CAPITAL ALLOCATION BY PERCENTILE LAYER

Step by Step Instructions and Calculations for Allocating Capital

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INTRODUCTION

- Why would anyone want to allocate capital?
  - to calculate how much premium to charge (prospective)
  - to measure profitability on a risk adjusted basis (retrospective)

- We'll focus on calculating target premium

INTRODUCTION

- Remember:
  - capital doesn't actually get allocated
  - we're allocating the cost of the total firm's capital
  - we're just saying "allocate capital" as shorthand

INTRODUCTION

- This presentation works in tandem with an accompanying spreadsheet workbook
- Each numbered step in the slides corresponds to a sheet in the workbook
- Let’s begin!

STEP BY STEP INSTRUCTIONS (1)

- Create or obtain simulation output
  - make sure output is at the desired degree of granularity for allocation
- We'll use simulated data for 3 Lines of Business
  - 3 lognormal distributions; mean =1m
  - CVs of 20%, 50%, and 100%
- Exact same data set as used for 2009 CAS Annual Meeting session of Mango, Venter, Bodoff

STEP BY STEP INSTRUCTIONS (2)

- For each simulated scenario
  - Track loss of each line of business or “component”
  - Track loss of the total firm
  - Track how much each component contributes to the total loss

STEP BY STEP INSTRUCTIONS (3)

- Sort total losses by order of magnitude
  - ascending or descending
  - we'll use descending order

STEP BY STEP INSTRUCTIONS (4)

- Select the “required capital rule” that underpins the amount of capital required to be held
- How one chooses the required capital affects the allocation
- For illustration we’ll use a “250 year return period”
  - also known as Value at Risk or VaR(99.6%)
  - in our simulated data, we have 1,000 simulated loss years
    - so we take the dollar amount such that one has a 99.6% probability of being less than or equal to that amount
      - 4 losses out of 1,000 (i.e. 1 out of 250) will be greater

STEP BY STEP INSTRUCTIONS (5)

- Now we can begin the “layering” process
- Goal
  - allocate total capital to loss scenarios
- Key
  - do not allocate all the capital in one fell swoop – the pitfall of all other capital allocation methods
  - rather, allocate each granular bit of capital
    - allocate uniquely each dollar of capital
    - or, more practically, allocate each layer of capital

STEP BY STEP INSTRUCTIONS (5)

- How should one select the boundary points of the layers of capital?
  - use a separate layer of capital for each loss scenario
    - 1,000 simulations = 1,000 layers of capital
    - 5,000 simulated losses = 5,000 layers of capital
    - etc.
- In case of VaR required capital, the largest losses typically do not correspond to larger capital
- So you should layer the capital based on the lesser of {loss, capital}
  - check that the layers of capital add up to the total capital

STEP BY STEP INSTRUCTIONS (6)

- We now have 2 columns of information:
  - column of insurer's loss scenarios
  - column of layers of capital
- Goal: allocate each layer of capital to loss scenarios
- Note:
  - each layer of capital will be allocated to numerous different loss scenarios
  - each loss scenario receives allocation of capital from numerous layers of capital

STEP BY STEP INSTRUCTIONS (7)

- For each layer of capital:
  - the layer of capital ought to be allocated only to losses that penetrate the layer
  - each loss scenario that does penetrate the layer of capital receives allocation from that layer
- Apply to a single layer of capital, then iterate
- Sheet 7 shows detailed demonstration
  - generates “expanding triangle of allocations”
  - useful for understanding, but not needed in practice
    - use shortcut formulas

STEP BY STEP INSTRUCTIONS (8)

- In practice, you don't need sheet 7
- Sheet 8 shows two methods of condensing sheet 7
  - calculation #1
    - do same formulas but condense into one column
  - calculation #2
    - iterative formula
      - starts with smallest loss, proceeds to next larger loss
    - each loss scenario receives allocation of capital that is
      - exactly the same amount as the next smaller loss scenario
      - plus an incremental amount of capital from the next layer of capital
  - for simulated losses, both calculations provide same answers

STEP BY STEP INSTRUCTIONS (9)

- We now have an initial allocation of capital to loss scenarios for the total firm
- Each loss scenario's allocated capital can be decomposed to individual line of business
  - based on each line's loss contribution to the total loss scenario
- Now we have allocated capital by loss scenario by line of business
- Sum each line of business across all loss scenarios

STEP BY STEP INSTRUCTIONS (10)

- So far
  - we've allocated capital based on loss scenarios
- But what about premium?
  - use allocated capital to calculate target premium
- Note:
  - Initially, we calculated the capital amount so that the firm has sufficient funds to satisfy VaR loss scenario
  - collecting target premium then contributes to the amount of needed funds, which reduces the amount of needed capital
  - capital → premium → reduces capital → reduces premium → increases capital etc.
  - need formula to solve the endless loop

STEP BY STEP INSTRUCTIONS (10)

- Premium = target premium net of expenses
- EL = expected value of company's loss
- r% = required rate of return on capital
- Net Capital = required capital - Premium

- Premium = EL + r% * Net Capital
- Premium = EL + r% * (required capital – Premium)

- **Premium = EL + r/(1+r) * (capital – EL)**

STEP BY STEP INSTRUCTIONS (10)

- Calculate target premium net of expenses via:
  - **Premium = EL + r/(1+r) * (capital – EL)**
- Let's use r% = 10%, for illustration purposes
- Remember that EL = expected value of loss to the company
  - losses capped at available capital
  - slightly less than simulated expected loss
- Calculate target premium for the whole portfolio

STEP BY STEP INSTRUCTIONS (11)

- Now apply to each LOB
  - Expected Value of Company Loss by LOB
  - Allocated Capital by LOB
  - Calculate target premium by LOB
  - Calculate target margin by LOB
  - Allocated capital – target premium = Net Capital
  - Target Margin / Net Capital balances to selected value of r% [required rate of return on capital] for all LOB
- Capital Allocation by Percentile Layer is complete!

STEP BY STEP INSTRUCTIONS (12)

- Optional Extra Credit
  - we can use an alternative method to perform step 11
    - apply to detailed scenarios
    - then roll up to LOB level
  - results are exactly the same, but achieves
    - enhanced granularity
    - insight into key drivers

STEP BY STEP INSTRUCTIONS (12)

- Optional Extra Credit
- For each LOB for each loss scenario, we can calculate
  - expected company loss
  - allocated capital
  - target premium
  - target margin
  - net capital
  - return on capital
  - target loss ratio by scenario
  - pain function / utility function / Mango capital consumption formula by scenario

SUMMARY

- Capital allocation by percentile layer
  - allocates from “firm level” to “scenario level” to “component level”
  - component can be as granular as you like
    - LOB, state, individual policy, etc.
  - allocates based on “which losses cause the firm to hold each dollar of capital”?
  - rooted in “equitable cost allocation”
  - takes the real world cost of holding capital and assigns it to the LOBs and policies that cause the firm to incur this cost
  - takes exposure, risk, and capital and interconnects them to risk adjusted pricing and to risk adjusted profitability measurement

CORRESPONDENCE

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