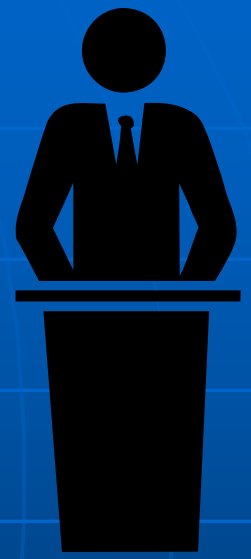
A Comparison of Actuarial Financial Scenario Generators



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Outline of Presentation

- Uses of Financial Scenario Generators
- An overview of the competing models
  - Interest rates
  - Equity returns
- A quick application of the models
- Conclusions

Applications of a Financial Scenario Generator

- Insurer performance sensitive to economic conditions
- Projecting insurer operations under different scenarios requires a financial “engine”
- Uses in many types of actuarial analyses
  - Ratemaking
  - Dynamic financial analysis
  - Capital requirements and allocation
  - Financial risk management solutions

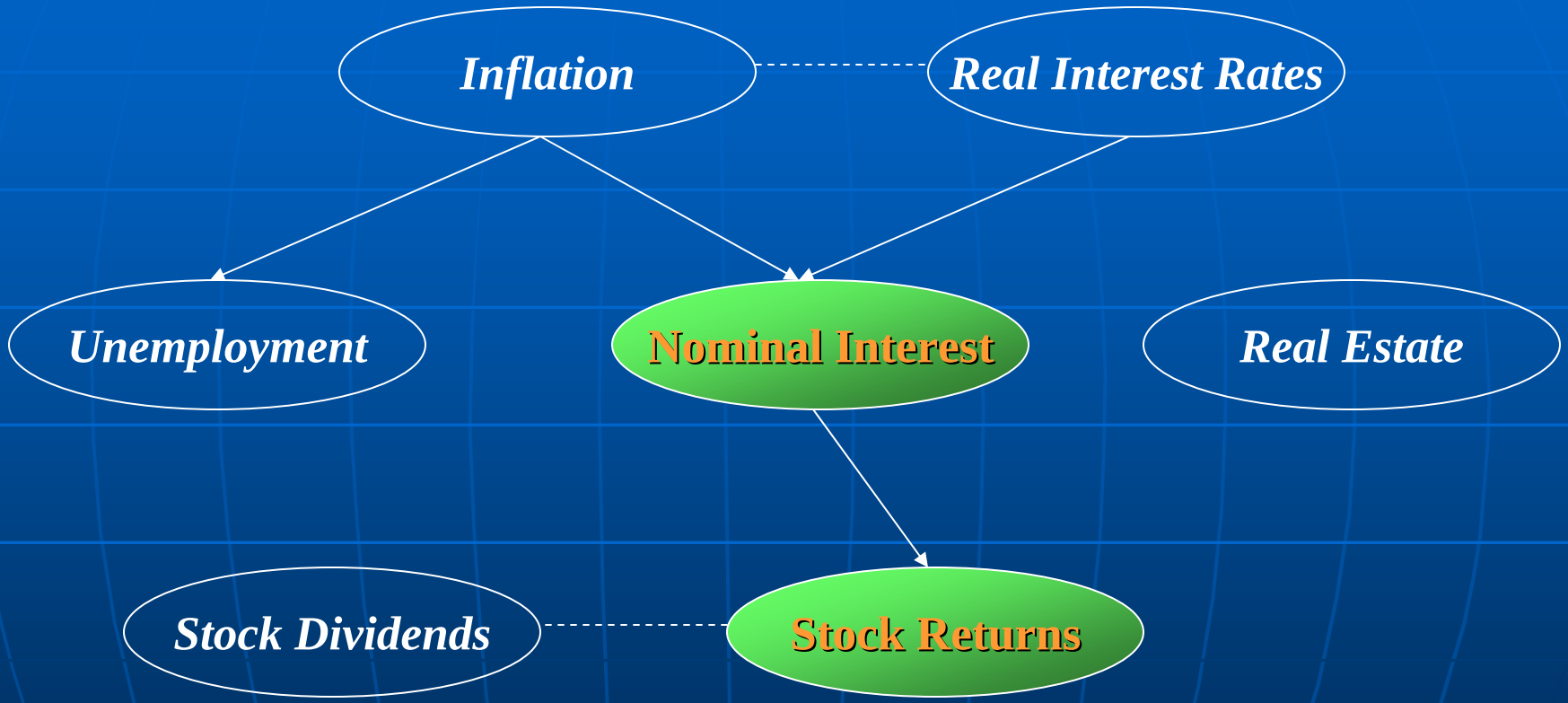
Prior Work

- Deterministic analysis with some scenarios
- Wilkie, 1986 and 1995
  - Widely used internationally
- Hibbert, Mowbray, and Turnbull, 2001
  - Modern financial tool
- CAS/SOA project (a.k.a. the Financial Scenario Generator) applies Wilkie/HMT to U.S.
- American Academy of Actuaries C-3 Phase II

CAS/SOA Model

- http://casact.org/research/econ/
- http://www.soa.org/ccm/content/areas-of-practice/finance/mod-econ-series-coor-int-rate-scen/
- Provides "actuaries with a model for ***projecting economic and financial indices***, ***with realistic interdependencies*** among the variables."
- Excel spreadsheet using @RISK simulation software
- Users can select different parameters, define new variables, and track any output

CAS/SOA Model
Modeled Economic Series



AAA Model

- http://www.actuary.org/life/phase2.asp
- Guidance for Risk-Based Capital (RBC) requirements for variable products with guarantees
- Focus is on annuities' major risks: interest rate risk (Phase I) and equity risk (Phase II)
- In absence of internally developed models, AAA created 10,000 scenarios of interest rates and equity returns

CAS/SOA Model: Nominal Interest Rates

- Combines inflation and real interest rates

$$1+i = \{(1+q) \times (1+r)\}$$

where i = nominal interest rate

q = inflation

r = real interest rate

Inflation (q)

- Modeled as an Ornstein-Uhlenbeck process
  - One-factor, mean-reverting

$$dq_t = \kappa_q (\mu_q - q_t)\, dt + \sigma\, dB_q$$

Real Interest Rates (r)

- Two-factor Vasicek term structure model
- Short-term rate (r) and long-term mean (l) are both stochastic variables

$$dr_t = \kappa_r (l_t - r_t)\, dt + \sigma_r\, dB_r$$

$$dl_t = \kappa_l (\mu_l - l_t)\, dt + \sigma_l\, dB_l$$

AAA Model

- Stochastic volatility model
- Long rate process

$$d(\ln \lambda_t) = \kappa_\lambda (\theta_\lambda - \ln \lambda_t) dt + a\varphi_t + \nu_t dB_t^\lambda$$

- Long and short rate spread process

$$d\varphi_t = \kappa_\varphi (\theta_\varphi - \varphi_t) dt + b\lambda_t + \sigma_\varphi dB_t^\varphi$$

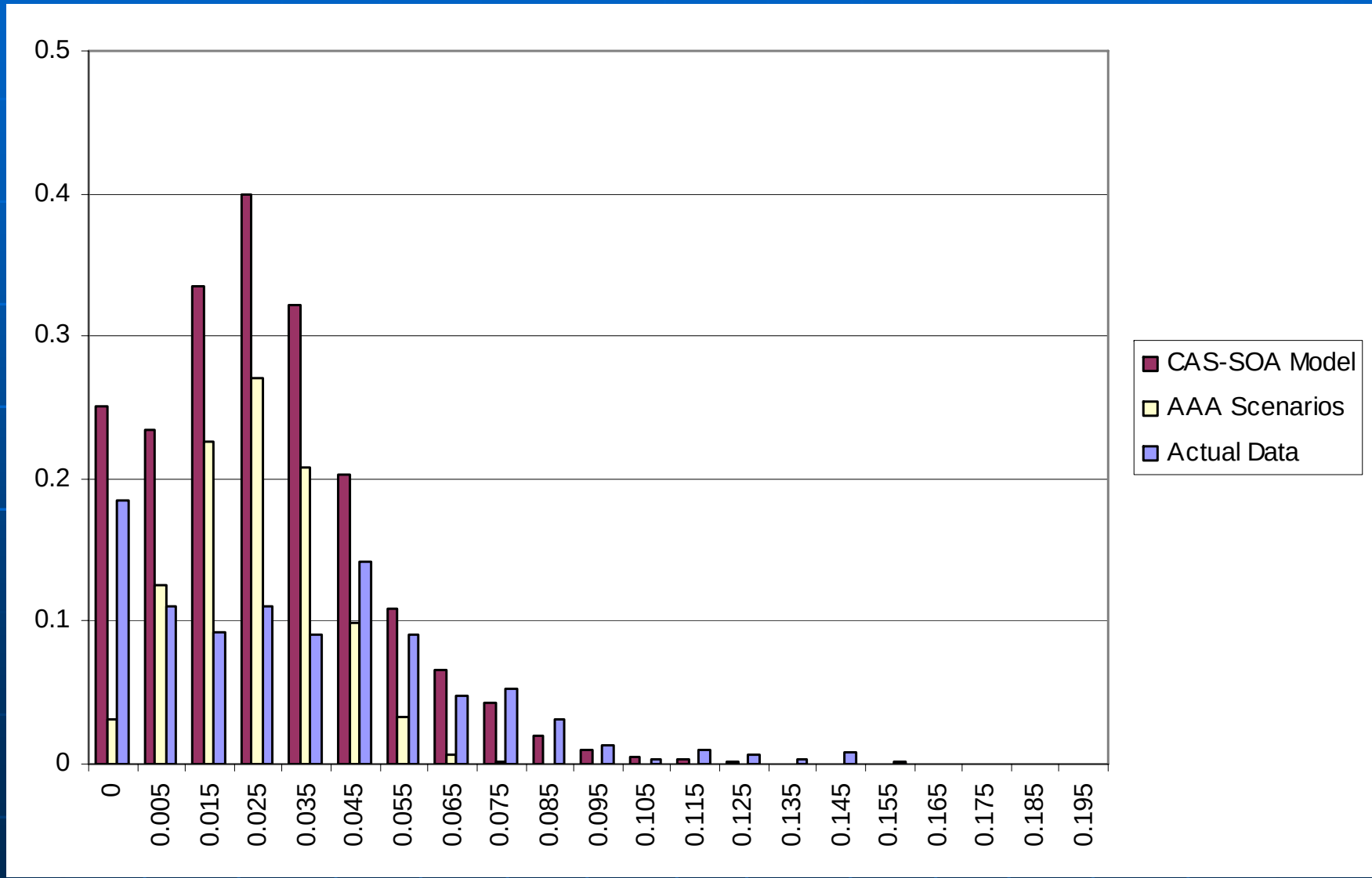
- Volatility process

$$d(\ln \nu_t^2) = \kappa_\nu (\theta_\nu - \ln \nu_t^2) dt + \sigma_\nu dB_t^\nu$$

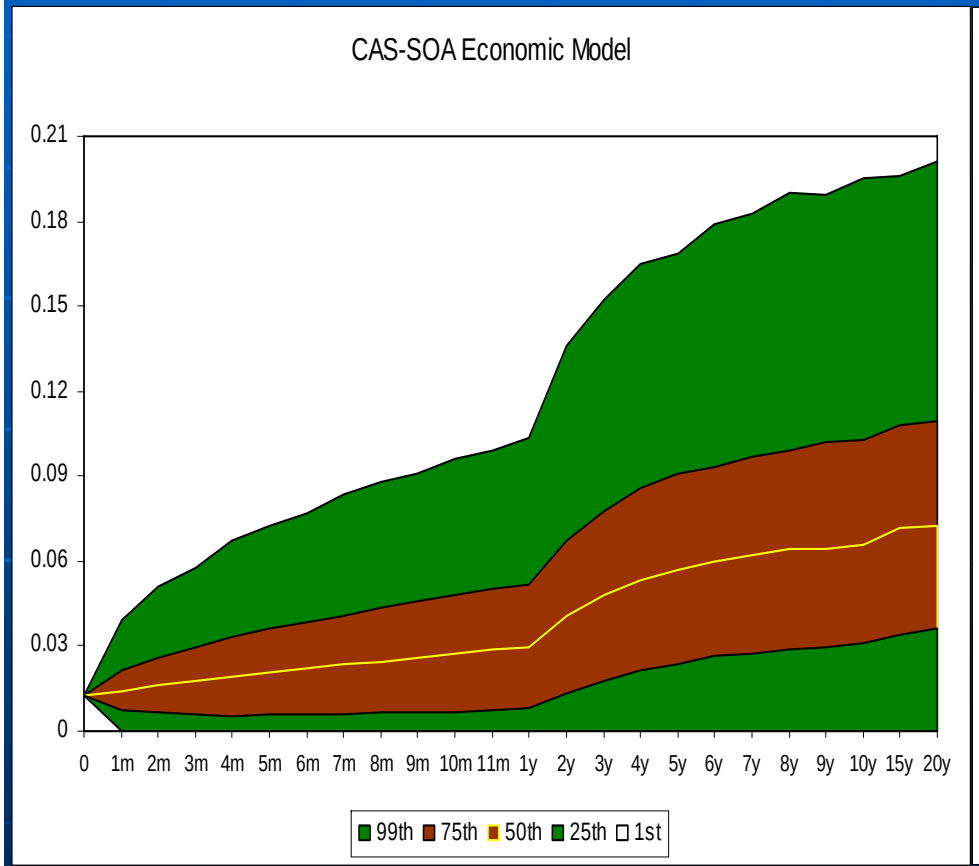
Comparative Statistics: Interest Rates

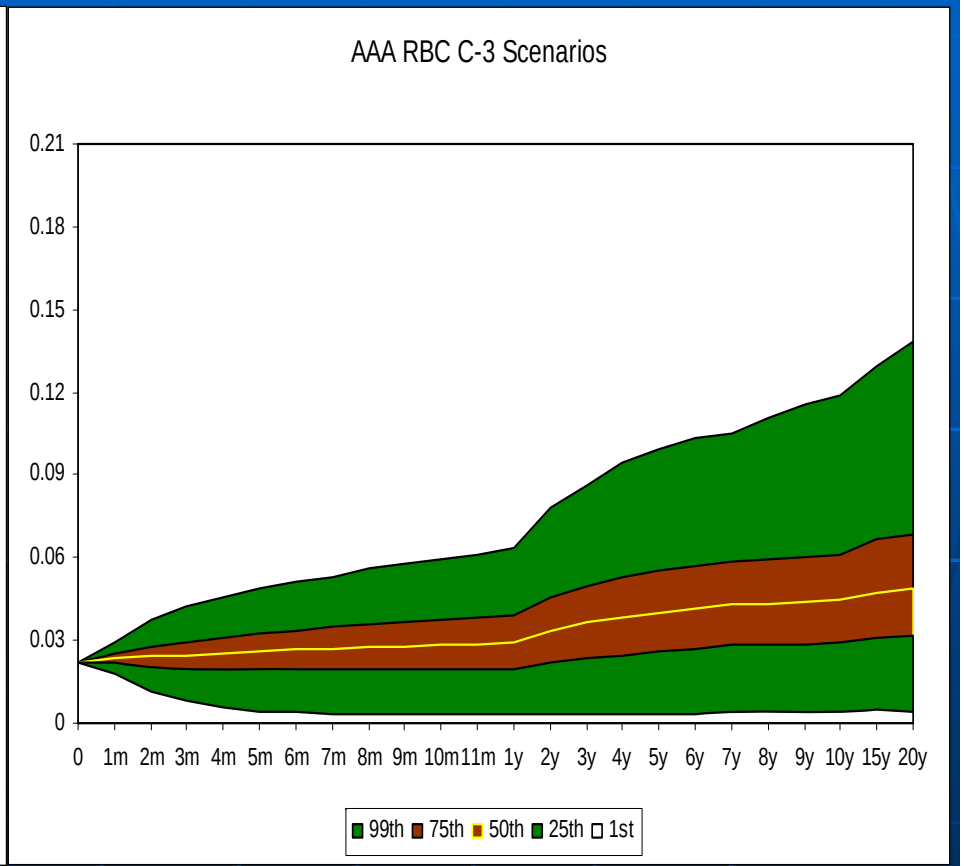
Table 2			
Descriptive Statistics - 3 Month Nominal Interest Rates			
	CAS-SoA	AAA	Historical
Mean	0.0328	0.0297	0.0391
Median	0.0298	0.0293	0.0352
Standard Deviation	0.0273	0.0140	0.0318
Kurtosis	-0.2302	-0.1894	0.9699
Skewness	0.6196	0.2492	0.9462
Range	0.1528	0.0890	0.1629
Minimum	0.0000	0.0020	0.0001
Maximum	0.1528	0.0910	0.1630
99th Percentile	0.1038	0.0635	0.147
75th Percentile	0.0514	0.0391	0.0566
25th Percentile	0.0078	0.0197	0.0114
1st Percentile	0.0000	0.0031	0.0003

Histogram of 3 Month Nominal Interest Rates Model Values and Actual Data (01/34-01/06)

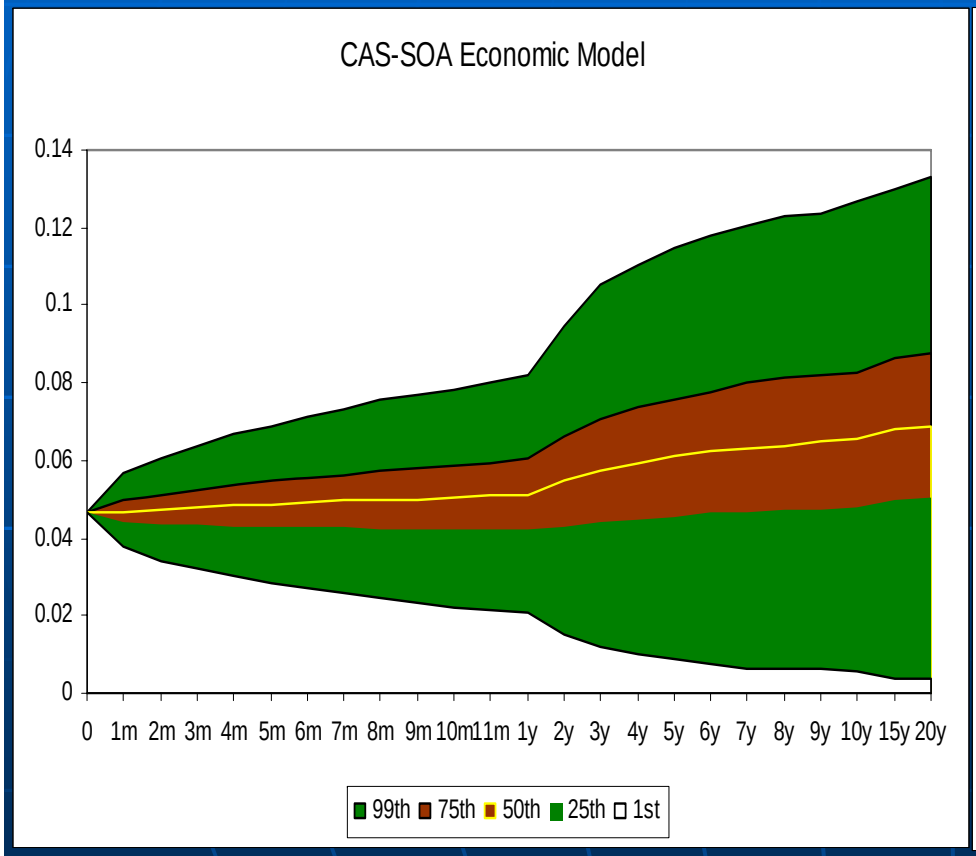


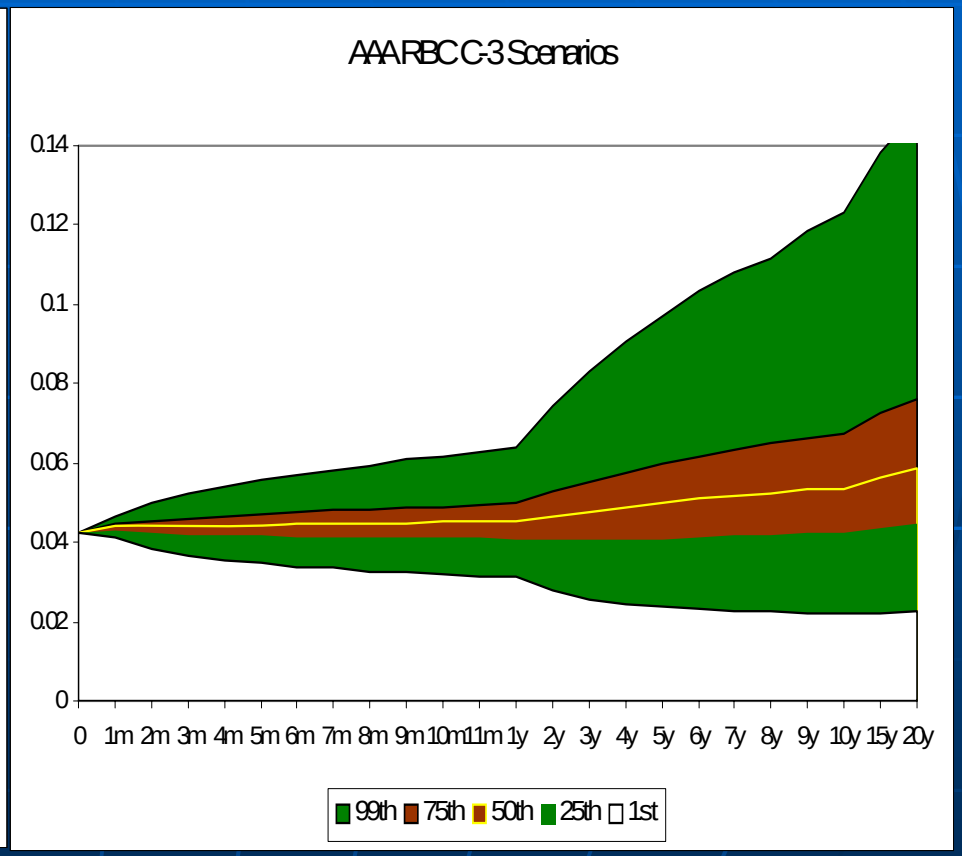
Funnel of Doubt Graphs
3 Month Nominal Interest Rates (U. S. Treasury Bills)





Funnel of Doubt Graphs
10 Year Nominal Interest Rates (U. S. Treasury Bonds)





CAS/SOA Equity Returns

- Empirical “fat tails” issue regarding equity returns distribution
- Thus, modeled using a “regime switching model”
  1. High return, low volatility regime
  2. Low return, high volatility regime
- Model equity returns as an excess return (x_t) over the nominal interest rate (i_t)

$$s_t = i_t + x_t$$

AAA Equity Returns

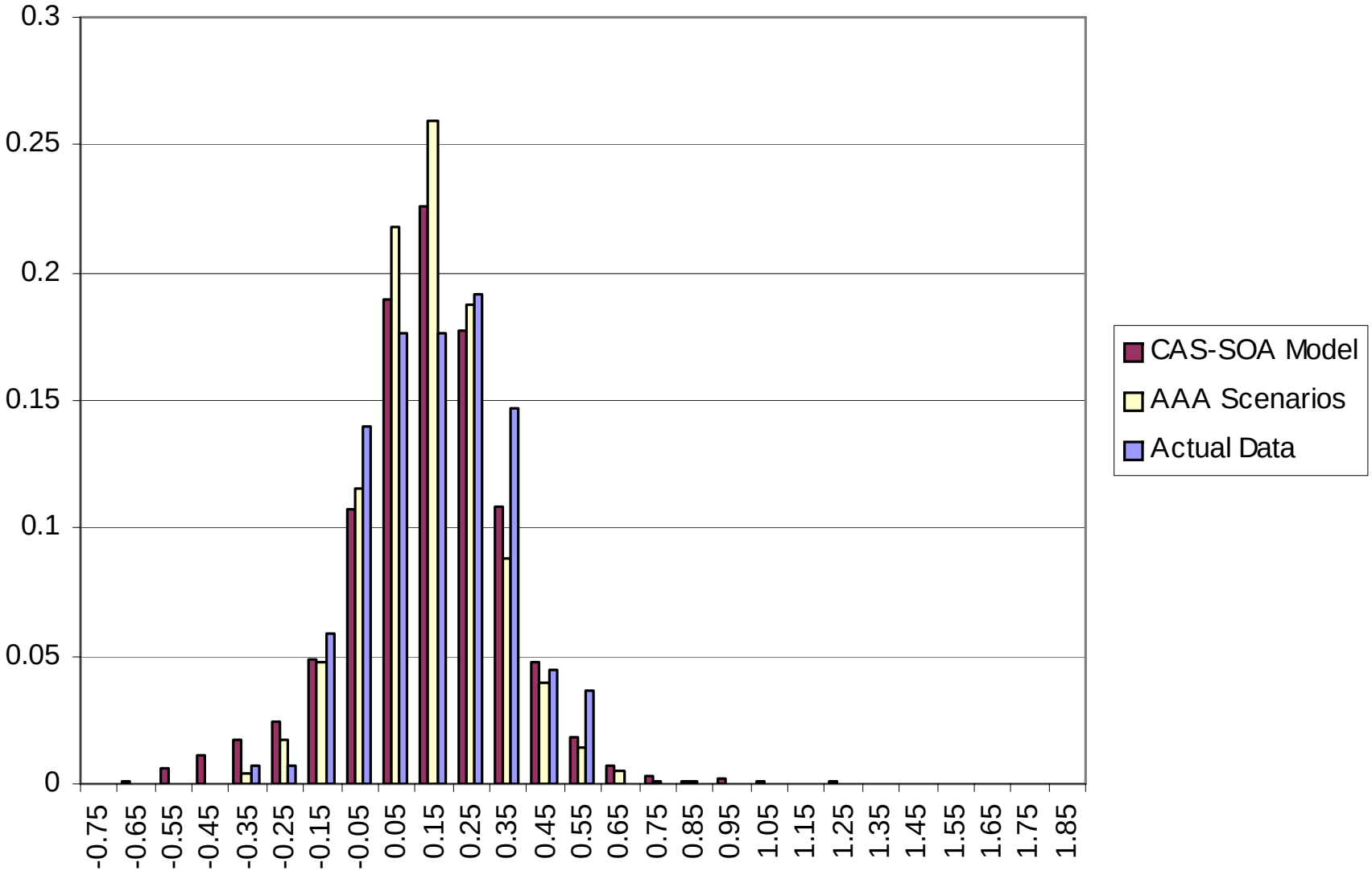
- Brownian motion with stochastic volatility and time dependent mean returns

$$d\left[S_t\right] = \mu_t dt + v_t dB_t^s$$

$$d\left(\ln v_t\right) = \phi \times \left[\ln \tau - \ln v_t\right] dt + \sigma_v dB_t^v$$

$$\mu_t = A + Bv_t + Cv_t^{\,2}$$

Histogram of Large Stock Return Model Values and Actual Data (1872-2006)



Comparative Statistics: Equity Returns

Table 4			
Descriptive Statistics - Large Stock Returns			
	CAS-SoA	AAA	Actual
Mean	0.0872	0.0895	0.1044
Median	0.0928	0.0886	0.0996
Standard Deviation	0.2205	0.1661	0.1781
Kurtosis	4.9511	0.7003	0.0407
Skewness	0.3639	0.1785	-0.0266
99th Percentile	0.6266	0.5173	0.5378
75th Percentile	0.2111	0.1895	0.2107
25th Percentile	-0.0295	-0.0157	-0.0235
1st Percentile	-0.5259	-0.2999	-0.3119

Two Applications

- *Single premium, 10-year term life insurance policy*
  - *Balance invested in T-bills and equities*
  - *Expected mortality settled each year*
- Property-liability reserve
  - Balance invested in T-bills and equities
  - $10 million of assets fund 10 years of payments

Table 12-A
Property-Liability Loss Reserve Example - CAS-SoA Model

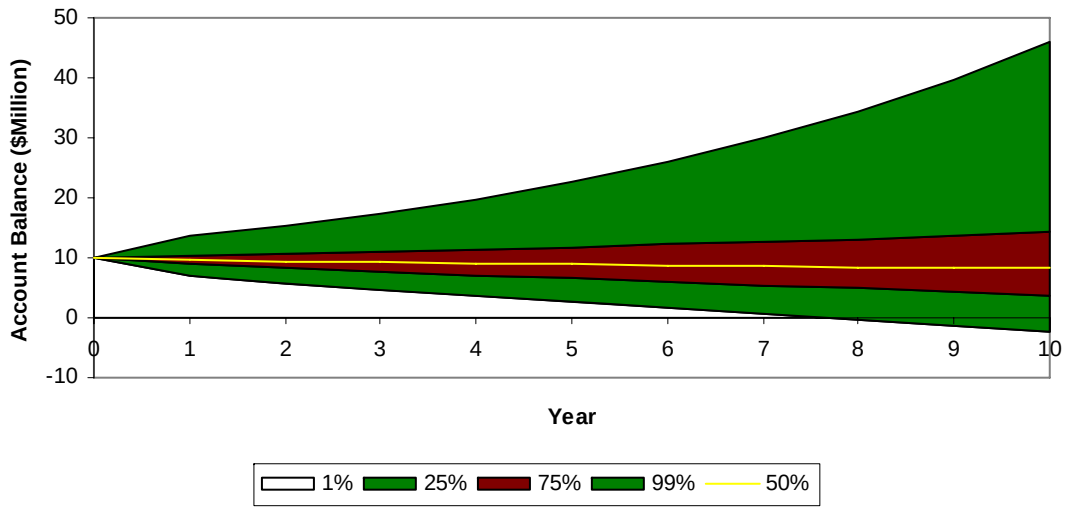
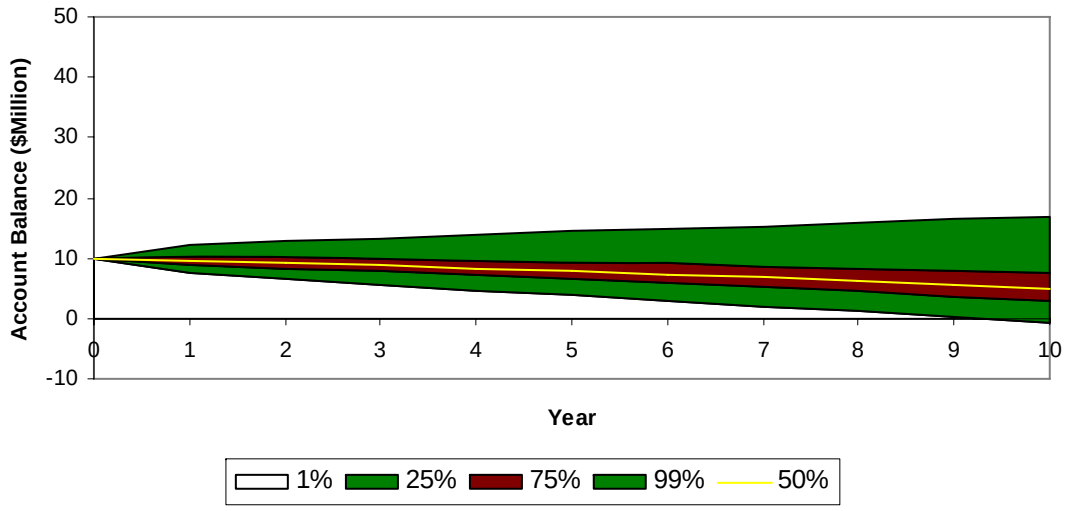


Table 12-B
Property-Liability Loss Reserve Example - AAA C3 Model



Property-Liability Insurance Example

Table 8-A CAS-SoA Model			
Year	*3*	*6*	*9*
Mean	9	10	10
Median	9	9	8
Standard Deviation	2.64	5.00	8.39
Kurtosis	2.44	3.10	6.45
Skew ness	0.92942	1.2833	1.8668
# of Negative Values	-	5	321
99th Percentile	17	26	40
75th Percentile	11	12	14
25th Percentile	8	6	4
5th Percentile	6	3	1

Table 8-B AAA-C3 Model			
Year	*3*	*6*	*9*
Mean	8.87	7.59	6
Median	8.78	7.32	6
Standard Deviation	1.64	2.52	3.39
Kurtosis	0.49	0.88	1.65
Skew ness	0.42	0.70	0.94136
# of Negative Values	0	0	72
75th Percentile	9.87	9.08	8
25th Percentile	7.74	5.84	4
5th Percentile	6.33	3.98	1
1st Percentile	5.45	2.90	0

Quantification of Model Fit

- Kolmogorov-Smirnov test

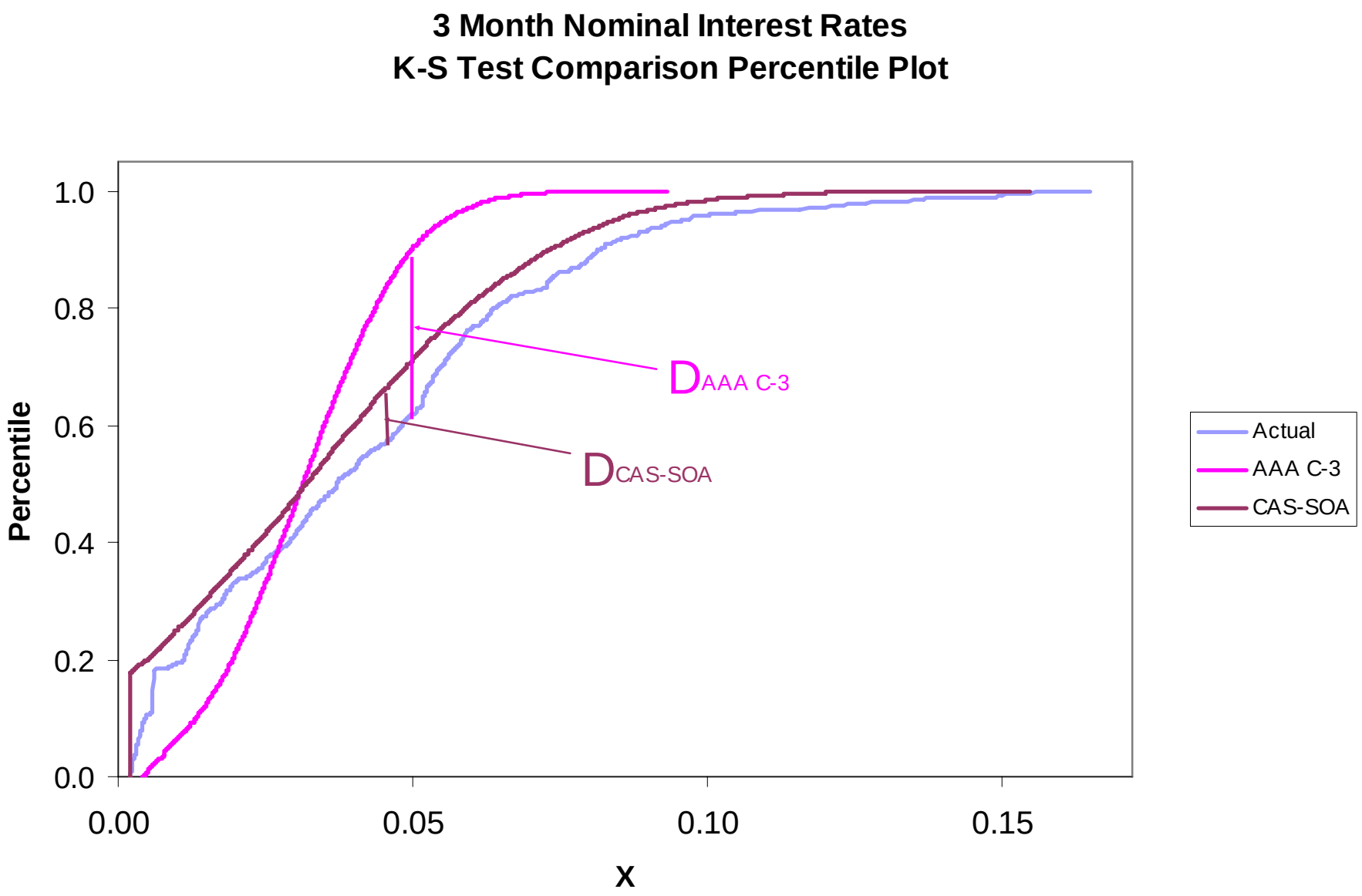
  Tries to determine if two datasets differ significantly

  Uses the maximum vertical difference between percentile plots of the data as statistic D

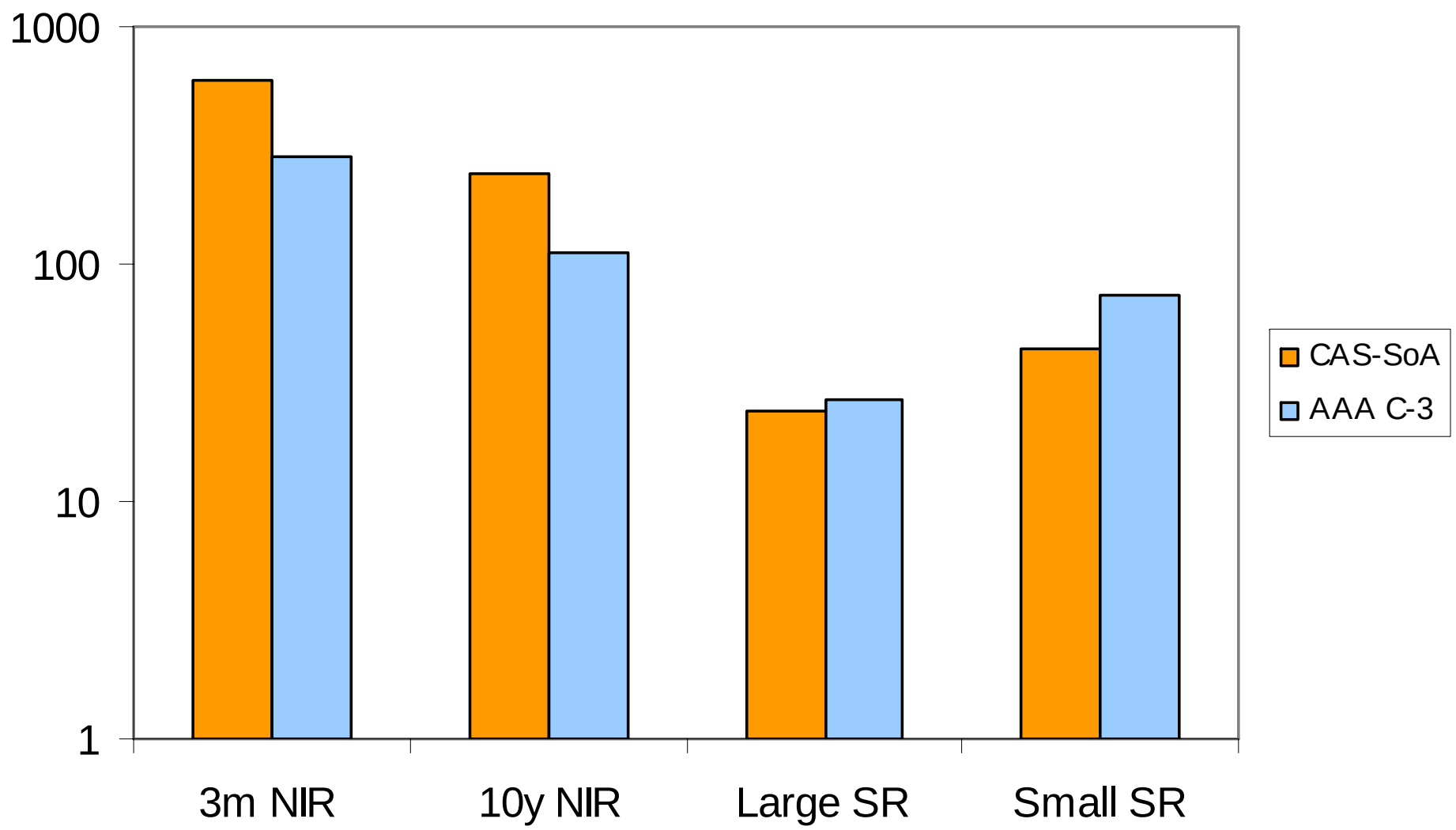
- Chi-square test

  Take the squared difference between observed frequency (**O**) and the expected frequency (**E**), and then divided by the expected frequency

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$


3 Month Nominal Interest Rates
K-S Test Comparison Percentile Plot
Percentile
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.05
0.10
0.15
X
DAAA C-3
DCAS-SOA
Actual
AAA C-3
CAS-SOA

Chi-square Test



Summary of Differences

- Kolmogorov-Smirnov test

  Statistic D of CAS-SOA model is smaller than that of AAA C-3 model

- Chi-square test

  For nominal interest rate, the Chi-square value of CAS-SOA model is higher than that of AAA C-3 model

  For small stock returns, both models are rejected at significant level of 0.025 while accepted at level of 0.1

  For large stock returns, both models are rejected at significant level of 0.05 while accepted at level of 0.1

How to Obtain Models

CAS-SOA model is posted on the following sites:

- http://casact.org/research/econ/
- http://www.soa.org/ccm/content/areas-of-practice/finance/mod-econ-series-coor-int-rate-scen/

Or contact us at: kahlgrim@ilstu.edu
s-darcy@uiuc.edu
gorvett@uiuc.edu

- AAA model is posted at:

http://www.actuary.org/life/phase2.asp

Conclusions

- CAS/SOA model is general; AAA model is more specialized
- Users of models should understand that differences exist
  - AAA interest rates appear restricted
  - Levels of equity returns and interest rates appear higher in CAS-SOA model
- Impact to specific application???