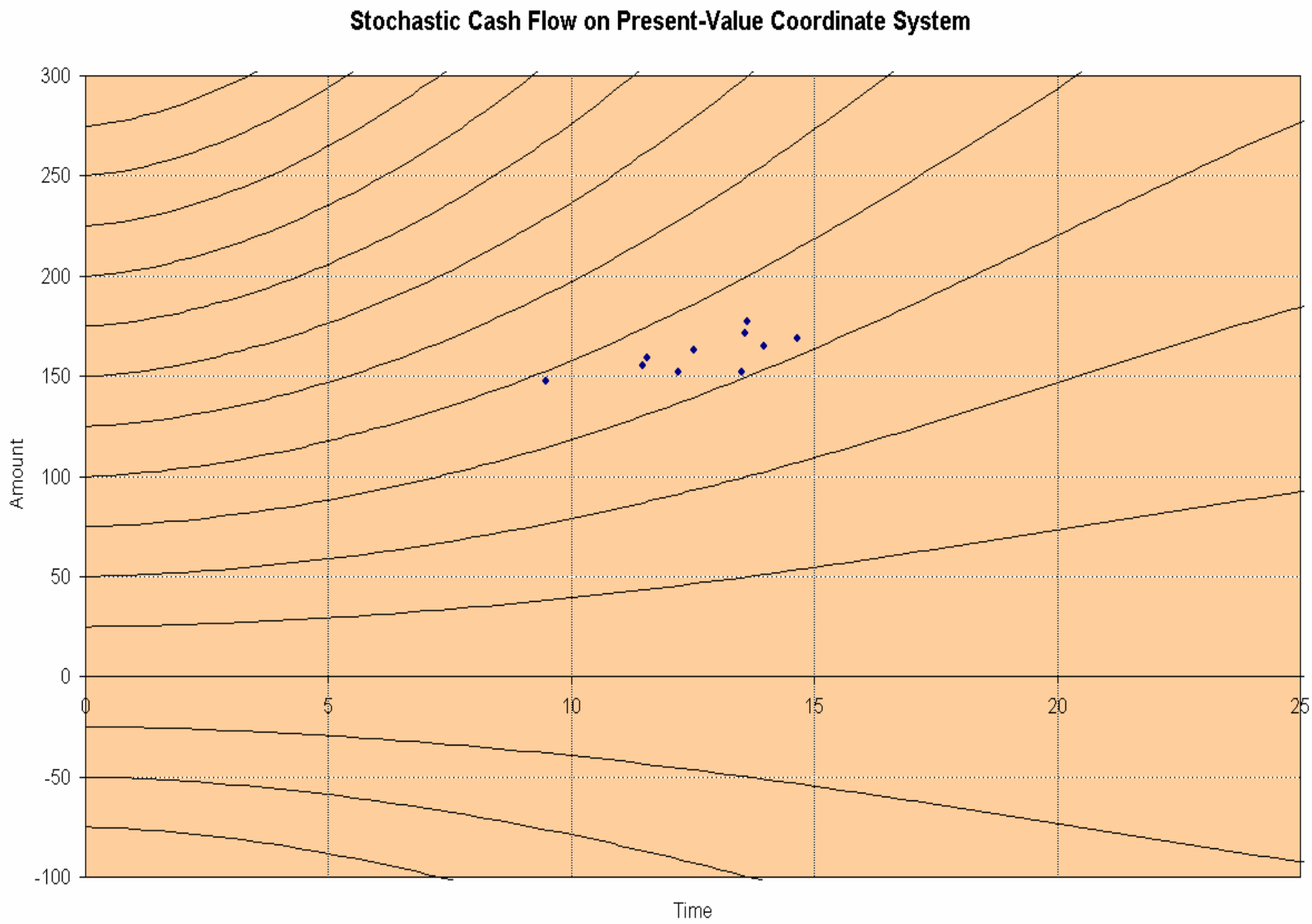
Leigh J. Halliwell, FCAS, MAAA
leigh@lhalliwell.com

CARe Limited-Attendance Seminar
New York, NY
September 26, 2005

# Outline

- The Four Principles of Valuing Stochastic Cash Flows (Underwriting)
  - Present Value as a Random Variable
    The Imposture of Risk-Adjusted Discounting
  - Expected Utility and Preferences of Stochastic Wealth
    Comments on Alternative Theories
  - Optimizing Expected Utility given Price
  - Market Price as Optimizing Everyone's Expected Utility
    Capital Markets as Information Providers?

- Examples in Excel (Utility-Theoretic Underwriting.xls)
  - Coin Toss for $100,000
  - Insurance against a Lawsuit
  - Simple Reinsurance Market
  - Capital-Consumption Example


Stochastic Cash Flow on Present-Value Coordinate System
300
250
200
150
100
50
0
-50
-100
Amount
0
5
10
15
20
25
Time

# Implications of Present-Value Coordinates

- Present value is a random variable. Exhibit 2 (VSCF, 40) graphs the CDF of the PV of this stochastic cash flow.
- The value of this stochastic cash flow should lie between 50 and 100. If all points lay on the same isobar, the value would be that of the isobar. Risk-adjusted discounting can break out of the envelope.
  - Objection: What if the coordinate system changes?
- If two PV random variables are (almost surely) equal, then they must have the same value. In symbols:

$$\text{Prob}\{PV[X] = PV[Y]\} = 1 \quad \Rightarrow \quad Value[X] = Value[Y]$$

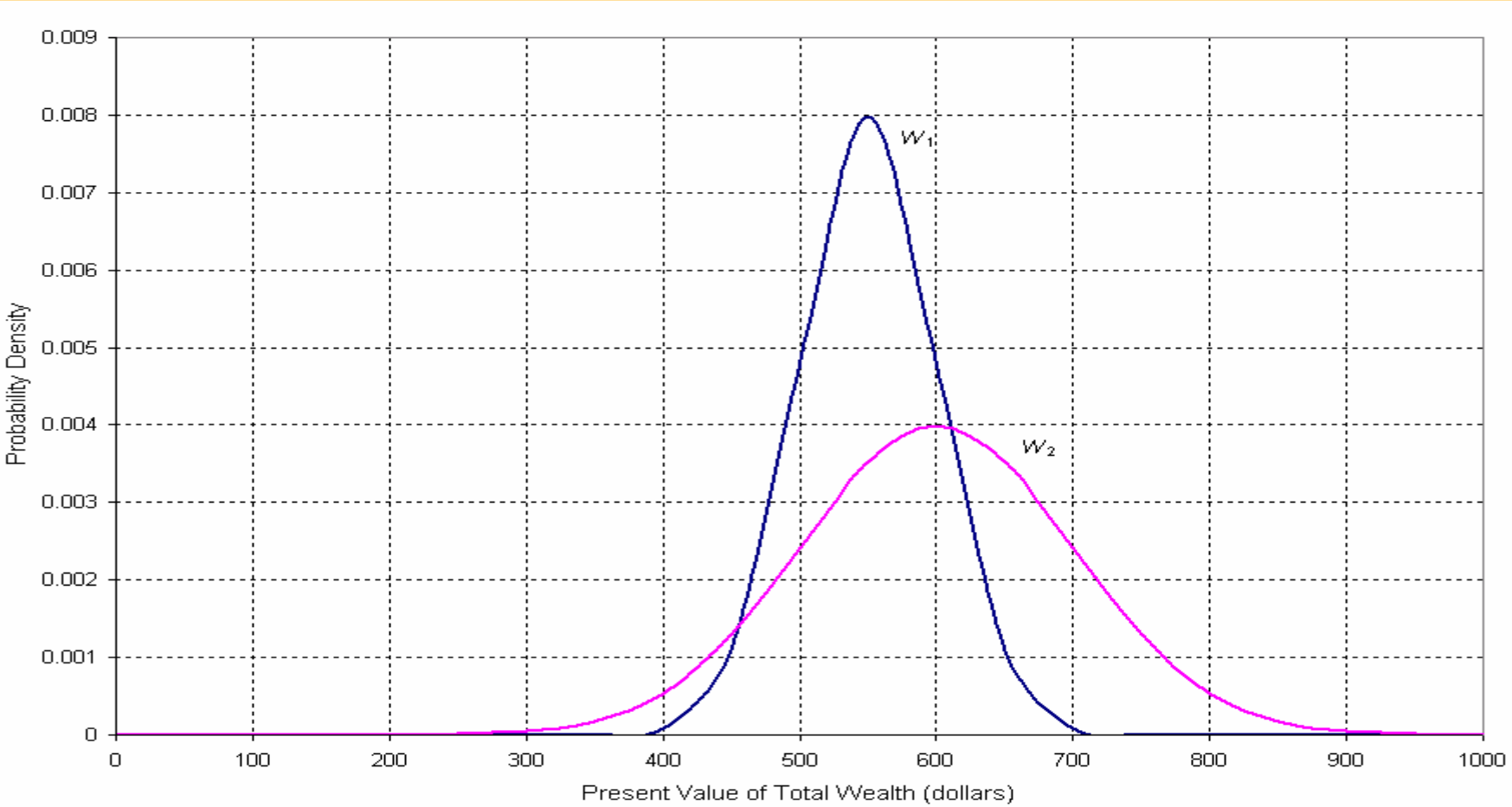
- PVs of outcomes are like sufficient statistics. Negative PVs as legitimate as positive; solvency and bankruptcy are constraints, not criteria (footnotes 25 and 26).

# Just Say “No!” to Capital Allocation

- Capital (or money) does not work, despite advertising slogans.
- Would capital allocated to a one-year hurricane treaty work off season (Dec-May)? Can we make capital moonlight? Some try to have it both ways.
- Capital allocation inevitably confuses rate of return (% per year) with return (%).
- Example of short-lived exposure:

  25% per year is 0.5% per week

# Question: Which Wealth is Better, $550 ± 50 or $600 ± 100?




Answer: Which is greater, $E[u(W_1)]$ or $E[u(W_2)]$?

# Ordering Projects by Expected Utility

- Current stochastic wealth $W$

  Project has present value $X$ and cost $q$.

  Wealth with project is $W+X-q$.

  Compare $E[u(W+X-q)]$ with $E[u(W)]$.

- Borch's insurance formulation, $W-L+p$, equivalent (VSCF, 16)

- Instantaneous formulation Value$=H[X]$ frequent in our actuarial literature. (e.g., Bühlmann, Gerber)

  Some are so future-oriented that they must displace instantaneous results to the end of an accounting period (with interest)! Why is a future time better than the present? Is an instantaneous problem illegitimate?

# Comments on Alternatives

## Capital Consumption Pricing Example

| a) Premium | b) Expenses | c) Probability of Loss | d) NPV Amount of Loss | e) (a-b-d) Loss or Profit | f) Adjusted Amounts |
|---|---|---|---|---|---|
| $1,000,000 | $175,000 | 20% | $0 | $825,000 | |
| | | 15% | $100,000 | $725,000 | |
| | | 15% | $250,000 | $575,000 | |
| | | 15% | $350,000 | $475,000 | |
| | | 10% | $500,000 | $325,000 | |
| | | 10% | $750,000 | $75,000 | |
| | | 10% | $1,000,000 | -$175,000 | -$350,000 |
| | | 2.5% | $1,500,000 | -$675,000 | -$1,350,000 |
| | | 2.5% | $2,500,000 | -$1,675,000 | -$3,350,000 |
| | | | | **$395,000** | **-$1,016,667** |
| | | | | **Penalty Charge** | **200.0%** |
| | | | | **Mean** | **$395,000** |
| | | | | **Risk-Adjusted Mean** | **$318,750** |

**Downside (Capital Consumed) Amounts Increased**

16

## Wang Pricing Transform Modifies the Probabilities

| Premium | Expenses | Probability of Loss | NPV Amount of Loss | (a-b-d) Loss or Profit | Downside | Upside |
|---|---|---|---|---|---|---|
| $1,000,000 | $175,000 | 20% | $0 | $825,000 | | $825,000 |
| | | 15% | $100,000 | $725,000 | | $725,000 |
| | | 15% | $250,000 | $575,000 | | $575,000 |
| | | 15% | $350,000 | $475,000 | | $475,000 |
| | | 10% | $500,000 | $325,000 | | $325,000 |
| | | 10% | $750,000 | $75,000 | | $75,000 |
| | | 10% | $1,000,000 | -$175,000 | -$175,000 | |
| | | 2.5% | $1,500,000 | -$675,000 | -$675,000 | |
| | | 2.5% | $2,500,000 | -$1,675,000 | -$1,675,000 | |
| | | | | $395,000 | -$508,333 | $554,412 |

**Applies a Greater Weight to Downside .... By Modifying Probabilities**

| Premium | Expenses | Probability of Loss | Cumulative Probability | NORMSINV | Lambda 0.75 Transform | Adjusted Probability | Implied Prob | NPV Amount of Loss | Loss or Profit |
|---|---|---|---|---|---|---|---|---|---|
| $1,000,000 | $175,000 | 20.0% | 20.0% | (0.84) | (1.59) | 5.6% | 5.6% | $0 | $825,000 |
| | | 15.0% | 35.0% | (0.39) | (1.14) | 12.8% | 7.2% | $100,000 | $725,000 |
| | | 15.0% | 50.0% | - | (0.75) | 22.7% | 9.9% | $250,000 | $575,000 |
| | | 15.0% | 65.0% | 0.39 | (0.36) | 35.8% | 13.1% | $350,000 | $475,000 |
| | | 10.0% | 75.0% | 0.67 | (0.08) | 47.0% | 11.2% | $500,000 | $325,000 |
| | | 10.0% | 85.0% | 1.04 | 0.29 | 61.3% | 14.3% | $750,000 | $75,000 |
| | | 10.0% | 95.0% | 1.64 | 0.89 | 81.5% | 20.2% | $1,000,000 | -$175,000 |
| | | 2.5% | 97.5% | 1.96 | 1.21 | 88.7% | 7.2% | $1,500,000 | -$675,000 |
| | | 2.5% | 100.0% | | | 100.0% | 11.3% | $2,500,000 | -$1,675,000 |
| | | | | | | | | Exp Value -- Unadjusted | $395,000 |
| | | | | | | | | Exp Value -- Adjusted | -$9,100 |

**Target adjusted ENPV**

19

- "NPV Amount of Loss": Instantaneous approach; no risk-adjusted discounting
- Broken-line utility function?
- CC-adjusted $\mu_{profit}$ = $318,750;
  $\therefore$ is fair premium $681,250?
  No, it's $739,000.

---

- WPT-adjusted $\mu_{profit}$ = ($9,099);
  Yes, fair premium is $1,009,099?
- Would half the deal cost half the price?

# New Perspective:
# How much to buy at a given price?

- Comparing $E[u(W+X-q)]$ with $E[u(W)]$ can give you only a ceiling price (for one unit of $X$), not the appropriate price.
- An economic agent should be free to choose how much of $X$ to purchase at unit price $q$. Cash flows are scaleable.
- New objective: Maximize $f(\theta)=E[u(W+\theta X-\theta q)]$. State-price form: $q = E[\Psi X]$.
- "Fundamental Theorem" of Appendix B: For a given $q$, $f(\theta)$ has one and only one maximum. The curve looks like an upside-down parabola. (cf. VSCF, Exhibit 5, p. 43)
- Two or more agents together find the unique price $q$ at which each maximizes its expected utility and all of $X$ clears. This is a Pareto optimum.
- The agents constitute a market, but each agent is entitled to its own beliefs. The market is an epiphenomenon.

# Exponential Utility

$$u(x) = \left(1 - e^{-ax}\right)/a, \quad u'(x) = e^{-ax}$$

- Just about the only game in town, as argued by Hans Gerber and in VSCF, Appendix C. Has all the desirable properties, including absolute risk aversion (ARA)
- Some argue for power-curve utility and relative risk aversion (RRA).
  - RRA is appropriate for bundles of physical goods, e.g, apples and oranges, which come in non-negative amounts.
  - With SCFs we are dealing with one unit (dollars) in random outcomes that can be positive or negative.
  - Only exponential utility is defined for all real numbers.
  - Only exponential utility allows an *X* independent of *W* to be valued by itself. Otherwise, one might have to know everything in order to value anything.

# Notes on Excel Examples

- Ex 1: Normal approximations (analytic solution) are very good.

  Overall $a = 3.33E{-}06$ is harmonic sum of individuals. Market persona exists because every has the same risk assessment, but market derives from individuals

  Interpretation of a "zero-$\beta$" stochastic cash flow

- Ex 2: Counter intuitively, all risk is insured for E[*X*].

  Insurer disagrees over lawsuit probability; yet Pareto optimum achieved.

  Try the example with risk-neutral insurer.

  Is the pooling theory of insurance valid?

- Ex 3: Insurer will pay any price at which reinsurers will sign for 100%.

  Reins A and B agree on E[*X*]; but B has correlated exposure. Rein C is pessimistic about the risk, estimating twice the pure premium.

  Part A: A and B rashly assume 100% for $5.1 million; C on sidelines at zero.

  Part B: C allowed a short position, and 100% clears for $6.1 million. Market stabilizes; everyone happy except for the insurer.

# Quotable Quotes

- [This theory] sets the agents to the virtuous task of extracting value from projects, rather than from one another. (“Valuation of Stochastic Cash Flows,” 2)
- Risk-adjusted discounting has misled many to elevate [solvency] from the status of a constraint to that of a valuation method. (VSCF, 31)
- One should make sound economic decisions and let the accounting chips fall where they may. (33, footnote 18)
- The business of insurance should be to underwrite well, not to underwrite to generate funds to invest well. … In companies that understand this theory and the near idealness of its application to insurance chief actuaries will be kings. (34)
- “Asking a valuation formula to depend on [wealth level] is like asking a shopkeeper to charge lower prices to the poor than to the rich.” (63)

- To him whose only tool is a hammer everything looks like a nail. (anonymous)
- If I have seen farther, it is by standing on the shoulders of giants. (Isaac Newton)

# Bibliography


- Borch, Karl H., “The Utility Concept Applied to the Theory of Insurance,” *ASTIN Bulletin*, 1(5), 1961, 245-255.
- Bühlmann, Hans, “An Economic Premium Principle,” *ASTIN Bulletin*, 11(1), 1980, 52-60.
- Debreu, Gerard, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*, New Haven, Yale, 1987.
- Gerber, Hans U., *An Introduction to Mathematical Risk Theory*, Philadelphia, S. S. Huebner Foundation, 1979.
- Gerber, Hans U., and Pafumi, Gérard, “Utility Functions: From Risk Theory To Finance,” *North American Actuarial Journal*, Volume 2 (July 1998), 74-100.
- Halliwell, Leigh J. “The Valuation of Stochastic Cash Flows,” *CAS Forum* (Reinsurance Call Papers, Spring 2003), 1-68.
  - “Valuing Stochastic Cash Flows: A Thought Experiment,” *CAS Forum* (Winter 2004), 291-294.
- Longley-Cook, Alastair G., “Risk-Adjusted Economic Value Analysis,” *North American Actuarial Journal*, Volume 2 (Jan 1998), 87-100.
- Panjer, Harry H., editor, *Financial Economics: with Applications to Investments, Insurance and Pensions*, Schaumburg, IL, Actuarial Foundation, 1998.
- Von Neumann, John, and Morgenstern, Oskar, *The Theory of Games and Economic Behavior*, Princeton, 1972.