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The Use of Dynamic Financial Analysis to Determine Whether an Optimal Growth Rate Exists for a Property-Liability Insurer

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Abstract

Prior research on the aging phenomenon has demonstrated that new business for property-liability (P-L) insurers generates high loss ratios that gradually decline as a book of business goes through successive renewal cycles. Although the experience on new business is initially unprofitable, the renewal book of business eventually becomes profitable over time. Within this context, insurers need to manage their exposure growth in order to maximize long run profitability. Dynamic financial analysis (DFA), a relatively new tool for P-L insurers, utilizes Monte Carlo simulation to generate the overall financial results for an insurer under a large number of scenarios. This article uses a publicly available DFA model—along with the estimated market value of an insurer, based on 1990–2001 data for stock P-L insurers and underlying financial variables—to determine optimal growth rates of a P-L insurer based on mean–variance analysis, stochastic dominance, and constraints on leverage.

Introduction

Managers of property-liability (P-L) insurance companies must make decisions regarding a broad range of important, and often challenging, strategic issues. Examples of these decisions include what lines of insurance to write, in which geographic territories coverage should be written, how to manage an investment portfolio effectively, and how to determine an optimal reinsurance program. The answers to these questions—and to some extent, the identification of the questions themselves—depend significantly upon the specific corporate, insurance, and economic environment in which the company operates. Selections among strategic alternatives may vary depending upon the financial condition of the company and its competitors, the

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current position of the industry within the insurance market cycle, the interest rate environment, and a myriad other circumstances at any given time.

One of the critical decisions facing a P-L insurer involves targeting a level of exposure growth for its underwriting portfolio. The decision regarding how much a company should attempt to grow its business in the future will depend heavily on the environmental conditions referred to above. For example, it may be easier to achieve a high growth rate during one phase of the underwriting cycle than another. There are also trade-offs involving the process of growing a book of business. If a high growth target can only be achieved by lowering underwriting standards, there is a potential trade-off between growth and profitability.

How, then, can an *optimal* exposure growth rate be determined for a P-L insurer? That question is the subject of this article. In light of the relatively strained capital position of the insurance industry in the early 2000s, especially after the events of September 11, 2001, this is a particularly important issue. Using a process called "dynamic financial analysis (DFA)," which allows for the systematic evaluation of strategic alternatives, the possibility of identifying an optimal growth rate with respect to a given set of environmental conditions is examined.

The remainder of this article is structured as follows. The next section introduces and describes the process called (in the P-L insurance industry) DFA, and how it can assist managers in making effective strategic planning decisions. The following section describes what is known as the "aging phenomenon," the understanding and inclusion of which is critical to an appropriate decision regarding growth. The next section presents issues involved in determining the market value of an insurer, and describes how the authors parameterized this calculation relative to publicly available company data. This is important since the expected value and the potential volatility of a company's future market value under various strategic assumptions will form the basis for making an optimal growth decision. The next three sections describe the analysis of the impact of growth on company value, under a variety of environmental conditions. This analysis is performed using a public-access DFA model, which is described in greater detail in the Appendix. This model provides for the stochastic simulation of future company financial and operating conditions. The final two sections describe certain caveats regarding the findings, provide suggestions for future research, and summarize the conclusions of this research.

DYNAMIC FINANCIAL ANALYSIS

Dynamic financial analysis is a new approach to modeling insurance companies that developed in the 1990s as a result of the convergence of several trends. One trend was the increase in financial risk that had begun in the 1970s as first inflation and then interest rates became increasingly volatile. Another trend was the increased access to computers powerful enough to accommodate the sophisticated mathematical techniques involved in DFA. A third trend was the adoption of similar types of analyses in banks and other financial institutions.

The general approach of DFA was applied first by insurers in Europe, then in Canada and the United States. The first applications were to life insurers, under a process termed "dynamic solvency testing" or "dynamic financial condition analysis." The primary impetus for the initial applications of these techniques was the interest rate volatility of the late 1970s and early 1980s that led to the financial distress of some life insurers. The focus of these applications was to improve solvency monitoring and reduce the likelihood of insurers becoming insolvent. DFA was envisioned as a powerful new regulatory tool, and in fact, elements of the DFA process are used in Canada in its dynamic capital adequacy-testing (DCAT) requirement. In DCAT, scenario testing is employed to provide insights into future company solvency potential, both for regulatory purposes and to better inform management (Canadian Institute of Actuaries, 1999). Although DFA is used by some regulators, the complexity of the modeling process and the number of adjustments that need to be made to apply DFA to a specific company have limited the regulatory applications in the United States. Instead, DFA has developed into a versatile strategic planning tool (CAS, 1995, 1996; Correnti, Sonlin, and Issac, 1998; Hodes et al., 1996).

The term *dynamic financial analysis* conveniently explains the methodology underlying these models. *Dynamic* indicates that this approach reflects the uncertainty involved in modeling an insurance company. Stochastic variables are used to represent factors that will affect the company's operations. This method leads to a range, or distribution, of possible outcomes, along with their associated probabilities, rather than simply a single best estimate of the outcome. Factors that will affect the operations or balance sheet of a company are allowed to vary, according to selected parameters, rather than being estimated as a single deterministic value. Interest rates, for example, are represented by sophisticated mathematical models rather than a single estimate of the future rate. DFA is *dynamic* in that it reflects the range of possible outcomes relative to underlying stochastic variables, rather than producing just one or a few point estimates of possible outcomes.

The term *financial* reflects the integration of underwriting and investment, with variables such as interest rates impacting both investment values and underwriting returns. Historically, insurance companies focused on the underwriting side of their operations, and neglected or ignored the investment side. The top managers of insurance companies typically had strong backgrounds in underwriting, sales, claims, or actuarial work, and rarely rose from the investment area. Most management attention was paid to underwriting issues, and investments, although expected to produce a steady return, were not given much consideration. For many companies, this changed in the 1980s as investment income began to dwarf underwriting returns and as investment returns became more volatile. Now insurers tend to coordinate their underwriting and investment operations and pay close attention to both assets and liabilities.

The final term of DFA is *analysis*, which is defined in Webster's Seventh New Collegiate Dictionary (1965) as "An examination of a *complex*, its elements and their relations." *Complex* is in turn defined as "A whole made up of complicated or interrelated parts." An insurance company is indeed a complicated structure with many interrelations, and DFA provides a method for studying these factors and their relationships.

By now, all major actuarial consulting firms have developed their own DFA models that can be applied to clients' operations. However, the proprietary nature of the models and the need for the consulting firms to keep key aspects of their models confidential (to protect their value) lead to somewhat of a "black box" structure.

Clients see the input values and the output values, and are given a general explanation of how the model operates, but the exact inner workings of the model are not seen. To get around this limitation, many large insurers have developed their own DFA models. Again, for proprietary reasons, these models are not available for other firms or individuals to use. The development of a realistic DFA model takes several years and a significant commitment of actuarial, programming, and other staff time, both on the underwriting and financial sides. After such an extensive investment, it is reasonable for consultants or companies to want to keep their models confidential.

Researchers wanting to utilize DFA to analyze insurance operations have faced the daunting task of having to invest extensive time and effort to, in essence, duplicate the DFA models currently in use. This hurdle has dissuaded most researchers from applying DFA. Given the importance of DFA modeling and the valuable contributions that it provides in the analysis of insurance operations, this deterrent has had a negative effect on actuarial research. However, one consulting firm has made a DFA model publicly available for use by companies, regulators, students, researchers, and even other consulting firms. This model, the latest version of which is termed DynaMo3, is available at no charge at http://www.pinnacleactuaries.com/. By providing open and free access to this model, this firm has changed the nature of DFA for users, from being a mysterious black box to being a transparent model. While any DFA model could be used to study the impact of growth on P-L insurers, *DynaMo3* is used here since it is publicly available. *DynaMo3* is described in detail in the Appendix. The opportunity to use a publicly available model advances the potential for education and peer review of the overall DFA process, and should enhance the profile of DFA as a widely used tool for the insurance industry.

Although the initial intent of DFA was to serve as a tool for regulators to monitor solvency, the current models can be far more useful for strategic planning. DFA models can allow managers to test various operational strategies, and adopt the ones that are expected to be the most successful (according to whatever measure of "success" the user chooses). Managers can observe the effect of adding a new line of business or entering into a new underwriting territory. The impact of raising or lowering prices, implementing new underwriting rules, changing the reinsurance program, instituting a new claim processing strategy, shifting the investment portfolio, or altering the mix of business can all be tested before being adopted. DFA can have a revolutionary impact on the insurance industry, once the models are sufficiently refined and widely accepted. However, there remains a long way to go before the models reach that level. Hopefully, this article can be one step in this process.

AGING PHENOMENON

A well-known, but not extensively reported, trait of P-L insurance is that new business has a very high loss ratio, often in excess of the initial premium. The loss ratio then declines with each renewal cycle. After a book of business has been in force for several years, the loss ratio has generally declined to the point that the book of business is profitable. Longer-term business has an even lower loss ratio, making it very profitable, and hence valuable, for most insurers. In the aggregate, the profits on this long-term business tend to offset the losses incurred on the book in the early years, so that, over the life of the book of business, the insurer earns a reasonable profit.

The aging phenomenon appears to occur for every P-L line of business for every insurer that has examined this trend. Since long-term business is extremely profitable for an insurer, it is understandable that most insurers do not want to share the results of their internal studies on this experience with competitors. One reason proposed for this phenomenon is that errors occur in the initial underwriting reviews or initial policyholder classification that result in underpricing new policies, and these errors are gradually remedied during subsequent renewals. Another reason proposed is that there is initial information asymmetry, and underwriters gradually weed out the undesirable policyholders as additional information is revealed. This reason would suggest that the more aggressive insurers are with regard to renewal underwriting the faster the improvement in loss ratio. However, the aging phenomenon also occurs in states that do not allow nonrenewal of policyholders except for nonpayment of premium. Thus, the aging phenomenon is not solely the result of renewal underwriting. Another possible explanation is the tendency of high-risk (but otherwise unidentifiable) policyholders to be dissuaded from renewing their policies after experiencing problems during claim settlements. These policyholders, therefore, go to other insurers, causing their new business loss ratios to be high.

Only a few studies on the aging phenomenon have been published. D'Arcy and Doherty (1989) demonstrate how the aging phenomenon affects pricing strategies as interest rates change. D'Arcy and Doherty (1990) document the automobile experience of seven unidentified insurers, and explain how asymmetric information between insurers and policyholders leads insurers to "lowball" their initial price to gain access to the private information that will be revealed during the contract term. Feldblum (1996) proposes that insurers analyze profitability on a cohort basis over the life of a book of business, rather than aggregating all business together. Cohen (2001) provides an extensive analysis of the experience of more than 200,000 policies over a 5-year period for one insurance company that clearly demonstrates and quantifies the aging phenomenon.

Since the original research on the aging phenomenon has been published, a new service has developed for P-L insurers in the United States to help provide companies with more information on applicants for auto insurance. The service, termed the "Comprehensive Loss Underwriting Exchange" (CLUE), allows participating insurers (which write approximately 95 percent of auto insurance premiums) to access the claim histories of any new applicant that has previously been insured by another participating insurer. By sharing claim histories, insurers reduce the informational asymmetry on new business. Anecdotal evidence suggests that CLUE has reduced the loss ratio for new business. However, the aging phenomenon is still clearly evident in company experience. For example, many new applicants have no insured claim history because they were either uninsured, insured under a parent's or someone else's policy, in the military, had licenses but never drove or purchased insurance, or lived internationally. In such situations, there may be no relevant information available from CLUE.

As a result of the aging phenomenon, a P-L insurer's exposure growth rate has a significant effect on the insurer's profitability. A rapidly growing company is likely to have a much higher loss ratio than a more slowly growing insurer simply due to the greater proportion of new business being written. Thus, the growth rate is a key strategic variable for an insurer. In addition, the persistency rate (the proportion of policies a company renews each cycle) also affects profitability. The more long-term business an insurer is able to retain, the more profitable this business will be.

MARKET VALUE OF A PROPERTY-LIABILITY INSURANCE COMPANY

Although there are over 3,000 P-L insurers writing business in the United States, determining the market value of a hypothetical P-L insurer is not a simple task. Many insurers are mutuals or reciprocals, who are owned by their policyholders and do not have a stated market value. Other P-L insurers are subsidiaries of other companies or a part of an organization that includes life insurers and other financial service companies, so the market value of the P-L segment of the operations is not readily determinable. Some P-L insurers are privately held entities. Only a few P-L insurers are stand-alone companies that are publicly traded, allowing the market value of the firm to be observed. It is these companies that are used to generate the parameters for estimating the market value of a P-L insurer.

The sample of companies used in this study includes all P-L insurers identified by Value Line or Standard & Poor's as P-L insurers for which information on the premium, losses, expenses, operating ratio, surplus, and the market value of the company was available during the period 1990–2001. One company included in the sample, Hartford, was part of the ITT group until 1996 so that the market value of the P-L segment of the business could only be determined for 1996–2001.

A number of different approaches are in use or have been proposed to determine the value of a company. Investment banks and insurers tend to use a valuation approach that generates a range for the value of a company or portion of a company based on a single variable, such as book value, sales, cash flows, or earnings. Several recent articles calculate a terminal value of a firm based on the assumption that cash flows will grow at a constant nominal rate in perpetuity (Kaplan, 1989; Kaplan and Ruback, 1995). The cost of equity capital has been measured based both on the capital asset pricing model (CAPM) and the Fama–French three-factor model (Fama and French, 1992). Ibbotson Associates, after an extensive analysis of several different methods of valuation, including the Fama–French three-factor approach, concludes (Ibbotson, 2002, p. 159):

> What conclusions can we draw from this analysis? The Fama–French model provided a different approach to calculating the cost of equity. It is not possible to say whether the numbers provided by the Fama–French model are better or more reliable than the cost of equity estimates provided by the CAPM. Both models fail to produce logical results for a large number of entities. The Fama–French model should be viewed as an additional tool available to analysts in determining the cost of equity.

Based on this conclusion, there is little evidence that either the Fama–French model or the CAPM is the ideal approach. One of the problems with using a discounted cash flow approach based on predicted growth rates is the sensitivity of the results to the selected growth rate and the discount rate, neither of which can be measured exactly. In lieu of this approach, which is found to produce illogical results in many cases, we decided to use an improvement over the commonly used method of taking a multiple of a single value. Our method uses multiple regression to determine which financial measures for a P-L insurer have been significantly related to market value. We found that three measures, Policyholders' Surplus, Net Written Premium, and Combined Ratio, are significant, all in the expected direction. These do correspond to the Fama–French framework: policyholders' surplus corresponds to book value, and net written premium corresponds to size. This approach may not be the ideal method for valuation, but it is the one considered the most appropriate for this purpose. Hopefully, this work can encourage additional research on this important topic.

The market value of an insurer is assumed to be a function of the surplus, the size of the book of business, and the profitability of the company. The only value for "surplus" that could be consistently obtained from available data over the period 1990–2001 was the statutory surplus. Statutory surplus, which is the statutory value of assets minus the statutory value of liabilities, is generally recognized as being conservative, with the statutory value less than the true economic value. Statutory assets do not include such items as agents' balances over 90 days due, reinsurance recoverables over 90 days due, furniture, and equipment. Real estate investments are valued at the lower of net book value (cost less depreciation) or market value. The only exception to the conservative valuation approach for assets is that bonds that are not held for trading (investment purposes) are valued at amortized cost, which is based on the interest rate that was in effect when the bond was purchased. If interest rates have risen in the interim, the amortized value of a bond would exceed the market value.

Statutory liabilities are also conservatively valued, in this case by having the statutory value exceed the economic value. The primary element of conservatism in statutory liabilities is the use of undiscounted loss and loss adjustment expense reserves. Another element of conservatism is the unearned premium reserve, which is the *pro rata* portion of the entire written premium for each policy representing the unexpired portion of the policy period. Since most expenses associated with writing a policy are incurred at the inception of the policy, this segment of the unearned premium reserve is recognized as being excessive. The only significant exception to the conservative valuation for liabilities is that whereas the market value of equities is included in statutory assets, there is no provision in liabilities for taxes that will be incurred when unrealized capital gains are eventually realized.

The size of the book of business of an insurance company is also assumed to be associated with the market value of an insurer. The larger the share of the market that an insurer controls, the more the company would be worth, since this business could generate future profits. Especially in light of the aging phenomenon, where an established book of business is known to be more profitable than new business, the size of an insurer's book of business should be an important factor in determining the company value. For this study, size is measured based on the net written premiums. Net written premium is the total of direct written premium minus reinsurance ceded plus reinsurance assumed. Other potential measures of the size of an insurer's book of business, such as the total number of policies or exposure units, are not publicly available.

It is assumed that a profitable insurance company would be valued more highly than a less profitable insurer. Measuring the true profitability of a P-L insurer is a complex process requiring accident year data on losses and risk-adjusted investment returns, neither of which is publicly available. However, two readily available measures, the combined ratio and the operating ratio, are commonly used to express profitability for P-L insurers. The combined ratio is the sum of the loss and loss adjustment expense ratio (losses and LAE over earned premium) and the expense ratio (other expenses over written premium). The combined ratio does not reflect investment income; the operating ratio does. The operating ratio is determined by subtracting the ratio of net investment income to earned premium from the combined ratio. (The operating ratio does not reflect realized or unrealized capital gains.) Since both the combined ratio and operating ratio are inversely related to profitability (lower values indicate higher profitability), then each of these ratios should be negatively related to market value.

The companies included in this analysis are listed in Table 1, along with total revenue of the company, the Policyholders' Surplus, P-L Net Written Premiums, and the Operating Ratio for the year 2001. This table also shows the relationship between the Net Written Premium for P-L business and the total revenue of the firm for 2001. This measure provides information regarding the importance of P-L business within the overall company.

Traditionally, valuation tests use the natural log of the market value and other relevant variables to determine which factors have a significant impact on the value of a company. This adjustment is made to reduce the impact of extremely large companies on the results, since the least squares criteria used in the multiple regression gives greater weight to similar percentage errors of large components than of smaller ones. In order to determine the effect of surplus, size, and profitability on the market value of an insurer, a similar test is performed on the 15 insurers in the sample. Table 2 shows the results of multiple regression analysis with the natural log of Market Value as the dependent variable and the natural log of Statutory Policyholders Surplus, the natural log of Net Written Premium, and the (actual value) Combined Ratio (Equation (1)) or the Operating Ratio (Equation (2)) as independent variables. Each of the variables is significant in the expected direction. The results of the combined ratio regression (Equation (1)) are slightly better than for the operating ratio, with an R^2 of 0.938, compared to 0.932, and a t-statistic of 6.9 for the combined ratio compared to 5.5 for the operating ratio. Thus, the combined ratio will be used on the following tests.

Although the use of the natural logs of variables with large differences is an effective method of determining the significant values in a relationship, the remainder of the article will use the actual values instead of logs. The reason the actual values must be used is due to the focus of the analysis. DFA will be used to simulate a large number of potential outcomes for an insurer. Each set of outcomes will be the result of a different operating strategy. The results of each strategy will then be compared to determine which one leads to the best outcome. For each individual result, the value of the company needs to be determined. The mean values and the variances are then calculated to apply a mean–variance selection process. The process would be distorted if the calculations of natural logs were used, since this would discount the impact of

TABLE 1
Company Data (in Millions)

Year 2001	Total Revenue	Total Admitted Assets	Market Value	Policyholders' Surplus	Net Written Premium (P/C)	Operating Ratio	NWP/Revenue
Acceptance	176	440	70	129	93	1.150	0.529
Allstate	28,865	39,290	31,723	13,796	21,991	0.950	0.762
AIG	61,766	52,458	216,528	15,362	14,007	0.910	0.227
Berkshire	37,668	73,400	94,628	27,103	11,656	1.025	0.309
Chubb	7,754	18,218	14,250	3,526	5,997	0.990	0.773
Cincinnati Fin.	2,561	6,873	7,029	2,530	2,591	0.888	1.012
CNA	13,203	32,446	6,919	6,089	7,663	1.409	0.580
Hartford	15,147	23,997	16,289	5,804	5,209	0.971	0.344
HCC Ins.	505	904	1,520	398	300	0.925	0.594
Ohio Casualty	1,902	3,830	1,030	768	1,472	0.985	0.774
Progressive	7,488	10,391	7,846	2,641	7,263	0.917	0.970
SAFECO	6,863	9,658	4,033	2,280	4,439	1.103	0.647
Selective	1,059	2,209	590	519	927	0.958	0.875
St. Paul	8,943	22,577	12,257	4,132	6,136	1.090	0.686
United F&C	473	748	215	198	366	0.960	0.774

TABLE 2
Market Value Estimation

Equation 1: $LN(MV) = a + b * LN(PHS) + c * LN(NWP) + d * CR;$
Equation 2: $LN(MV) = a + b * LN(PHS) + c * LN(NWP) + d * OR.$

Equation	*a*	S.E.	*b*	S.E.	*c*	S.E.	*d*	S.E.	R^2
1	0.88	0.48	1.01	0.05	0.16	0.05	−2.37	0.35	0.938
2	0.51	0.49	0.86	0.06	0.28	0.06	−1.99	0.36	0.932

Note: Least-squares linear regression, based on the experience of 15 companies over the period 1990–2001. MV = market value, PHS = statutory policyholders' surplus, NWP = net written premium, CR = combined ratio, and OR = operating ratio.

extremely good results. In this analysis, the actual size of the outcomes is important, so the parameter values must be determined without adjusting the components.

The regression was determined based on the following form:

$$MV = a + b * PHS + c * NWP + d * CR + \epsilon, \quad (1)$$

where MV is market value of the firm, PHS is statutory policyholders' surplus, NWP is net written premium, and CR is combined ratio.

The results of the regressions for each company separately, and for all 15 companies combined, are shown on Table 3. However, since no adjustment is made for size, the parameter values are heavily influenced by the largest insurers, particularly by AIG, due to its high market value. Although AIG has a significant amount of P-L insurance business, the ratio of P-L NWP to total revenue for this company was much lower than most of the other companies in the sample (see Table 1). For this reason, two sets of coefficients were used, one set based on the entire sample, and the other based on all companies other than AIG.

OPTIMAL GROWTH RATE

The objective of this study is to determine the optimal growth rate for XYZ Company. This is a fictitious P-L insurance industry company, created for illustrative purposes, which is embedded in the default version of *DynaMo3*. XYZ Company is a $59 million (in premium) writer of workers' compensation and homeowners insurance. (See the end of the Appendix of this article for additional details.)

The first step in this process is to determine the appropriate metric to optimize. One alternative would be the net income of the insurer over the time period of the simulation, in this case the next 5 years. However, this ignores the value of the company at the end of this period, which could differ based on the operating strategy selected. Another alternative would be the policyholders' surplus at the end of the simulation period. This value, though, focuses only on a statutory value, which is recognized as being conservative at a given point in time. Combining these ideas, and adjusting for the above difficulties, the metric selected for this analysis is the net income over the

TABLE 3
Market Value Estimation

$$MV = a + b * PHS + c * NWP + d * CR$$
Dependent Variable: MV;
Independent Variables: PHS, NWP, CR.

Company	*a*	S.E.	*b*	S.E.	*c*	S.E	*d*	S.E.	R^2
Acceptance	447,067	91,359	1.32	0.30	0.07	0.21	−442,500	80,883	0.969
Allstate	−45,842,210	73,592,068	5.86	2.11	−3.01	1.87	60,912,024	73,496,027	0.835
AIG	60,741,483	507,769,771	0.10	7.49	32.87	13.64	−282,623,500	528,222,750	0.910
Berkshire	10,310,535	18,250,118	0.48	0.25	6.61	0.82	−5,794,440	15,089,729	0.983
Chubb	565,774	7,472,749	−2.40	2.67	4.34	1.69	−3,209,198	7,346,370	0.861
Cincinnati	10,802,307	8,247,771	1.83	0.49	0.89	0.90	−10,827,410	8,177,836	0.876
CNA	−631,440	2,630,205	0.37	0.17	0.29	0.22	2,115,101	1,583,555	0.768
Hartford	42,030,565	271,590,558	−0.34	4.90	−1.16	13.35	−18,526,345	147,906,538	0.030
HCC Ins.	465,647	724,190	3.88	1.71	−0.23	1.71	−720,770	947,986	0.571
Ohio Casualty	3,479,368	951,537	0.91	0.26	−0.54	0.39	−2,201,743	971,712	0.788
Progressive	13,816,591	13,403,390	5.03	11.60	−0.34	3.93	−15,002,413	13,898,864	0.698
SAFECO	−366,248	4,954,621	2.68	0.82	−0.89	0.52	1,475,766	4,565,682	0.791
Selective	1,922,264	1,321,201	0.81	0.54	0.22	0.51	−1,819,411	1,282,157	0.786
St. Paul	−5,654,781	6,380,946	1.38	0.56	0.98	0.90	3,563,808	7,427,642	0.889
United F&C	57,765	180,966	3.07	0.37	−1.19	0.24	−24,077	167,854	0.899
All Companies	22,701,635	18,248,833	2.13	0.29	1.57	0.46	−23,787,168	17,244,028	0.444
All Except AIG	1,906,580	3,832,821	1.85	0.06	0.28	0.10	−2,076,192	3,623,035	0.900

Note: Least squares linear regression based on the experience of each company over the period 1990–2001. MV = market value, PHS = statutory policyholders' surplus, NWP = net written premium, and CR = combined ratio.

projection period plus the terminal value of the company at the end of the 5-year period. The terminal value of the company is based on the parameter values determined in the valuation section.

The basic approach used to test the sensitivity of results to various strategies is to assume several different growth rates within the range of reasonable values. In this case, the growth rates ranged from 0 to 15 percent, in increments of 2.5 percentage points. For each growth rate selected, 500 simulations were run.[1] Both the mean values for each growth rate and the distribution of results were analyzed. The mean values of statutory policyholders' surplus, net written premium, and combined ratio 5 years out (i.e., the year 2007), and the net income for the 5-year projection period (years 2003–2007) are shown on Table 4, as well as the total of the 5-year net income plus the estimated terminal value of the company based on the two regression models. Looking just at the mean values of the results, the optimal growth rate for the company is 0 percent based on the parameters determined when AIG is excluded (column 8). The standard deviation is also the lowest for the 0 percent growth rate in column 8, confirming the optimality of this growth rate based on mean–variance analysis. Thus, in order to maximize its value, the company should not attempt to grow. The primary reason for this result is that if the company is not attempting to grow, its book of business gradually ages (on average, the policyholders have been with the company through more renewal cycles), and so the loss ratio declines. This generates a higher net income in the near future and increases policyholders' surplus.

However, for the parameters based on all companies, including AIG, based on the mean values of the results, the optimal growth rate for the company is 10 percent (column 6). Note that *both* the standard deviations and the mean values increase from the 0 percent growth rate to the 10 percent growth rate, making the choice of an optimal growth rate more complicated. In essence, the growth rates from 0 to 10 percent represent the efficient growth frontier, in line with the efficient investment frontier defined by Markowitz (1952). Mean–variance does not allow the selection of a single optimal growth rate.

One benefit of a DFA model is that the output provides the entire distribution of results, so the decision about the optimal growth rate does not need to be made only on the expected value and standard deviation. Stochastic dominance is a process for decision making under uncertainty that utilizes the entire distribution of potential results, rather than only the first two moments (the mean and standard deviation).[2] The optimal choice is made based on a pairwise comparison of the cumulative distribution function (CDF) for each alternative. Under first-degree stochastic dominance,

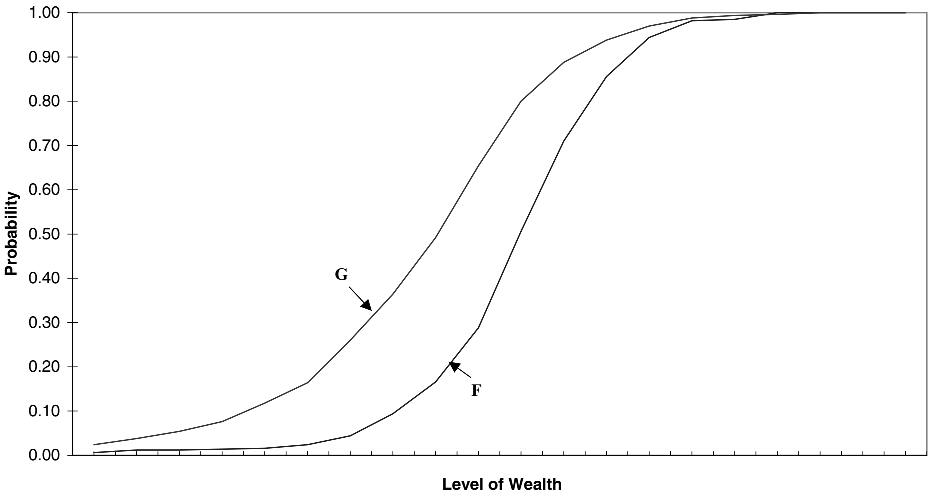
[1] In order to examine the level of uncertainty introduced by running only 500 simulations, an additional set of runs was performed for the base case scenario generating 5,000 simulations for each growth rate. The mean values of most of the figures in columns 6 and 8 in Table 4 (the metric used for optimization) changed by less than 1 percent. There was no impact on the efficient frontier or optimal growth rate under either measure.

[2] For a detailed description of stochastic dominance, see Porter (1973) and Bawa (1975). For an extensive list of references on the topic, see Bawa (1982). For examples of applications of stochastic dominance to insurance, see Cummins and Weiss (1993) and Heyer (2001).

TABLE 4
Mean Values of 500 Simulations

					Base Case				
					All Companies		Without AIG		
Growth Rate (%) 1	PHS in 2007 (000) 2	NI From 2003 to 2007 (000) 3	NWP in 2007 (000) 4	CR in 2007 5	NI + 22,701,635 + 2.13 ∗ PHS + 1.57 ∗ NWP − 23,787,168 ∗ CR (000) 6	Standard Deviation (Column 6) 7	NI + 19,065,80 + 1.85 ∗ PHS + 0.28 ∗ NWP − 20,761,92 ∗ CR 8	Standard Deviation (Column 8) 9	Unacceptable Premium to Surplus Ratio (%) 10
0	55,234	13,239	68,956	1.057	236,706	17,621	134,442	17,968	0.6
2.5	52,252	10,547	78,531	1.060	242,633	19,941	128,908	20,171	1.2
5	48,632	7,243	89,079	1.063	248,091	24,181	121,853	23,745	3.0
7.5	44,059	3,012	100,661	1.069	252,180	30,556	112,394	28,896	15.2
10	38,277	−2,400	113,292	1.076	254,112	39,253	99,807	35,801	42.0
12.5	31,028	−9,247	127,027	1.085	253,178	50,543	83,376	44,672	76.8
15	22,117	−17,732	141,934	1.096	248,855	64,099	62,558	55,345	91.6

FIGURE 1
First-Degree Stochastic Dominance Cumulative Distribution Function



which is based only on the assumption that more wealth is preferred to less, as long as the cumulative distribution function of one alternative lies entirely to the right of another, the alternative generating the rightmost distribution is said to dominate the other. First-degree stochastic dominance is illustrated in Figure 1. Since the unfavorable outcomes are low-wealth positions, to the left along the x-axis, then any distribution that always has a lower likelihood of generating the less desirable outcomes is clearly preferred over a distribution that has a greater chance of these adverse outcomes.

Second-degree stochastic dominance requires risk aversion (specifically, the second derivative of the utility function is negative). In this case, a cumulative distribution function that begins to the right of another cumulative distribution function but later crosses the other function can dominate the second function if the area where the first function is below the second function is at least as large as the area where the first function exceeds the second function. Second-degree stochastic dominance is illustrated in Figure 2. The logic behind second-degree stochastic dominance is that the distribution that begins to the right (F) has a lower probability than the other distribution (G) of the most adverse outcomes. Since the entity is assumed to be risk averse, then the value of having a lower probability of the worst outcomes is greater than missing out on the possibility of exceptionally good outcomes (those to the right along the x-axis) by an equal probability. As long as distribution F has a cumulative advantage over G, as measured by comparing the area where F is to the right of G (area A) to the area where G is to the right of F (area B), then F would be the preferred choice over G.

To illustrate this process, the standard probability distributions of the net income plus the terminal value of the company (based on the parameters for all companies) are displayed in Figure 3 for the five growth rates that cannot be ranked based on mean–variance criteria: 0, 2.5, 5, 7.5, and 10 percent. The higher growth rates generate

FIGURE 2
Second-Degree Stochastic Dominance Cumulative Distribution Function

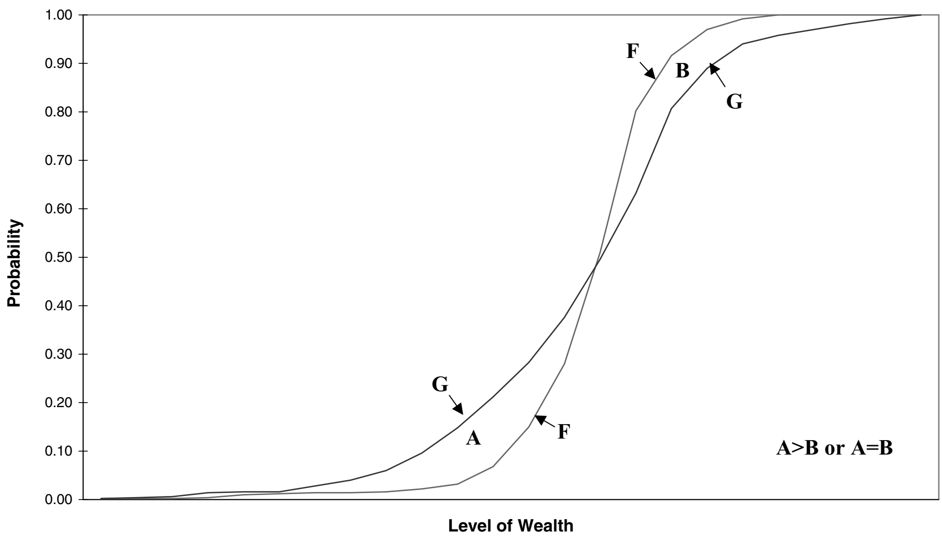
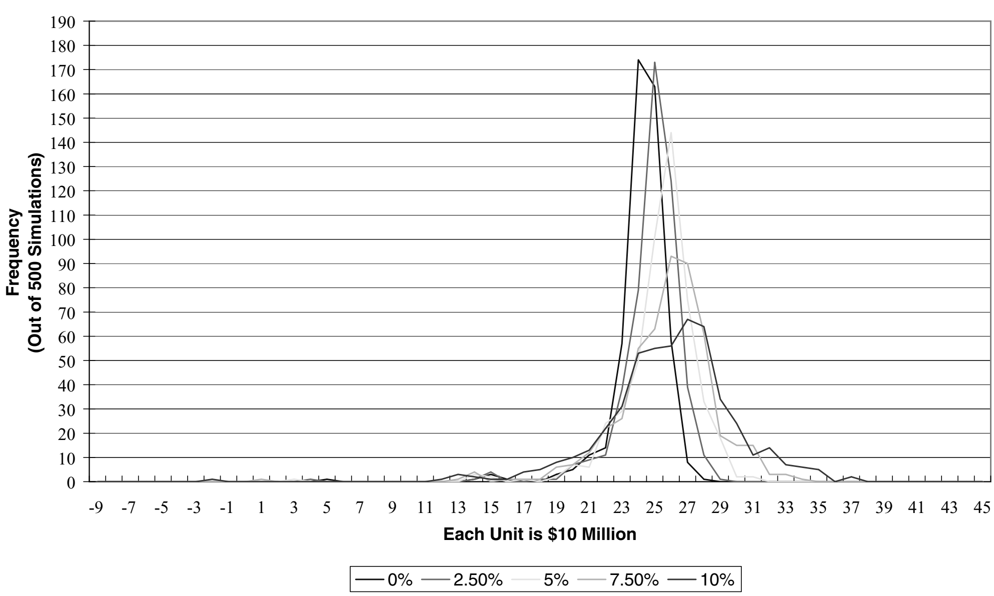


FIGURE 3
Histogram of Company Values Under Different Projected Growth Rates



wider distributions, with more very favorable results *and* more unfavorable results compared with the 0 percent growth rate. The CDFs are shown in Figure 4. The results of the pairwise comparisons of the alternative cumulative distribution functions, where the growth rate labeled distribution F is the CDF that begins to the right of the

FIGURE 4
Commulative Distribution of Company Values Under Different Projected Growth Rates

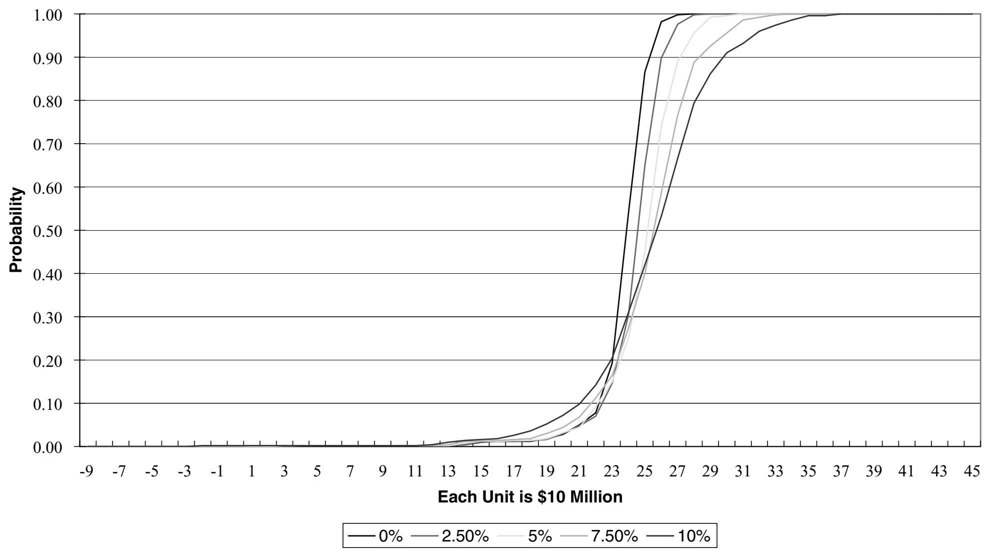


TABLE 5
Test for Second-Degree Stochastic Dominance

		Base Case			
F (%)	G (%)	Intersection Point (Millions)	Area of A	Area of B	Second-Degree Stochastic Dominance
0	2.5	208	5.0	2,977.5	No
0	5.0	222	106.5	5,824.0	No
0	7.5	225	510.5	8,259.5	No
0	10.0	231	1,433.5	10,153.5	No
2.5	5.0	232	157.0	2,904.5	No
2.5	7.5	236	656.0	5,434.5	No
2.5	10.0	241	1,790.0	7,537.0	No
5.0	7.5	246	573.5	2,600.5	No
5.0	10.0	249	1,872.5	4,871.5	No
7.5	10.0	252	1,367.5	2,336.5	No

distribution labeled G, are displayed in Table 5. However, the area where the CDF of the lower growth rate falls below the CDF of the higher growth rate (area of A) is not larger than the area where the higher growth rate lies to the right of the lower growth rate (area of B) for any of the comparisons. Thus, no distribution can be selected based on second-degree stochastic dominance. The optimal growth rate for the parameters based on all companies cannot be determined from this analysis based on either mean–variance analysis or first- or second-degree stochastic dominance. Instead, managers of a company would have to select the optimal growth rate based on a more definitive understanding of their company's relative preferences for risk and return.

As neither mean–variance analysis nor second-degree stochastic dominance produces a single optimal growth rate based on the parameters for all companies, the managers of the insurer may want to utilize an additional factor to determine the appropriate growth rate. A number of additional elements could be introduced to help select the single optimal growth rate for an insurer. The premium-to-surplus ratios vary as a result of the different growth rates; higher growth rates lead to both higher levels of written premiums and lower levels of surplus. Thus, the net written premium-to-statutory surplus ratio, which averages 1.25 over the 500 simulations for 0 percent growth, is on average 2.96 for the 10 percent growth rate. Although this average level is generally acceptable to regulators, values in excess of 3.0 frequently raise concern. (Even regardless of its specific value, significant increases in the ratio over a short period of time would also likely raise concern and invite additional regulatory investigation.) The proportion of outcomes that lead to unacceptable premium-to-surplus levels, which would be larger for the higher growth rate strategies, can be added as a constraint in the maximization process. Column 10 of Table 4 indicates the percent of time that the simulations generated an unacceptable premium-to-surplus ratio, which is defined as exceeding 3 or being less than 0 (a negative premium-to-surplus ratio would occur if the surplus were negative, indicating insolvency). For a growth rate of 0 percent, only 3 of the 500 simulations (0.6 percent) produced an unacceptable premium-to-surplus ratio; however, for a 10 percent growth rate, the premium-to-surplus ratio is at an unacceptable level 42 percent of the time. The company could select an upper limit to the likelihood of experiencing an unacceptable premium-to-surplus ratio (analogously to selecting a target maximum probability of ruin). For example, if a value of 5 percent were selected, then the insurer would not want to grow at the 7.5 percent rate, despite that being part of the mean–variance efficient growth frontier, since such a growth rate leads to a 15.2 percent chance of having an unacceptable premium-to-surplus ratio. In this example, the optimal growth rate would be 5 percent since that would produce the highest company value ($248 million), but still keep the percentage of unacceptable premium-to-surplus values below the predetermined 5 percent level.

Similarly, *DynaMo3* provides the results of 8 of the 11 Insurance Regulatory Information System (IRIS) tests.[3] Another constraint could be to put a limit on the number of failed tests allowed—for example, specifying that no more than three of these tests can have failing values in more than 1.0 percent of the runs. In each case, the insurer would be using additional constraints to determine an optimal value from the efficient growth frontier. One might also be able to create a "penalty metric," which would quantify the potential loss to the company (e.g., in terms of lost future business, loss of reputation, etc.) associated with failure of IRIS tests, downgrades of financial strength ratings, or other possible undesirable events caused by excessive growth.

[3] The three tests involving loss reserve adequacy are not determined. The reserve levels in *DynaMo3* are set at the indicated values with no margin for inadequacy or redundancy.

COMPARATIVE STATICS

The initial parameters that are included in *DynaMo3* were selected to be useful starting points to illustrate the functions of the model. Relative to these default values, many of the parameters should change to reflect individual company experience or current economic conditions. Thus, the results determined throughout this article should be viewed as illustrative, and not applicable in all circumstances to all companies. The default parameters in *DynaMo3* are meant to be illustrative, but reasonable; however, it is expected that any user will verify the appropriateness of each parameter in the model prior to an actual application.

Regardless of the specific values of these default parameters, a DFA model such as *DynaMo3* provides an opportunity to evaluate the sensitivity of future company value to alternative initial conditions. In this section, key parameter values affecting four key areas—the insurance market cycle, the aging phenomenon, policy renewal rates, and interest rates—are varied to examine this sensitivity. This analysis indicates the tradeoffs that exist between likely outcomes and the decision variables.

The first change examined is the initial state of the insurance market. In the base case, both homeowners and workers' compensation were assumed to start in a *mature soft market*. (While this situation may have been appropriate for the late 1990s when *DynaMo3* was first developed, market conditions change over time; the program thus allows for different starting conditions, as well as user-specified transition probabilities between future states.) To determine the impact of a change in market conditions, the current market condition (cell W9 on the XYZ Company HMP-I and WC-I worksheets) were changed to the three other possible market regimes (*immature hard*, *mature hard*, and *immature soft*). The results of this analysis are displayed in Table 6. Although the numerical values change, the relationships are fairly constant. In all cases for the without-AIG parameters, the 0 percent growth rate is still optimal. For the all-companies parameters, the efficient growth frontier runs from 0 to 10 percent for mature hard and immature soft, but only from 0 to 7.5 percent for the immature hard market. In each case, if the company wanted to limit the likelihood of experiencing an unacceptable premium-to-surplus ratio to less than 5 percent, then the optimal growth rate would be 5 percent.

The next change examined is the acuity of the aging phenomenon. Acuity measures the rate of improvement in experience as business matures. In *DynaMo3*, the aging phenomenon is addressed via the loss frequency, which can vary according to the maturity of the business. The loss frequency for the second and subsequent renewal years (cell G158 on the XYZ Company HMP-I and WC-I worksheets) is an input value. In the base case, the frequency for the first renewal year (cell G157) is 1.1 times the value in cell G158. Similarly, the new business frequency (cell G156) is 1.1 times the calculated frequency for the first renewal years. This calculation applies to both HMP and WC. In order to test the sensitivity of results to changes in initial conditions, the acuity parameters were changed as follows: for slower acuity, the multipliers (1.1 in the base case, in both cells G157 and G156) are 1.05; for faster acuity, the multipliers are 1.15. The total profitability was then rebalanced by selecting a new value for cell G158 that kept the total loss ratio by line the same as it was in the base case. The results are displayed in Table 7. The different acuities do not affect the results significantly. In this case, the efficient frontier for the all-companies parameters, the optimal growth

TABLE 6
Mean Values of 500 Simulations

Market Condition	Growth Rate (%) 1	PHS in 2007 (000) 2	NI From 2003 to 2007 (000) 3	NWP in 2007 (000) 4	CR in 2007 5	All Companies: NI + 22,701,635 + 2.13 * PHS + 1.57 * NWP − 23,787,168 * CR (000) 6	All Companies: Standard Deviation (Column 6) 7	Without AIG: NI + 1,906,580 + 1.85 * PHS + 0.28 * NWP − 2,076,192 * CR 8	Without AIG: Standard Deviation (Column 8) 9	Unacceptable Premium to Surplus Ratio (%) 10
Mature Soft	0	55,234	13,239	68,956	1.057	236,706	17,621	134,442	17,968	0.6
	2.5	52,252	10,547	78,531	1.060	242,633	19,941	128,908	20,171	1.2
	5	48,632	7,243	89,079	1.063	248,091	24,181	121,853	23,745	3.0
	7.5	44,059	3,012	100,661	1.069	252,180	30,556	112,394	28,896	15.2
	10	38,277	−2,400	113,292	1.076	254,112	39,253	99,807	35,801	42.0
	12.5	31,028	−9,247	127,027	1.085	253,178	50,543	83,376	44,672	76.8
	15	22,117	−17,732	141,934	1.096	248,855	64,099	62,558	55,345	91.6
Immature Soft	0	55,112	13,109	68,841	1.058	236,110	17,845	134,051	18,133	0.8
	2.5	52,359	10,661	78,625	1.059	243,139	19,643	129,248	19,944	1.2
	5	48,998	7,632	89,435	1.061	249,874	23,248	123,024	22,993	3.6
	7.5	44,723	3,724	101,310	1.064	255,422	29,275	114,525	27,783	14.0
	10	39,225	−1,378	114,305	1.070	258,889	37,522	102,879	34,227	41.4
	12.5	32,066	−8,122	128,388	1.077	258,822	48,086	86,819	42,479	74.4
	15	23,039	−16,716	143,631	1.088	254,685	61,214	65,770	52,980	93.6

(Continued)

TABLE 6
(Continued)

						All Companies		Without AIG		
Market Condition	Growth Rate (%) 1	PHS in 2007 (000) 2	NI From 2003 to 2007 (000) 3	NWP in 2007 (000) 4	CR in 2007 5	NI + 22,701,635 + 2.13 * PHS + 1.57 * NWP − 23,787,168 * CR (000) 6	Standard Deviation (Column 6) 7	NI + 1,906,580 + 1.85 * PHS + 0.28 * NWP − 2,076,192 * CR 8	Standard Deviation (Column 8) 9	Unacceptable Premium to Surplus Ratio (%) 10
Mature Hard	0	55,399	13,400	68,884	1.057	237,101	17,572	134,888	17,939	0.8
	2.5	52,170	10,467	78,573	1.060	242,445	20,137	128,688	20,284	1.2
	5	48,255	6,875	89,262	1.063	247,214	24,393	120,839	23,865	3.8
	7.5	43,351	2,327	100,993	1.068	250,519	30,449	110,493	28,807	16.0
	10	36,993	−3,656	113,785	1.075	250,907	38,087	96,316	34,985	48.0
	12.5	28,808	−11,447	127,617	1.085	247,166	47,182	77,235	42,361	81.2
	15	18,505	−21,342	142,507	1.097	238,407	57,279	52,422	50,652	96.6
Immature Hard	0	55,486	13,492	68,991	1.057	237,561	17,503	135,171	17,884	0.6
	2.5	52,089	10,384	78,527	1.060	242,113	20,163	128,442	20,329	1.2
	5	47,941	6,552	89,033	1.064	245,832	24,590	119,869	24,136	3.8
	7.5	42,730	1,681	100,560	1.070	247,815	30,648	108,573	29,187	17.4
	10	36,028	−4,660	113,095	1.079	246,674	38,262	93,324	35,460	48.8
	12.5	27,475	−12,832	126,673	1.090	241,335	47,415	73,109	42,941	83.8
	15	16,900	−23,012	141,318	1.104	231,298	57,553	47,438	51,293	97.0

TABLE 7
Mean Values of 500 Simulations

						All Companies		Without AIG		
Different Acuities	Growth Rate (%) 1	PHS in 2007 (000) 2	NI From 2003 to 2007 (000) 3	NWP in 2007 (000) 4	CR in 2007 5	NI + 22,701,635 + 2.13 * PHS + 1.57 * NWP − 23,787,168 * CR (000) 6	Standard Deviation (Column 6) 7	NI + 1,906,580 + 1.85 * PHS + 0.28 * NWP − 2,076,192 * CR 8	Standard Deviation (Column 8) 9	Unacceptable Premium to Surplus Ratio (%) 10
Base Case	0	55,234	13,239	68,956	1.057	236,706	17,621	134,442	17,968	0.6
	2.5	52,252	10,547	78,531	1.060	242,633	19,941	128,908	20,171	1.2
	5	48,632	7,243	89,079	1.063	248,091	24,181	121,853	23,745	3.0
	7.5	44,059	3,012	100,661	1.069	252,180	30,556	112,394	28,896	15.2
	10	38,277	−2,400	113,292	1.076	254,112	39,253	99,807	35,801	42.0
	12.5	31,028	−9,247	127,027	1.085	253,178	50,543	83,376	44,672	76.8
	15	22,117	−17,732	141,934	1.096	248,855	64,099	62,558	55,345	91.6
Slower	0	54,561	12,587	69,565	1.058	235,554	18,262	132,712	18,559	0.8
	2.5	51,698	10,008	78,964	1.060	241,577	20,572	127,465	20,767	1.4
	5	48,237	6,856	89,293	1.064	247,183	24,776	120,793	24,302	3.6
	7.5	43,889	2,842	100,599	1.069	251,537	31,036	111,891	29,329	15.4
	10	38,441	−2,244	112,914	1.076	254,017	39,467	100,161	35,972	41.6
	12.5	31,650	−8,639	126,298	1.085	253,963	50,477	84,931	44,574	74.4
	15	23,362	−16,507	140,794	1.095	250,949	63,703	65,768	54,902	90.2
Faster	0	55,872	13,856	68,352	1.056	237,749	17,076	136,071	17,470	0.6
	2.5	52,758	11,038	78,101	1.059	243,542	19,440	130,216	19,695	1.2
	5	48,987	7,590	88,876	1.063	248,889	23,682	122,802	23,280	3.0
	7.5	44,197	3,150	100,718	1.068	252,711	30,176	112,804	28,544	14.8
	10	38,078	−2,590	113,669	1.075	254,099	39,096	99,356	35,681	43.0
	12.5	30,356	−9,905	127,758	1.084	252,239	50,685	81,681	44,845	78.8
	15	20,816	−19,015	143,063	1.096	246,565	64,537	59,183	55,840	92.8

rate for the without-AIG parameters, and the values considering the constraint for unacceptable premium-to-surplus ratios, all remain the same.

The next sensitivity test examined is with respect to the renewal rate. Renewal rates reflect the proportion of policies that renew with the insurer. These values (listed in cells F34-L36 in the HMP-I and WC-I worksheets) vary by line and by the age of the business. For the base case, the renewal rate for homeowners business is 75 percent for new business, 90 percent for business that has been renewed once, and 95 percent for business that has been renewed at least twice. For workers' compensation, the renewal rates are 80 percent for new business and 90 percent for policies that have been renewed once or more. To test sensitivity to these parameters, the renewal rates for all years were increased to 87.5, 95, and 97.5 percent, respectively, for homeowners, and 90 and 95 percent for workers' compensation; the rates were then decreased to 72.5, 85 and 92.5 percent for homeowners and to 70 and 85 percent for workers' compensation. The results are displayed in Table 8. Once again, although the numerical values changed, neither the efficient growth frontier (for the all-companies parameters) nor the optimal values (for the without-AIG parameters) were affected. The likelihood of an unacceptable premium-to-surplus ratio for a 5 percent growth rises somewhat (to 4.8 percent) for the lower renewal rate, but it would still not change the optimal growth rate if the cutoff were 5 percent.

Finally, the impact of different starting interest rates was tested. *DynaMo3* uses a one-factor Cox–Ingersoll–Ross (CIR) interest rate model. In the base case, the initial parameters (listed in cells E9–E14 in the General Input worksheet) are a speed of reversion of 0.25, a long-run mean of 6 percent, a standard error of 5 percent and an initial short-term interest rate of 4.91 percent. Although the first three parameters are still reasonable, the short-term interest rate has declined sharply over the last few years, since the model was originally developed. In line with this fact, and in order to test the sensitivity of results to this variable, the impact of reducing the initial short-term interest rate to 2 percent, and increasing it to 8 percent, leaving the rest of the parameters unchanged, was examined. The results are displayed in Table 9. These changes had the most significant impact on the results of any of the parameter changes tested. For the lower starting interest rate, the efficient growth rate frontier expanded to run from 0 to 15 percent. This occurred because the favorable impact of a lower interest rate, and correlated lower inflation rate, on underwriting results more than offset the reduction in investment income that the lower interest rate produced.[4] Since interest rates and inflation are correlated in this model and rate increases are subject to regulatory constraints in some states, a lower rate of inflation leads to more adequate rates. Note that, compared to the base case, the combined ratio (column 5) declines for the lower interest rate and increases for the higher interest rate. On the other hand, investment income is not markedly affected by the change in the starting interest rate parameter, since many of the investments are long-term bonds. Only new investments and reinvested capital earn the current interest rate; other investments continue to earn the original interest rate. The lower premium levels

[4] For example, in the case of the 7.5% growth rate, the average underwriting gain (loss) for the 5 year period increased by $9,869,000 when the starting interest rate was changed from 4.91 percent to 2 percent, but the investment income declined by only $653,000.

TABLE 8
Mean Values of 500 Simulations

						All Companies		Without AIG		
Renewal Rate	Growth Rate (%) 1	PHS in 2007 (000) 2	NI From 2003 to 2007 (000) 3	NWP in 2007 (000) 4	CR in 2007 5	NI + 22,701,635 + 2.13 * PHS + 1.57 * NWP − 23,787,168 * CR (000) 6	Standard Deviation (Column 6) 7	NI + 1,906,580 + 1.85 * PHS + 0.28 * NWP − 2,076,192 * CR 8	Standard Deviation (Column 8) 9	Unacceptable Premium to Surplus Ratio (%) 10
Base Case	0	55,234	13,239	68,956	1.057	236,706	17,621	134,442	17,968	0.6
	2.5	52,252	10,547	78,531	1.060	242,633	19,941	128,908	20,171	1.2
	5	48,632	7,243	89,079	1.063	248,091	24,181	121,853	23,745	3.0
	7.5	44,059	3,012	100,661	1.069	252,180	30,556	112,394	28,896	15.2
	10	38,277	−2,400	113,292	1.076	254,112	39,253	99,807	35,801	42.0
	12.5	31,028	−9,247	127,027	1.085	253,178	50,543	83,376	44,672	76.8
	15	22,117	−17,732	141,934	1.096	248,855	64,099	62,558	55,345	91.6
Higher	0	56,050	14,027	68,024	1.057	237,772	17,158	136,479	17,533	0.6
	2.5	53,171	11,435	77,488	1.059	243,846	19,407	131,204	19,629	1.2
	5	49,724	8,304	87,929	1.063	249,685	23,407	124,613	22,958	2.6
	7.5	45,405	4,328	99,389	1.068	254,381	29,517	115,845	27,817	11.4
	10	39,908	−798	111,912	1.075	257,044	37,994	104,043	34,489	36.0
	12.5	33,006	−7,300	126,298	1.083	257,011	48,950	88,565	43,058	70.8
	15	24,480	−15,399	140,312	1.904	253,712	62,356	68,812	53,573	89.8
Lower	0	54,317	12,351	69,947	1.058	235,403	18,226	132,133	18,547	0.8
	2.5	51,177	9,503	79,635	1.060	241,013	20,709	126,183	20,942	1.4
	5	47,338	5,980	90,305	1.064	245,973	25,231	118,537	24,814	4.8
	7.5	42,485	1,468	101,993	1.070	249,345	31,906	108,308	30,265	18.0
	10	36,372	−4,275	114,740	1.077	250,418	40,850	94,811	37,411	48.4
	12.5	28,722	−11,523	128,598	1.086	248,411	52,536	77,271	46,632	81.8
	15	19,369	−20,455	143,625	1.098	242,876	66,287	55,220	57,520	93.8

TABLE 9
Mean Values of 500 Simulations

Interest Rate	Growth Rate (%) 1	PHS in 2007 (000) 2	NI From 2003 to 2007 (000) 3	NWP in 2007 (000) 4	CR in 2007 5	All Companies: NI + 22,701,635 + 2.13 ∗ PHS + 1.57 ∗ NWP − 23,787,168 ∗ CR (000) 6	All Companies: Standard Deviation (Column 6) 7	Without AIG: NI + 1,906,580 + 1.85 ∗ PHS + 0.28 ∗ NWP − 2,076,192 ∗ CR 8	Without AIG: Standard Deviation (Column 8) 9	Unacceptable Premium to Surplus Ratio (%) 10
Base Case	0	55,234	13,239	68,956	1.057	236,706	17,621	134,442	17,968	0.6
	2.5	52,252	10,547	78,531	1.060	242,633	19,941	128,908	20,171	1.2
	5	48,632	7,243	89,079	1.063	248,091	24,181	121,853	23,745	3.0
	7.5	44,059	3,012	100,661	1.069	252,180	30,556	112,394	28,896	15.2
	10	38,277	−2,400	113,292	1.076	254,112	39,253	99,807	35,801	42.0
	12.5	31,028	−9,247	127,027	1.085	253,178	50,543	83,376	44,672	76.8
	15	22,117	−17,732	141,934	1.096	248,855	64,099	62,558	55,345	91.6
Lower	0	56,504	14,743	65,618	1.053	235,758	16,344	137,367	16,261	0.2
	2.5	54,956	13,487	75,436	1.051	246,686	17,583	136,003	17,493	1.0
	5	53,255	12,121	86,388	1.048	258,949	20,552	134,561	19,616	1.4
	7.5	51,308	10,566	98,557	1.046	272,407	25,813	132,817	23,014	2.2
	10	48,935	8,624	112,040	1.044	286,621	33,423	130,264	27,660	11.0
	12.5	45,932	6,108	126,930	1.043	301,110	43,815	126,364	34,238	34.8
	15	42,158	2,878	143,331	1.043	315,584	56,576	120,743	42,267	64.6
Higher	0	53,744	11,542	72,549	1.061	237,378	19,765	130,987	20,579	1.0
	2.5	49,782	7,873	82,330	1.066	240,502	22,819	122,714	23,621	2.8
	5	44,954	3,362	93,086	1.072	242,453	27,241	112,271	27,837	9.8
	7.5	38,974	−2,275	104,863	1.080	242,387	32,724	98,854	32,927	33.4
	10	31,585	−9,294	117,714	1.089	239,579	39,053	81,742	38,666	64.6
	12.5	22,490	−18,002	131,638	1.101	233,085	45,551	60,083	44,513	92.2
	15	11,671	−28,413	146,707	1.114	222,972	52,048	33,850	50,350	99.2

that accompanied the lower interest rates also reduced the unacceptable premium-to-surplus ratios for all growth rates, bringing the ratio for a 7.5 percent growth rate to 2.2 percent. Thus, based on this constraint the optimal growth rate would be 7.5 percent. Conversely, for the higher initial short-term interest rate, the efficient growth frontier was reduced to 0–5 percent, and the optimal growth rate based on a 5 percent unacceptable premium-to-surplus ratio would be reduced to 2.5 percent, since the 5 percent growth rate produced an unacceptable ratio 9.8 percent of the time.

ANALYSIS OF RESULTS

In order to understand if the no- or low-growth strategies are indeed optimal, the relationships inherent in the DFA model need to be fully understood. Thus, additional discussion of the modeling that underlies the above results is warranted.

One important factor to consider is the implied rate change variable. This value, which goes into the rate level calculation, is a function of the desired growth rate, which represents the strategic variable, and market conditions. One prominent feature of the P-L insurance market is the underwriting cycle.[5] Market conditions shift over time between soft markets, in which strong competition limits the ability of insurers to raise prices, and hard markets, in which insurance coverage is difficult to obtain and insurers are freer to raise prices without losing market share. As alluded to in the previous section, *DynaMo3* provides for four distinct market conditions: mature hard, immature soft, mature soft, and immature hard. The market is allowed to move between these different states based on a stochastic function that reflects differing probabilities of movement from one condition to another. Depending on the specific market condition and the targeted growth rate, an implied rate change value is determined (determined as a rate change in excess of the change needed to compensate for loss cost inflation). The model allows the company to achieve its targeted growth rate by implementing a rate level which, according to the demand curve parameters associated with each market condition, will attract enough new policyholders to achieve overall targeted exposure growth. Thus, the higher the targeted growth rate, the lower the implied rate change, since it is more difficult to attract new business if rates are increasing. Implied rate changes are higher in hard markets and lower in soft markets. Based on the 500 simulated runs for the base-case parameters, the implied rate change for the 0 percent growth rate averaged 1.3 percent, compared to an average implied rate change of −1.2 percent for a 10 percent growth rate. Essentially, if the insurer is not trying to grow, then it can raise rates 1.3 percent per year (on average across the 500 simulations) over the loss cost inflation rate to achieve this target. In order to grow by 10 percent per year, the insurer would have to restrain rate levels 1.2 percent per year below loss cost inflation. The higher rates would lead to greater profitability and the lower rates to reduced profitability. This effect compounds the impact of the aging phenomenon. In following a selected growth strategy, the insurer needs to adjust premium levels in line with targeted growth rates.

Another consideration of rate changes is the potential impact on persistency, or renewal rates. As described previously, *DynaMo3* separates business into three age

[5] Recent analyses of the underwriting cycle include Lamm-Tennant and Weiss (1997), Fung et al. (1998), and Lai et al. (2000). An extensive analysis of the issue is provided in Cummins, Harrington, and Klein (1992).

categories: one for new business, one for first renewal business, and one for second and subsequent renewals. A separate persistency rate can be applied to each category. Rate changes are likely to affect the renewal rates, with higher rate changes reducing the renewal rates. Lower renewal rates reduce the profitability of long-term business, because fewer policies would remain with the insurer. If rates are being increased more rapidly in the low-growth strategies, then persistency will also be affected. In the current version of *DynaMo3*, persistency is not directly affected by the level of rate changes.

Another limitation of *DynaMo3* is that whatever growth rate is targeted is applied in every type of market condition. Since it is more difficult to grow at a particular rate in a soft market than in a hard market, a company has to temper its rate levels more in a soft market to achieve the targeted growth. An alternative strategy for managing growth would be to vary the growth rate target based on the market conditions, growing more slowly in soft markets and more rapidly in hard markets. Examining the effect of this type of strategy would require making modifications to the DFA program. Since *DynaMo3* is a public-access model, based on Excel with clear documentation, making such a revision is relatively straightforward. *DynaMo3* was written with the recognition that individual insurers or researchers would want to modify the basic program to meet their varying needs. Thus, the program can be readily customized to meet particular goals.

CAVEATS

Any application of a financial model should include the warning that all models are simplified versions of reality, and cannot be relied upon to mimic reality in all circumstances. In order to be useful, a model includes a number of simplifying assumptions about the real world to focus attention on particular relationships. No model can include all possible relationships or anticipate all feasible developments. Each model has both parameter and process risk. Parameter risk deals with the use of the correct value for a variable; process risk refers to the relationships being correctly reflected. This particular DFA model deals with quantifiable risk only. In addition, insurers face other significant risks that cannot be accurately quantified in advance, but if they occurred would have a major impact on the industry. One example of an excluded risk is the possibility of a line of business being socialized, with the government taking over the coverage. Automobile insurance in British Columbia is an example of this occurrence. Management fraud is another risk that is excluded from this DFA model. Although fraud plays a large role in insurer insolvencies, all insurers are not equally exposed to this problem. Also, quantifying the actual risk of its management perpetrating a fraud against the company is beyond the scope of actuarial analysis, and is, therefore, not included in the model. This means, though, that the actual risk of financial difficulty for an insurer is higher than the model would indicate, because there is some, albeit unknown level of, risk of fraudulent behavior by management.

Other catastrophic risks are also left out of this model. A devastating meteor strike, major terrorist attacks, a new global weather pattern that leads to unprecedented windstorms and flooding, a novel legal determination making employers liable for currently uncompensated employee losses, all would impact the insurance industry adversely. None of these are reflected in the model because the likelihood of their occurrence and the severity of their impact could not be readily quantified when the model was initially developed. Leaving them out of the model does not mean that they cannot happen, or that the modelers do not think they could occur—it only means that these risks were too difficult to incorporate in this model. However, recent advances in modeling some of these risks, especially relating to terrorism, could allow the model to be modified to incorporate certain additional risks. AIR Worldwide, a subsidiary of ISO that specializes on modeling catastrophes, has already developed a model for terrorism losses. Woo (2002, 2003) and Gollier (2002) provide a useful analysis of the issues involved in quantifying this type of risk. The DFA process would benefit from further research into quantifying additional areas of catastrophe risk.

Although DFA can be a very useful tool for both solvency testing and strategic planning, DFA should not be considered the ultimate solution to all the problems of operating, or regulating, an insurance company. Following the strategies that appear to be optimal based on DFA models will help an insurer, but will not guarantee long-term success. DFA allows insurers to position themselves to take better advantage of opportunities and avoid potential problems that can be accurately projected by the use of mathematical modeling. However, additional risks and developments beyond the ability of the model to quantify should be expected to occur. DFA can be a significant help in managing an insurance company, but it will not provide managers with answers to all the problems that face the insurance industry. Insurance is too complex to be modeled completely.

CONCLUSION

This research illustrates how DFA, a tool initially developed for regulatory purposes, can be effectively used for strategic planning by insurers. This work provides an example of how an insurer can utilize a DFA model to determine the optimal growth rate based on a combination of mean–variance efficiency, stochastic dominance and constraints of leverage. Over much of the range of growth rates tested, increasing the growth rate reduced statutory policyholders' surplus and current net income, but increased both the future market value of the insurer and the volatility of results. Depending on the strength of the aging phenomenon, policy renewal rates, insurance market conditions and current interest rates, the optimal growth rate for the modeled insurer varied from 0 to 7.5 percent. Growth rates of 10 percent or higher generated unacceptable premium to surplus ratios too frequently under all parameter values tested. Low initial interest rates increased the incentive for growth as the increase in underwriting gains more than offset the reduction in investment income. High initial interest rates lowered the optimal growth rate. Varying the other key parameters did not affect the optimal growth rate significantly.

APPENDIX A

Description of *DynaMo3*

The dual objectives considered by the team that developed this model were to create a model that was *realistic* enough to be useful, but at the same time *simple* enough to be understood. (Simplicity, along with public accessibility and transparency, was important since the model was also designed with educational purposes in mind.) Thus, when choices had to be made about the degree of complexity to incorporate in the model regarding a particular relationship, the guiding principle involved balancing realism and simplicity. Enhancements that added only a little to the accuracy of the model, but much to the complexity, were not included. However, when a relationship had to be complex to be at all useful, the complex relationship was included.

Several articles have been published describing the details of this model (D'Arcy et al., 1997a,b, 1998; Walling et al., 1999). These articles can be consulted for details; brief comments summarizing the model will be provided here. In essence, *DynaMo3* simulates the results of a P-L insurance company over the next 5 years for multiple stochastically-generated runs (sometimes also referred to as "trials" or "iterations"). The balance sheet, the operating statement, and the IRIS test results for the simulated company are calculated for each run. The user can select any cell in the *DynaMo3* spreadsheet as an output; the model will then compile statistics—including expected values, standard deviations, minimums, maximums, medians, and distributions and percentiles of all simulated results—for these selected outputs. The sheer volume of output information that is available requires careful selection of the items to view, since it is easy to get lost in the details. Commonly, users will select as outputs such items as the surplus (statutory and GAAP), loss ratio, combined ratio, operating ratio, net investment income, net written premiums, and gross or net income to analyze. For many output variables, it is important to examine results on a by-line basis, and the model allows this as well.

An example of an output graph from the DFA model is shown as Figure A1. In this situation, the user is evaluating the effect of different growth rates on the statutory policyholders' surplus of the company in the year 2007 (i.e., 5 years in the modeling future). Each curve represents a different growth rate. In this situation, the lowest growth rate generates the highest surplus distribution.

DynaMo3 consists of six separate but interrelated modules. The modules cover, respectively, investments, underwriting, the interest rate generator, catastrophes, taxation,

FIGURE A1
Statutory PHS 2007 Under Different Growth Rates

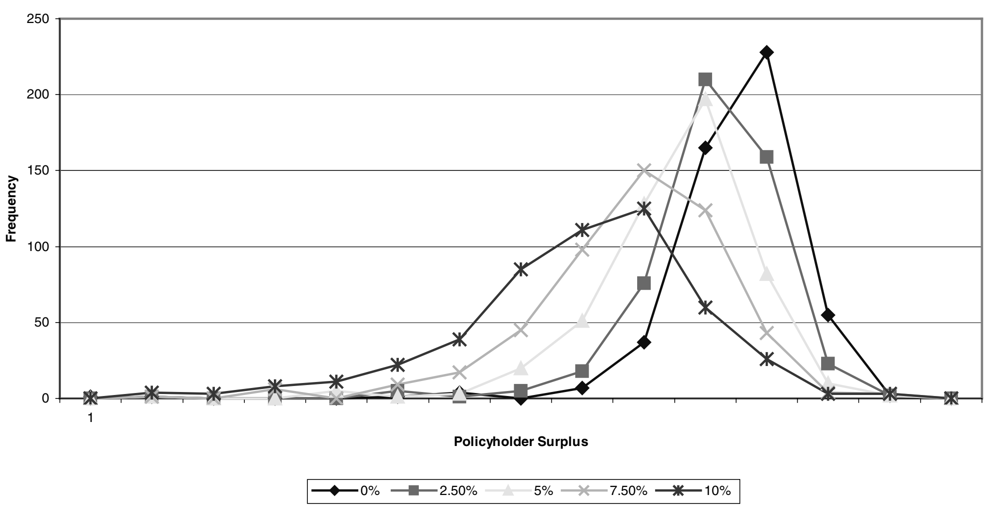


TABLE A1
Key Variables in *DynaMo3*

Financial	Underwriting
Short-term interest rate	Loss frequency and severity
Term structure of interest rates	Rates and exposures
Default potential of investments	Expenses
Equity performance	Underwriting cycle
Inflation	Loss reserve development
Mortgage prepayment patterns	Jurisdictional risk
	Aging phenomenon
	Payment patterns
	Catastrophes
	Reinsurance
	Taxes

and loss reserve development. The basic model contains two lines of business, although additional lines can be added. Each line of business is divided into three age categories: new business, first renewals, and second and subsequent renewals. The loss frequency, severity, and premium levels can vary by age category of the business.

The key variables in *DynaMo3* are listed on Table A1. The primary variable is the short-term interest rate, the simulated realizations of which affect the investment income, the market values of bonds and other investments, the inflation rate, loss severity, premium levels, and loss reserve development. Due to the importance of this variable, significant care was taken to model interest rates appropriately.

There is an extensive literature on interest rate modeling that is summarized very effectively by Chapman and Pearson (2001) and Hull (2003). Term structure models can be classified into two primary types, general equilibrium models and arbitrage-free models. General equilibrium models are based on proposed economic relationships that interact to determine interest rates. Although the initial shape of the term structure of interest rates in a general equilibrium model can be adjusted to take a variety of forms (upward sloping, inverted, humped), it rarely conforms exactly to the current yield curve. Alternatively, arbitrage-free models begin with the current term structure of interest rates and allow changes to evolve over time. Both types of models have been tested to determine their usefulness in matching historical and potential future interest rate movements (Ahlgrim, D'Arcy, and Gorvett, 1999). Arbitrage-free models can tend to drift into unrealistic levels (extraordinarily high or negative values) when run over an extended period of time. Although arbitrage-free models are preferred for pricing interest rate derivatives in which the time-frame for interest rate movements is very short, they are not as useful for generating interest rate scenarios over the next 20–30 years, as needed for a DFA analysis.[6] Thus, a general equilibrium model was selected for this DFA model.

[6] Even though operating results are only projected for 5 years in this model, interest rate scenarios for 30 years need to be generated in order to price long term bonds held by the insurer.

TABLE A2
DynaMo3 Worksheets

Disclaimer	General Input
Investment Input	Reinsurance Input
XYZ Company-HMP-I	XYZ Company-WC-I
XYZ Company-HMP-O	XYZ Company-WC-O
LINE-SUMMARY	Cat Generator
Bond Summary	Bond 1
Bond 2	Bond 3
Bond 4	Bond 5
Stocks	Tax Calculator
Investment Distribution	Output
Statutory Summary	GAAP Summary
Simulation Data	Rnd Numbers

Term structure models are also classified based on the number of factors allowed to vary. One-factor models have only one variable, generally the short-term interest rate. Two-factor models allow the short-term interest rate and another factor, either the volatility or the long-term mean, to vary. Other models allow three or more factors to vary. Additional research has been conducted to determine the appropriate number of factors to include for various applications. Ahlgrim (2001) determined that a one-factor model was sufficient for a DFA model of a P-L insurer, but a two-factor model was more appropriate for a similar framework for a life insurer. Life insurance products and the financial structure of life insurers are more sensitive to interest rate fluctuations than P-L insurers, requiring a more complex term structure model. Based on this research, *DynaMo3* models interest rates based on the CIR one-factor model (Cox, Ingersoll, and Ross, 1985). This model has the advantages of avoiding negative interest rates (at least in its continuous-time format), being relatively easy to understand (in comparison with some other term structure models), and providing the flexibility needed to model the term structure of interest rates as appropriate for P-L insurers.

DynaMo3 consists of 24 different Excel worksheets, which are listed in Table A2. The first sheet listed in the table, "Disclaimer," indicates that the model contains preloaded parameters (to assist in its use as an educational tool), but that these parameters may or may not be appropriate in a given time frame or situation. Any user of the model is expected to review all inputs and programmed parameters and relationships, and change whatever is inappropriate, prior to use. Nevertheless, the model as downloadable from the Pinnacle website provides a tool with which studies of the impact of a variety of issues can be analyzed. In this article, we have analyzed exposure growth rates and their impact on a hypothetical insurer.

Many of the 24 worksheets in the model have input fields, which allow the user to document the company, financial, and economic environment relevant to the analysis, and to enter the parameters associated with all stochastic variables affecting the 5-year simulation of results. The General Input page allows the user to indicate the first year to be simulated, the parameters of the CIR interest rate model, and the level of mortgage prepayment activity (important in order to evaluate mortgage-backed securities). The "Investment Input" worksheet provides for documentation, along the lines of Schedule D in the Annual Statement required by regulators in the United States, of all the investments of the insurer being analyzed. The Reinsurance Input worksheet receives detailed information on the company's reinsurance program. The following two worksheets, XYZ Company-HMP-I and XYZ Company-WC-I, describe the current book of business of the insurer for each line of coverage, in this case homeowners and workers' compensation, including information on premiums, losses, exposures, expenses, frequency and severity, market conditions, growth targets, and renewal rates. The next three worksheets, XYZ Company-HMP-O, XYZ Company-WC-O, and LINE SUMMARY, provide detailed information about the output of the simulation for each of the next 5 years. The CAT Generator worksheet is the catastrophe module; it contains the parameters for simulating the number of catastrophes (based on a Poisson distribution), the size of each catastrophe (based on a lognormal distribution), and a geographic locator and a state contagion matrix, parameterized based on historical catastrophe activity.

The next six worksheets contain information that is used to calculate the values of bond investments. The five individual bond worksheets (Bonds 1–5) each provide for a different set of bonds, allowing the risk premium and effective tax rate to vary. The Bond Summary worksheet summarizes the book and market values of all bond investments. The Stocks worksheet is used to value the equity investments of the insurer, both common and preferred, and affiliated and unaffiliated. The Tax Calculator determines the tax liability of the insurer, after calculating the taxes under both the regular tax rate and the alternative minimum tax formula. The Investment Determination worksheet provides information on the insurer's cash flows and how they are being allocated to different types of investments. The Output worksheet shows the company's balance sheet for the 5-year projection period, along with the IRIS ratios. The Statutory Summary worksheet shows the financial statement and income statement for each of the projection years on a statutory basis. The GAAP Summary shows the same information based on Generally Accepted Accounting Principles. The Simulation Data worksheet is the sheet on which the user indicates the number of runs to be simulated, and selects the variables to be analyzed. The final worksheet, Rnd Numbers, lists the random numbers used in the simulation, to aid in checking the model.

DynaMo3 is set up with the data of a hypothetical insurer, XYZ Company, although users are expected to replace the example values with those that represent whatever insurer they wish to analyze. The XYZ Company is hypothesized to be a small insurer that wrote approximately $59 million of premium in the most recent year, equally split between Homeowners and Workers' Compensation. The company writes in two states, Florida and Illinois. The company has $102.5 million in assets ($93 million in bonds) and a statutory surplus of just under $50 million. Of the 58,000 homeowners exposures, 38,000 have been with the insurer through two or more renewal cycles. Of the 21,800 workers' compensation exposures, 16,650 have been with the company through two or more renewal cycles. These, along with all other hypothetical corporate, economic, and financial data embedded in *DynaMo3* as default values, are the characteristics that are used in the analysis of the impact of growth in this article.

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