Solvency, Capital Allocation and Fair Rate of Return in Insurance*

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Introduction

- Many different methods (risk measures) for measuring solvency risk - VaR, 1 year ruin probability, infinite horizon ruin probability, TailVaR, Expected Policyholder Deficit, Default Option value (Insolvency Exchange Option value)

- Many different approaches to allocating capital to line of business - proportional to selected risk measure, proportional to liabilities, marginal allocations (Merton and Perold and Myers and Read), equal expected returns to capital, covariance of losses

- Under what assumptions is capital allocation irrelevant for pricing and other financial decisions?

Aims of Paper

- To consider economic capital for a multi-line insurer, the fair pricing (arbitrage-free) of lines of business allowing for solvency and the fair (arbitrage-free) allocation of capital to lines of business

- Model assumes assets, liabilities and insolvency exchange option are fairly priced (arbitrage-free), complete markets and no market frictions (strong assumptions but no different to most other capital allocation papers)

- Results illustrated with a simple numerical example

Economic Balance Sheet

- Value of the assets (Q risk-neutral valuation probability measure)

$$V_A = \sum_{j=1}^{J} E^Q\left[\frac{A_j}{1+r}\right] = E^Q\left[\frac{A}{1+r}\right]$$

- Determined by investment strategy of the company given by w_j the weight of asset $j = 1, \ldots, J$ in the insurer's portfolio and the end of period payoff distribution for the assets $j = 1, \ldots, J$

$$A = V_A\left(1 + R_A\right) = V_A\left(1 + \sum_{j=1}^{J} w_j R_{A_j}\right)$$

Economic Balance Sheet

- Insurer writes multiple (K) lines of business denoted by $k = 1, \dots, K$, could be individual policies

- Line of business k incurs the random claim amount L_k at the end of the period, assuming unlimited liability. L_k is not affected by the amount of capital, dividend policy, investment policy, reinsurance strategy and any other actions of the insurer that may impact on its ability to pay the liabilities under the insurance contracts.

Economic Balance Sheet

- The end-of-period total claim payments for the insurance company is

$$L = \sum_{k=1}^{K} L_k.$$

- The value of the liability, assuming full payment, can be written as

$$V_L = E^Q \left[\frac{L}{1+r} \right] = \sum_{k=1}^{K} E^Q \left[\frac{L_k}{1+r} \right]$$

Economic Balance Sheet

- Liability claim payments are still risky since the future pay-off is a random variable. Value of the liability in terms of real world or historical probabilities is

$$\begin{aligned} V_L &= E^P[mL] \\ &= E^P[m]\,E^P[L] + cov^P(m, L) \\ &= \frac{E^P[L]}{1+r} + cov^P(m, L) \end{aligned}$$

  where m is a stochastic discount factor.

- This value of the liabilities allows for relevant economic risk factors but does not take into account the insolvency of the insurance company

Economic Balance Sheet

- Let one plus the liability growth rate be denoted by $1 + R_L = \frac{L}{V_L}$ then

$$1 + E\left[R_L\right] = \frac{E^P\left[L\right]}{V_L} = (1 + r)\left[1 - cov^P\left(m, 1 + R_L\right)\right]$$

- For line of business k

$$1 + E\left[R_{L_k}\right] = (1 + r)\left[1 - cov^P\left(m, 1 + R_{L_k}\right)\right]$$

Economic Balance Sheet

- Competitive premium policyholders, under perfect market assumptions, will pay in total is $V_L - D$ where D is the value of the insolvency exchange option for the insurer

$$D = \frac{E^Q\left[\max\left(L - A, 0\right)\right]}{1+r} = \frac{E^Q\left[L - A \middle| \frac{A}{L} < 1\right] \Pr{}^Q\left[\frac{A}{L} < 1\right]}{1+r}$$

- Insolvency exchange option value reflects both the probability of insolvency and the expected severity of the insolvency based on the risk neutral probabilities. For extreme events, assuming risk aversion, the risk neutral probabilities will usually exceed the actual historical or real world probabilities.

Economic Balance Sheet

- Denote the solvency ratio by s so that $V_A = (1 + s)\, V_L$. A solvent insurer will have $s > 0$.

- The market value of the initial actuarial surplus is given by

$$S = V_A - V_L > 0$$

where the asset values allow for the issuer default but the actuarial liability values of the insurer do not allow for the insurer default risk.

Economic Balance Sheet

- The market value of the equity will be the actuarial surplus plus the value of the insolvency exchange option. Since the market value of the equity is the market value of the assets less the market value of the liabilities we have

$$
\begin{aligned}
V_X &\equiv V_A - (V_L - D) \\
&= sV_L - D > 0
\end{aligned}
$$

- This is the same balance sheet value as in Myers and Read (2001)

- To allocate capital to line of business we must allocate V_A, V_L, and D

Economic Balance Sheet

- End of the period payoffs on the balance sheet of the insurer

Balance Sheet	Initial Value	End of Period Payoff
Assets	V_A	A
Liabilities	$V_L - D$	$\left[\begin{array}{l} \min(L, A) \\ = L + \min(A - L, 0) \\ = L - \max(L - A, 0) \end{array}\right]$
Equity	$S + D$	$\left[\begin{array}{l} \max(A - L, 0) \\ = A - L - \min(A - L, 0) \\ = A - L + \max(L - A, 0) \end{array}\right]$

Economic Balance Sheet

- At the start of the period the insurer sets its investment policy $\left(w_j \text{ for all } j\right)$, determines the liability risks that the company will underwrite and its solvency ratio, s. This information is assumed known and reflected in the valuation of cashflows.

- The distribution of liability risks, L, is known and the value of these liabilities ignoring the insurer default option, V_L, is given by the risk neutral Q probabilities, or equivalently the stochastic discount factor, since we assume a complete market.

- The total value of the assets is determined from the liability value and the solvency ratio $(1+s)\,V_L$.

Economic Balance Sheet

- Given the distribution of both A and L, the value of the insolvency exchange option is

$$D = \frac{E^Q\left[\max\left(L - A, 0\right)\right]}{1 + r}$$

- The total market premiums for the policyholders is $V_L - D$, and the capital subscribed is $V_A - (V_L - D)$.

Fair Rate of Return

- The premiums charged are fair, allowing for the insolvency of the insurer, and the balance sheet structure is determined by the liabilities underwritten and the target solvency ratio.

- Capital earns a fair rate of expected return since all assets and liabilities, including the insolvency exchange option, are fairly priced under the risk neutral Q-measure. The fair rate of return reflects the leverage of the insurer balance sheet.

Capital Allocation to Lines of Business

- Assume all lines of business rank equally in the event of default

- Policyholders who have claims due and payable in line of business k will be entitled to a share $\frac{L_k}{L}$ of the assets of the company where the total outstanding claim amount is $L = \sum_{k=1}^{K} L_k$.

- The end-of-period payoff to line of business k is well defined based on this equal priority as

$$\begin{array}{ll} \frac{L_k}{L} A & \text{if } L > A \text{ (or } \frac{L}{A} > 1) \\ L_k & \text{if } L \leq A \text{ (or } \frac{L}{A} \leq 1) \end{array}$$

In either case the payoff on the assets will be A.

Capital Allocation to Lines of Business

- Let the value of the exchange option allocated to line of business k be denoted by D_k, then this is given by the value of the pay-off to the line of business in the event of insurer default

$$D_k = \frac{1}{1+r} E^Q \left[L_k \max \left[1 - \frac{A}{L}, 0 \right] \right]$$

- Value of the insolvency exchange option for each line of business "adds up" to the total insurer value

$$\sum_{k=1}^{K} D_k = \frac{1}{1+r} \sum_{k=1}^{K} E^Q \left[L_k \max \left[1 - \frac{A}{L}, 0 \right] \right] = \frac{1}{1+r} E^Q \left[\max \left[L - A, 0 \right] \right]$$

Capital Allocation to Lines of Business

- Allocation of assets to lines of business is "irrelevant" for fair pricing or solvency

- Allocation of assets to line of business is an internal insurer allocation that will have no economic impact on the payoffs or risks of the insurer

- Assets are available to meet the losses of all lines of business (capital allocation irrelevance for pricing and risk measurement by-line)

- Allocation of the insolvency exchange option is required for fair (arbitrage-free) pricing by line of business

Capital Allocation to Lines of Business

- Possible methods of allocating assets
  - Surplus allocated to lines of business so that each line of business has the same solvency ratio as for the total insurer
  - Assets allocated so that the expected return on allocated capital will be equal across all lines of business and the same as for the total insurer.
- Latter used in practice and suggested by a number of researchers

Market Premiums by Lines of Business

- Many authors - for example Phillips, Cummins and Allen (1998), Myers and Read (2001) - implicitly or explicitly assume that price by line of business is given by

$$V_{L_k} - D_k = V_{L_k} - \frac{V_{L_k}}{V_L} D$$

  or

$$D_k = \frac{V_{L_k}}{\sum_k V_{L_k}} = \frac{V_{L_k}}{V_L} D$$

- This does not reflect the by-line payoffs for the default option

Market Premiums by Lines of Business

- Note that in Myers and Read (2001) d_k is a **sensitivity not an allocation**

$$d_k = \frac{\partial D}{\partial V_{L_k}}$$

and d_k must be the same for all lines of business in order for $\frac{D}{L}$ to be invariant for small changes in a line of business

Market Premiums by Lines of Business

- If used for capital allocation then

$$d_k = \frac{D}{V_L} \text{ for all } k$$

or

$$\begin{aligned} \frac{D_k}{V_{L_k}} &= \frac{D}{V_L} \text{ for all } k \\ D_k &= \frac{V_{L_k}}{V_L} D \text{ for all } k \end{aligned}$$

- Allocation should be proportional to the liability values by-line

Example - Discrete State, Discrete Time

- Single risky asset and two lines of business, assumed payoff for a unit of the risky and risk free asset and the payoff to the liabilities as well as the P and Q probabilities are

Table 1: Probabilities and Payoffs for Example Insurer

			Time 1 Payoffs			
State	P-probs	Q-probs	Risky Asset	Risk Free Asset	Liability 1	Liability 2
1	0.1	0.1	0.6	1.05	200	40
2	0.6	0.4	1.1	1.05	4	10
3	0.2	0.4	1.0	1.05	2	4
4	0.1	0.1	1.5	1.05	0	310
Time 0 Value			1.0	1.0	21.3333	38.6667

Example - Discrete State, Discrete Time

- Table 2 gives the payoffs for the assets and liabilities as well as the amount of liabilities not met because of insufficient assets. Note that the insurer defaults in both State 1 and State 4.

Table 2 Insurer Balance Sheet Payoffs

	Time 1 Insurer Balance Sheet Payoffs				
State	Assets	L_1	L_2	Total L	$max(L-A,0)$
1	120	200	40	240	120
2	220	4	10	14	0
3	200	2	4	6	0
4	300	0	310	310	10
Time 0 Value	200	21.3333	38.6667	60	12.381

Example - Discrete State, Discrete Time

- The surplus ratio for the insurer is

$$s = \frac{S}{L} = \frac{200 - 60}{60} = 2.3333$$

- The economic capital of the insurer at time 0 will be the value of the assets less the value of the liabilities ignoring the insolvency costs and plus the value of the insolvency exchange option which is $200 - 60 + 12.381 = 152.381$.

Example - Discrete State, Discrete Time

- Shortfall in the event of insolvency for each line of business are given in Table 3.

Table 3 Liability Shortfalls in the Event of Insolvency		
	Time 1 Liability Shortfalls	
State	$D_1 = L_1 \max(1-\frac{A}{L}, 0)$	$D_2 = L_2 \max(1-\frac{A}{L}, 0)$
1	100	20
2	0	0
3	0	0
4	0	10
Time 0 Value	9.5238	2.8571

Example - Discrete State, Discrete Time

- The allocation of the insurer shortfall of assets over liabilities is based on equal priority of the policyholders to the assets for each line of business. Thus in State 1 the shortfall of 120 is allocated in proportion to the outstanding liabilities so that $\frac{200}{240} \times 120 = 100$ is the shortfall for line of business 1 and $\frac{40}{240} \times 120 = 20$ is the shortfall for line of business 2.

- The premium for each line of business is determined allowing for the insurer insolvency exchange option value. For line of business 1 the premium will be $21.3333 - 9.5238 = 11.8095$ and for line of business 2 it will be $38.6667 - 2.8571 = 35.8095$.

Example - Discrete State, Discrete Time

Table 4 gives the insurer equity payoffs

Table 4 Insurer Equity Payoffs				
	Time 1 Insurer Equity Payoffs			
State	P-probs	Assets	Total L	Equity $= max(A - L, 0)$
1	0.1	120	240	0
2	0.6	220	14	206
3	0.2	200	6	194
4	0.1	300	310	0
Time 0 Value		200	60	152.3810

Example - Discrete State, Discrete Time

- The ratio of the insolvency exchange option value to the value of the liabilities for the insurer and for each line of business is

$$d = \frac{D}{V_L} = \frac{12.381}{60} = 0.2063$$

$$\begin{aligned} d_1 &= \frac{D_1}{V_{L_1}} = \frac{9.5238}{21.3333} = 0.4464 \\ d_2 &= \frac{D_2}{V_{L_2}} = \frac{2.8571}{38.6667} = 0.0739 \end{aligned}$$

The expected return to equity for the insurer is

$$\frac{0.1 \times 0 + 0.6 \times 206 + 0.2 \times 194 + 0.1 \times 0}{152.3810} - 1 = 0.06575$$

Example - Discrete State, Discrete Time

- Assets can be allocated so the same solvency ratio will apply to each line of business and for the total insurer. For this to hold we would allocate 71.1111 of the asset value to line of business 1 and 128.8889 to line of business 2. This would then give a capital allocation of $71.1111 - 21.3333 + 9.5238 = 59.3016$ to line of business 1 and $128.8889 - 38.6667 + 2.8571 = 93.0794$ to line of business 2.

- The solvency ratios for each line of business are then

$$
\begin{aligned}
\frac{71.1111 - 21.3333}{21.3333} &= 2.3333 \\
\frac{128.8889 - 38.6667}{38.6667} &= 2.3333
\end{aligned}
$$

Example - Discrete State, Discrete Time

- Can also allocate the capital to lines of business to equate the expected return to capital by line of business and to the insurer expected return to equity.

- If we allocate 50.3544 of the asset value to line of business 1 and 149.6456 to line of business 2 then this will equate the expected return to capital (equity) for each line of business.

- This would give a capital allocation of $50.3544 - 21.3333 + 9.5238 = 38.5449$ to line of business 1 and $149.6456 - 38.6667 + 2.8571 = 113.8361$ to line of business 2.

Conclusions

- Capital allocation in a multi-line insurer can be assessed using an arbitrage-free, perfect markets model

- Surplus allocation, ignoring the default put option value, is “irrelevant” for by-line financial decisions such as pricing

- Allocation of the insolvency exchange option is not “irrelevant” and is important for fair rate of return by-line pricing