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Capital Allocation by Percentile Layer

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Outline

- Why allocate capital? What are the ramifications?
- Description of situation and context
- A current approach to capital allocation
- Reformulating the meaning of Value at Risk (VaR)
- New approach: “Capital Allocation by Percentile Layer”
- Discussion and summary

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Why allocate capital?

- Risk adjusted performance measurement of
  - Operating units
  - Lines of business
  - Policies
  - Underwriters
  - Etc.

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What are the ramifications?

- Profitability measurement
- Compensation
- Incentives
- Employee behavior
- Composition of firm’s portfolio

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What are the ramifications?

- Allocate insufficient capital →
  - Incentive ("moral hazard") to take risk
    - Trading natural gas futures
    - Writing large book of Florida hurricane policies
    - Issuing subprime mortgages
- Allocate too much capital →
  - Firm misses out on profitable opportunities
- Conclusion: accurate capital allocation is critical

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Description of situation and context

- Risk taking financial firm
- Firm has several different business units
- Firm holds capital in order to deal with risk
- Firm incurs cost associated with this capital
- How should firm allocate this cost of capital to its various business units?

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Description of situation and context

- Allocating capital to business units applies to
  - Banks
  - Hedge funds
  - Insurance companies
  - Others

- Our discussion will focus on an insurance company

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Description of situation and context

- Publicly traded insurance company
  - Only covers property business (“short tail”)
- Insurance company must hold capital
  - Required by rating agencies, regulators, investors
- Capital is based upon
  - Value at Risk (VaR) at the 99th percentile
  - Single year time horizon
- Investors require rate of return on capital

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Description of situation and context

- Therefore, the operating environment dictates
  - How much capital to hold
  - How much it costs to hold this capital
- Management can only decide how to allocate this cost
- So we will focus only on allocation
  - Outside of scope:
    - Required capital amount
    - Required rate of return

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A current approach to capital allocation

- Critical feature of any allocation method:
  - Measure risk and capital within a holistic framework

- One notable example: “Co-measures” approach
  - Via Kreps, Mango, others

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A current approach to capital allocation

- Create simulation model for each “component” to be analyzed
  - Line of business
  - Peril
  - Policy
- Simulate profit and loss results for each component
  - Also tabulate results of total portfolio
- Keep track of components that contribute to each total loss

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A current approach to capital allocation

- Run simulation and then:
- Amount of capital = Value at Risk at 99th percentile
- Company holds capital for the 99th percentile loss event
  - Or the “1 in 100 year loss event”
- Allocate cost of capital to simulated “1 in 100 year event”
- Further allocate to perils and sublines that contribute to this loss event

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A current approach to capital allocation

- Assumption:
  - When firm holds VaR(99%) capital, it holds capital for the 99th percentile loss event
    - Or the “1 in 100 year loss scenario”
- Therefore →
  - Allocate cost of capital to lines of business that contribute to the “1 in 100 year loss scenario”

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A current approach to capital allocation

- Result:
  - Allocate all capital costs to risks that contribute to “tail” or “1 in 100 year” or “extreme” losses.
- This makes sense “intuitively”
- Or does it?

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Reformulating the meaning of VaR

- What does it mean to hold capital equal to VaR(99%)?
- Current formulation:
  - Holding VaR capital → to hold capital "for the 99th percentile loss event"
- Imprecise formulation leads to flawed assumption
  - Holding VaR capital → to hold capital "only for the 99th percentile loss event"
    - Therefore, allocate capital only to those who contribute to the 99th percentile loss event

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Reformulating the meaning of VaR

- Problem: is the 99th percentile loss event the only loss that uses capital?
- What about the 98th percentile loss event?
  - 98th percentile loss is still a substantial loss
  - Does it not "deplete" or "consume" capital?
  - Should it not receive an allocation of capital cost?

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Reformulating the meaning of VaR

Numerical example (insurance):

- 99th percentile loss event (San Francisco EQ) is 900M
  - Firm holds 900M of capital to satisfy VaR(99%)
- 98th percentile loss event (Florida Wind) is 600M
  - Does firm hold 900M of capital only for EQ event of 900M?
  - Doesn't Wind event of 600M use some capital as well?
- What about 97th percentile loss event (Midwest Tornado) of 400M?
  - Doesn't this loss event consume capital as well?

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Reformulating the meaning of VaR

- What does it mean to hold capital equal to VaR(99%)?
- Proposed new formulation:
  - Holding VaR(99%) capital →
    - To hold capital "even for the 99th percentile loss event"
    - But not "only for the 99th percentile loss event"

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Reformulating the meaning of VaR

- Apply new formulation to prior numerical example:
  - 98th percentile loss (e.g., Florida Wind) = 600M
  - 99th percentile loss (e.g., San Francisco EQ) = 900M

- Why hold 900M of capital? Why not hold 600M?

  - Company could hold 600M, have enough capital for Florida Wind event (98th percentile loss)

  - Capital of 900M = 600M + *additional 300M* in order to pay even for San Fran EQ event (99th percentile loss)

  - Large portion of capital (600M) used not only by 99th percentile loss event but also by 98th percentile loss event

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Proposed new approach to capital allocation

- Ramifications of new formulation:
  - Must analyze each percentile layer of capital
  - Hence, “Capital Allocation by Percentile Layer”

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Proposed new approach to capital allocation

- 99th percentile minus 98th percentile
  - Why hold this additional amount of capital?
    - Only used by loss events > 98th percentile loss
- 98th percentile minus 97th percentile
  - Why hold this additional amount of capital?
    - Attributable solely to loss events > 97th percentile
- And so on…

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Proposed new approach to capital allocation

- Almost all “percentile layers of capital” will be used by many different loss events.
- How should we allocate each layer to these loss events?
- Allocate using conditional exceedance probability
  - Probability of loss event / (probability of all loss events > lower bound of layer)
  - Allocate only to loss events that can “use” the layer of capital
  - Loss events that are “more likely” receive greater allocation
  - Allocation % on each layer always sums to 100%

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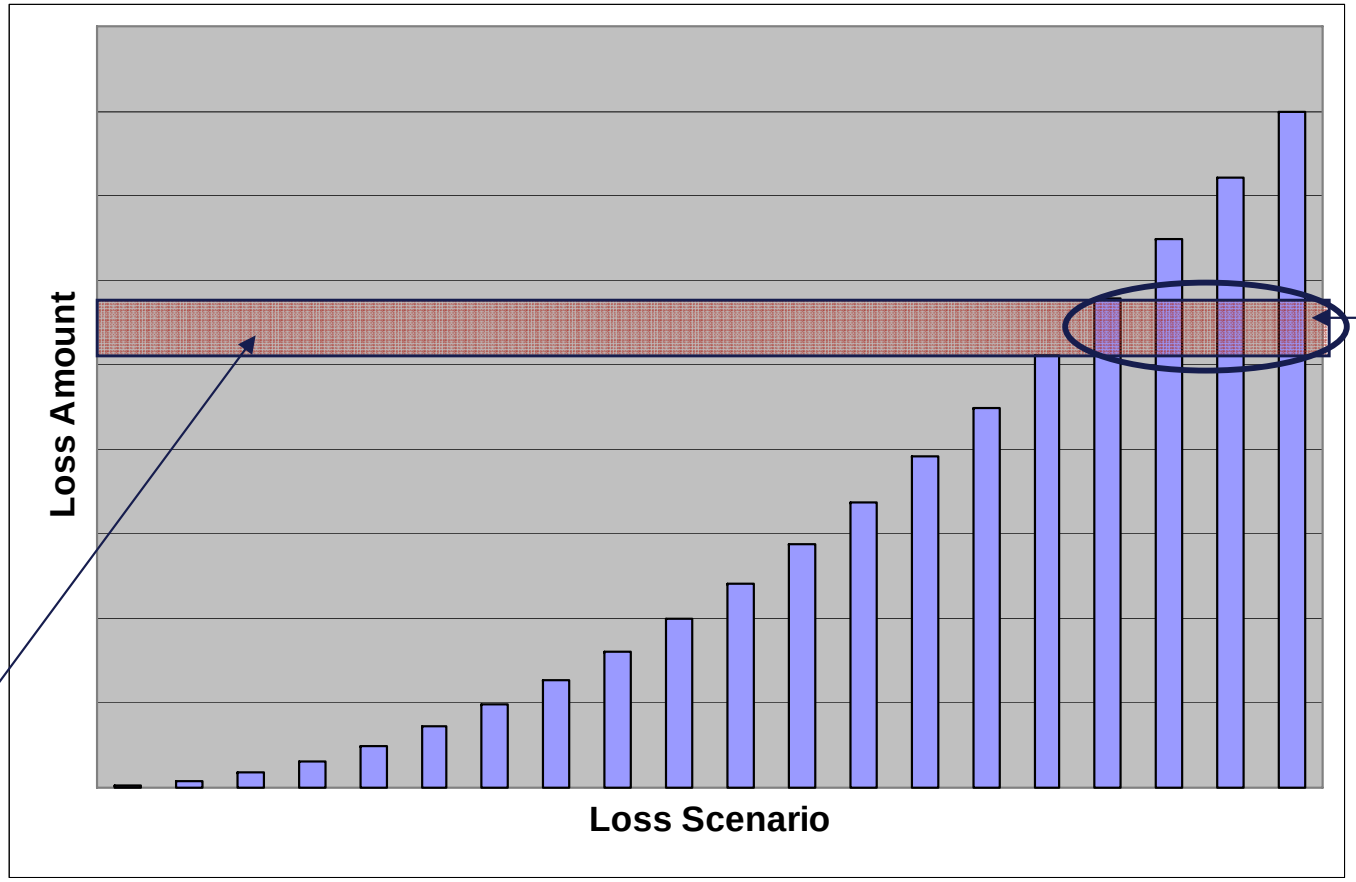
Proposed new approach to capital allocation



Layer of capital

Loss events (scenarios) that exceed the lower bound of the layer of capital

Allocate this layer of capital only to losses that “use” the layer of capital

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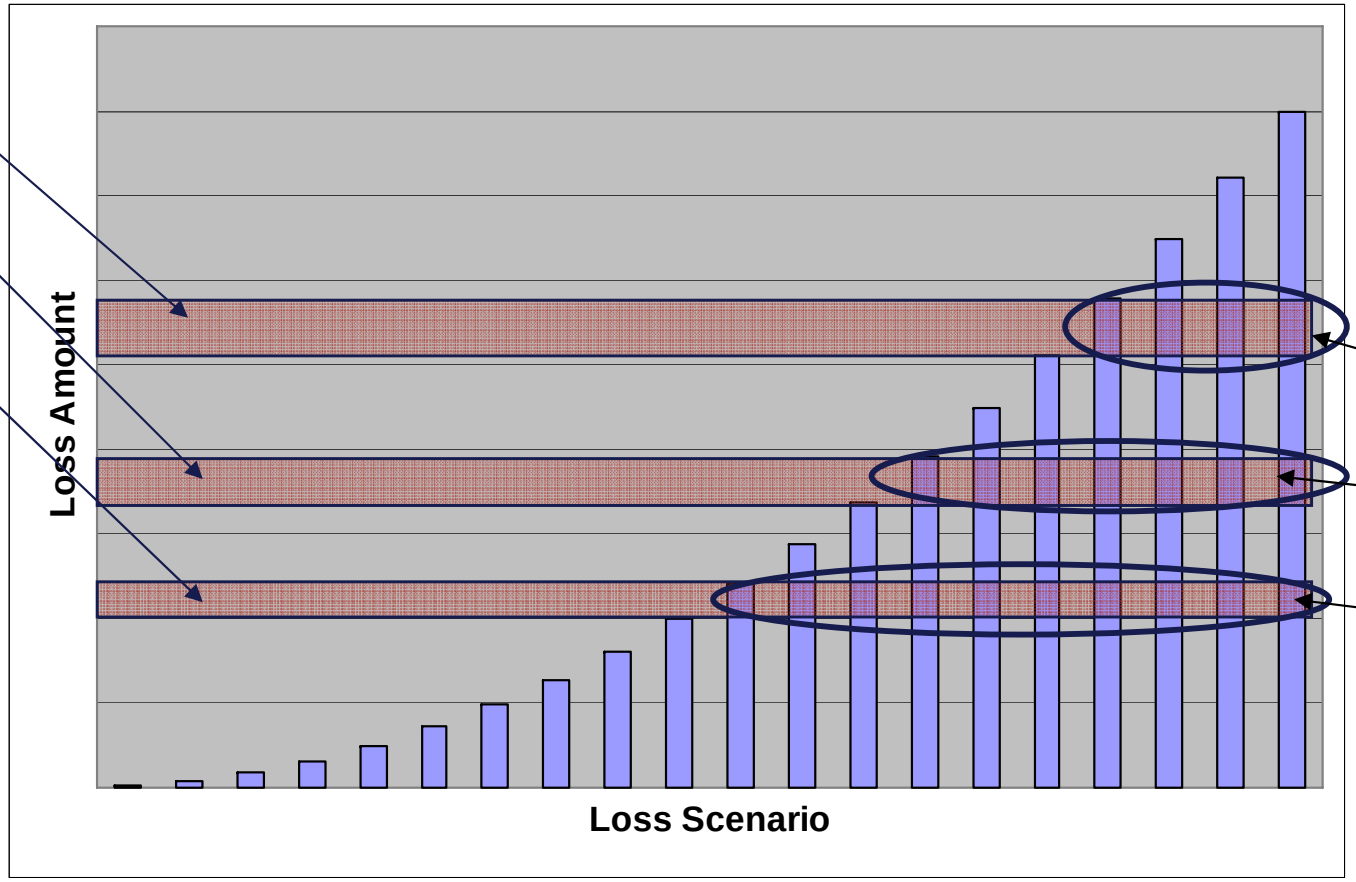
Proposed new approach to capital allocation

Layers of capital



Loss events that exceed the lower bound of each layer

Perform allocation for all layers of capital (up to required VaR capital)

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Proposed new approach to capital allocation

- Straightforward to implement with simulation output
  - Loss events from property catastrophe model
  - Loss scenarios from other simulation engines
- Width of each layer of capital =
  - Simulated loss event (i) – simulated loss event (i - 1)
- Number of layers of capital depends on number of simulations
  - If 1,000 simulations, then 1,000 layers
  - If 10,000 simulations, then 10,000 layers

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Proposed new approach to capital allocation

Simplified numerical example: simulation model

- LOB A: “Fire”
  - 25% chance of loss
    - If there is loss, severity is exponential, mean = 4m
- LOB B: “Wind”
  - 5% chance of loss
    - If there is loss, severity is exponential, mean = 20m
- LOB C: “EQ”
  - 1% chance of loss
    - If there is loss, severity is exponential, mean = 100m
- LOBs have equal expected loss of 1m but with dissimilar downside loss potential

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Proposed new approach to capital allocation

Simplified numerical example: allocation results

Allocation Method	LOB A (Fire)	LOB B (Wind)	LOB C (EQ)
coTVaR(99%)	0%	24%	76%
Capital allocation by percentile layer	17%	53%	30%

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Proposed new approach to capital allocation

Numerical example #2:

- 2007 Winter CAS Forum paper by Trent Vaughn
  - Multiline insurance company
  - Designed to have realistic loss behavior

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Proposed new approach to capital allocation

Numerical example #2:

Allocation Method	% Allocated to Homeowners	% Allocated to Personal Auto
coXTVaR(99%)	84.6%	8.0%
coXTVaR (all losses > mean loss)	16.8%	36.7%
Capital allocation by percentile layer	53.0%	22.6%

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Proposed new approach to capital allocation

- We have analyzed discrete layers of capital
- What if we set
  - The number of loss events infinitely large
  - The number of layers of capital infinitely large
  - The “width” of each layer of capital infinitesimally small
- Now we have a continuous case

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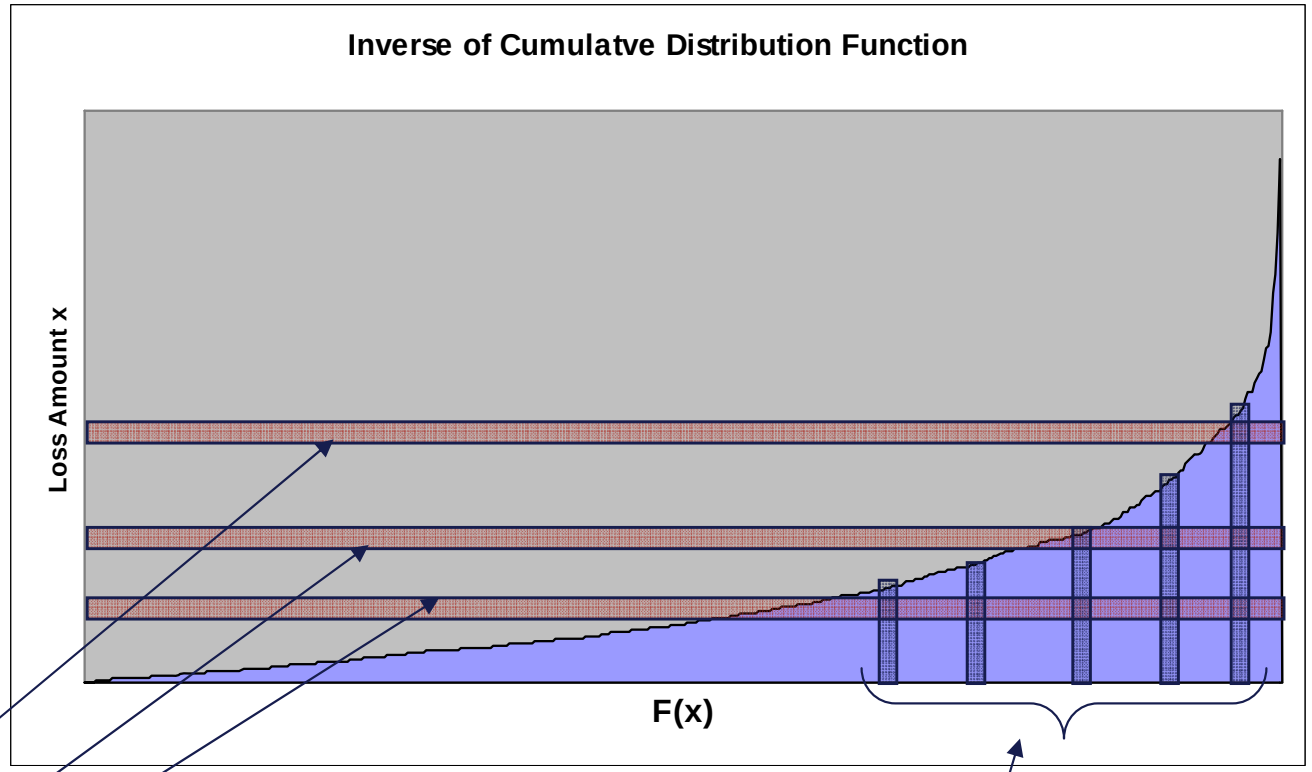
Proposed new approach to capital allocation



Layers of capital

Loss scenarios

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Proposed new approach to capital allocation

- Let x = loss amount
- Let y = capital
- Let layer of capital span (y, y+dy)
- dy = width of layer of capital

- Allocate to each loss event using conditional probability
  - Probability of loss event: $f(x)$
  - Probability of all losses that use the layer of capital: $1-F(y)$
  - Conditional probability of loss event: $\frac{f(x)}{1-F(y)}$

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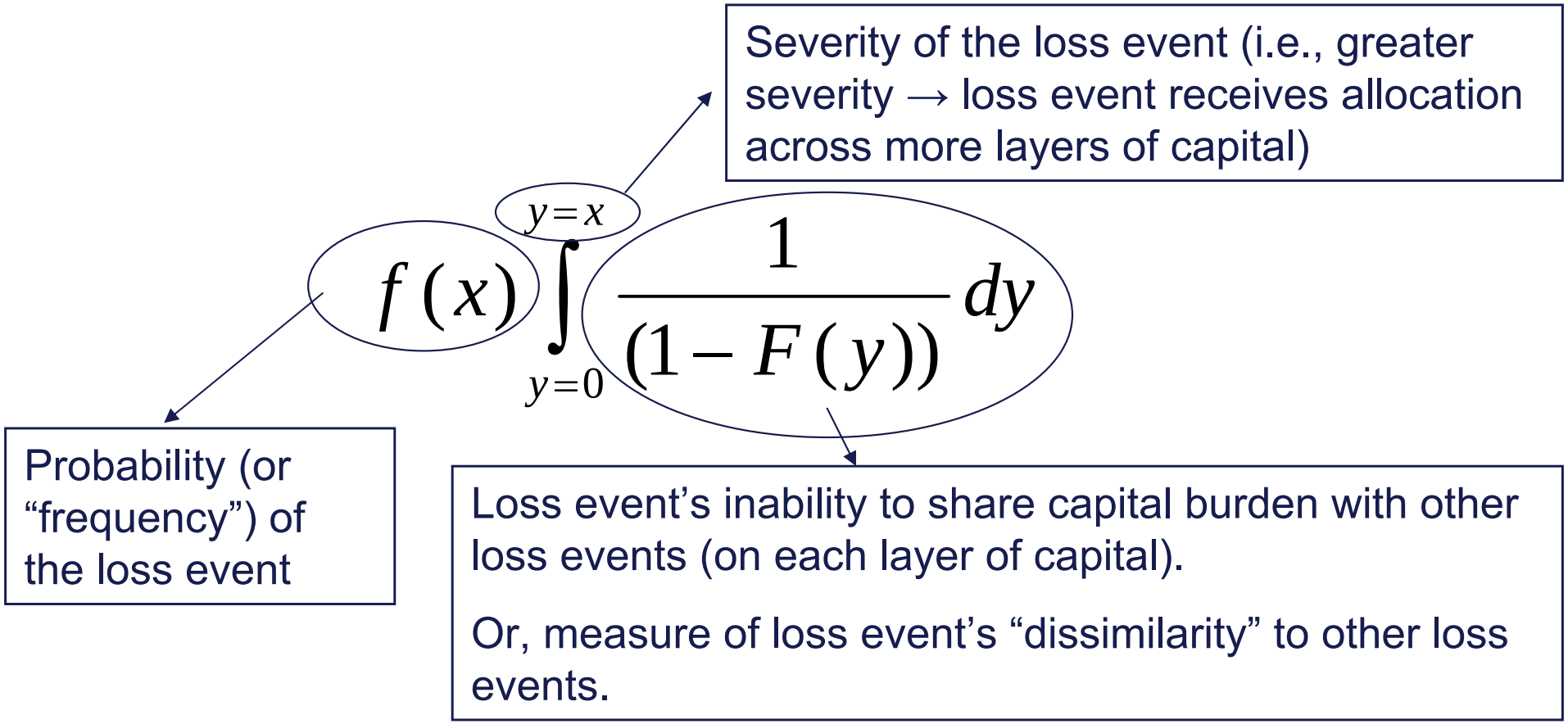
Proposed new approach to capital allocation

A loss event's allocated capital thus depends upon:



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Proposed new approach to capital allocation

Allocated capital to loss amount x = $$f(x)\int_{y=0}^{y=x}\frac{1}{(1-F(y))}dy$$

This equation also serves as:

- A new form of “risk measure” given loss amount x
- A measure of the “disutility” given loss amount x
  - If loss severity follows an Exponential probability distribution, then exponential disutility function
  - Disutility function may not necessarily have a closed form or “intuitive” formula other than the solution to the integral above

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Summary

- New formulation of the meaning of holding VaR capital
  - Hold capital to pay “even for” 99th percentile loss
  - But not “only for” this loss
- Similar logic extends to TVaR (tail value at risk)

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Summary

- Ramifications for allocating capital
  - Must allocate capital by layer or “percentile layer”
  - Allocate each layer of capital using conditional probability
    - Allocate only to loss events that exceed lower bound of the layer
  - Continuous results show new forms for
    - Risk measure
    - Disutility
    - Risk load
    - Transformed probability

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Summary

- Results of new proposal for capital allocation
  - Allocate capital to all loss events, not just in the tail
  - Smaller loss events below the tail percentile receive some allocation
    - Versus zero via some current methods
  - Largest loss events continue to receive large allocation
    - But less than current “tail based” allocation methods
  - Can alter the perceived profitability of various lines of business

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Comments?

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