An Introduction to the Munich Chain Ladder

based on 2008 Variance paper by Quarg and Mack

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Sometimes we are intimidated by seemingly complex new techniques

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Objectives

- Hands on introduction to the *Variance* paper "Munich Chain Ladder"
- Give simple illustration that participants can follow
- **Download triangle spreadsheet data from Spring Meeting web site**

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Data From Appendix
(see web site for download)

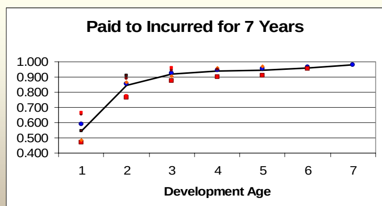
Paid Losses

Year	1	2	3	4	5	6	7
1	576	1,804	1,970	2,024	2,074	2,102	2,131
2	866	1,948	2,162	2,232	2,284	2,348	
3	1,412	3,758	4,252	4,416	4,494		
4	2,286	5,292	5,724	5,850			
5	1,868	3,778	4,648				
6	1,442	4,010					
7	2,044						

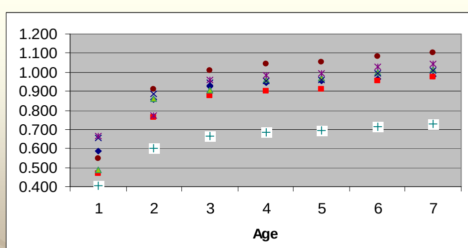
Incurred

Year	1	2	3	4	5	6	7
1	978	2,104	2,134	2,144	2,174	2,182	2,174
2	1,844	2,552	2,466	2,480	2,508	2,454	
3	2,904	4,354	4,698	4,600	4,644		
4	3,502	5,958	6,070	6,142			
5	2,812	4,882	4,852				
6	2,642	4,406					
7	5,022						

Paid to Incurred Ratios for 7 AYs



Chain Ladder P/I Ratio Estimates



Why SCL (Separate Chain Ladder) Results Are Surprising

- Ratio of Projected (P/I) to average are the same as ratio of current (P/I) to current (P/I) average

$$\left(\frac{P}{I}\right)_{i,j} = \frac{P_{i,c}}{I_{i,c}} \frac{\dfrac{\sum_1^n P_{j,t}}{\sum_1^n P_{j,c}}}{\dfrac{\sum_1^n I_{j,t}}{\sum_1^n I_{j,c}}} \rightarrow \frac{(P/I)_{i,t}}{(P/I)_t} = \frac{(P/I)_{i,c}}{(P/I)_c}$$

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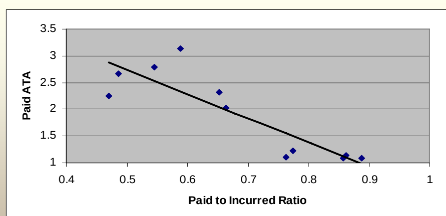
Are Paid ATAs Correlated With PTI Ratios?



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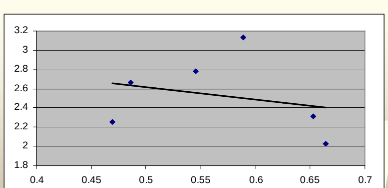
Paid ATAs vs PTI, Age 1-2, Sample Data



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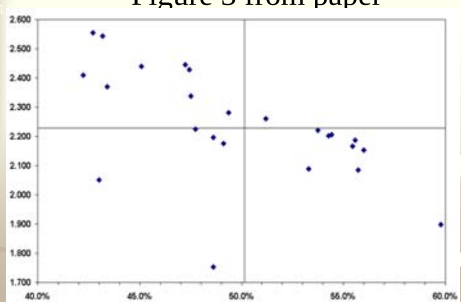
Paid factors vs. preceding P/I ratios: Figure 3 from paper



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Incurred ATAs, Age 1 Using Download Data

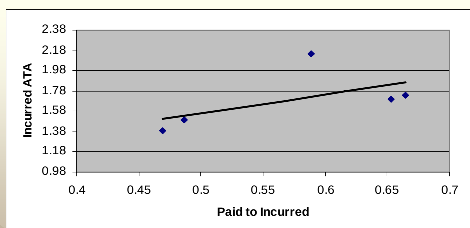

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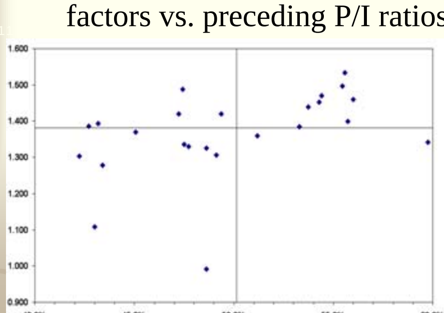
Figure 4: Incurred development factors vs. preceding P/I ratios



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ATAs Under PTI Correlation

- Depending on whether prior paid to incurred ratio is below average or above average, the paid age to age factor should be above average or below average
- Depending on whether prior paid to incurred ratio is below average or above average, the incurred age to age factor should be below average or above average

The residual approach

- Problem: high volatility due to not enough data, especially in later development years
- Solution: consider all development years together

  Use residuals to make different development years comparable.

  Residuals measure deviations from the expected value in multiples of the standard deviation.

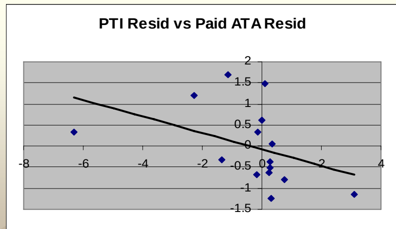
Compute Residuals

Resid, Age To Age for Paid Losses

Year	1	2	3	4	5
Avg	2.527	1.129	1.030	1.022	1.021
SD	0.406	0.060	0.007	0.004	0.010
1	1.490	-0.621	-0.380	0.756	-0.707
2	-0.683	-0.322	0.324	0.378	0.707
3	0.331	0.040	1.201	-1.134	
4	-0.522	-0.795	-1.144		
5	-1.242	1.697			
6	0.625				

e = (x – E(x))/sd(x)

Residual Graph



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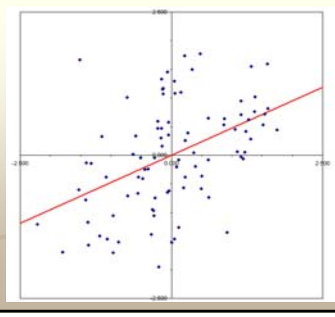
Paper: Residuals of paid development factors vs. I/P residuals



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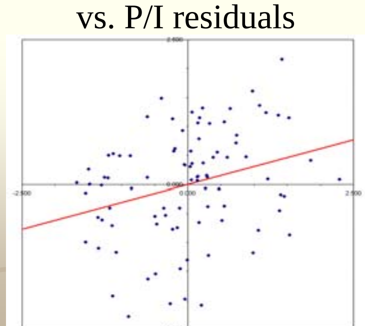
Residuals of incurred dev. factors vs. P/I residuals



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Required model features

$$\mathbf{E}\left(\frac{P_{i,k+1}}{P_{i,k}} | P(k), I(k)\right) = \,??\qquad \mathbf{E}\left(\frac{I_{i,k+1}}{I_{i,k}} | P(k), I(k)\right) = \,??$$

or equivalently

$$\mathbf{E}\left(\mathbf{Res}\left(\frac{P_{i,k+1}}{P_{i,k}}\right) | P(k), I(k)\right) = \,??$$

$$\mathbf{E}\left(\mathbf{Res}\left(\frac{I_{i,k+1}}{I_{i,k}}\right) | P(k), I(k)\right) = \,??$$

where Res ( ) denotes the conditional residual.

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The new model: Munich Chain Ladder

- Interpretation of lambda as correlation parameter:

$$\mathbf{Corr}\left(\frac{P_{i,k+1}}{P_{i,k}}, (I/P)_{i,k} \mid P(k)\right) = \lambda^P$$

$$\mathbf{Corr}\left(\frac{I_{i,k+1}}{I_{i,k}}, (P/I)_{i,k} \mid I(k)\right) = \lambda^I$$

- Together, both lambda parameters characterise the interdependency of paid and incurred.

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The new model: Munich Chain Ladder

- The Munich Chain Ladder assumptions:

$$\mathbf{E}\left(\mathbf{Res}\left(\frac{P_{i,k+1}}{P_{i,k}}\right) | P(k), I(k)\right) = \lambda^P \cdot \mathbf{Res}((I/P)_{i,k})$$

$$\mathbf{E}\left(\mathbf{Res}\left(\frac{I_{i,k+1}}{I_{i,k}}\right) | P(k), I(k)\right) = \lambda^I \cdot \mathbf{Res}((P/I)_{i,k})$$

- Lambda is the slope of the regression line through the origin in the respective residual plot.

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Classic Regression Formula

$$E(Y / X = \mu_y + \rho \frac{\sigma_x}{\sigma_y}(x - \mu_x)$$

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The new model: Munich Chain Ladder

- The Munich Chain Ladder recursion formulas:

$$\widehat{P_{i,k+1}} := \widehat{P_{i,k}} \cdot \left(\widehat{f_k^P} + \widehat{\lambda^P} \cdot \frac{\widehat{\sigma_k^P}}{\widehat{\rho_k^P}} \cdot \left(\frac{\widehat{I_{i,k}}}{\widehat{P_{i,k}}} - \widehat{q_k}^{-1} \right) \right)$$

$$\widehat{I_{i,k+1}} := \widehat{I_{i,k}} \cdot \left(\widehat{f_k^I} + \widehat{\lambda^I} \cdot \frac{\widehat{\sigma_k^I}}{\widehat{\rho_k^I}} \cdot \left(\frac{\widehat{P_{i,k}}}{\widehat{I_{i,k}}} - \widehat{q_k} \right) \right)$$

- through the origin in the residual plot, sigma and rho are variance parameters and q is the average P/I ratio.

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Standard deviations

- Var of Paid ATA

$$\left[\sigma_{s,t}^P\right]^2 = \frac{1}{n-s-1} * \sum_1^{n-s} P_{i,s}\left(\frac{P_{i,t}}{P_{i,s}} - \hat{f}_{s-t}^P\right)^2, n-s = \# \ factors, \hat{f} = E(ATA)$$

- Var of paid ATA, sd = sqrt(Var)

$$\left[\sigma_{s,t}^P\right]^2 = \frac{1}{n-s-1} * \sum_1^{n-s} I_{i,s}\left(\frac{I_{i,t}}{I_{i,s}} - \hat{f}_{s-t}^P\right)^2, n-s = \# \ factors, \hat{f} = E(ATA)$$

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More Standard Deviation Parameters: PTI Ratio

$$(\hat{\rho}_s^I)^2 = \frac{1}{n-s} * \sum_{1}^{n-s+1} I_{j,s}(Q_{j,s} - \hat{q}_s)^2,$$

$$qhat_s = \frac{1}{\sum_{1}^{n-s+1} I_{j,s}} * \sum_{1}^{n-s+1} I_{j,s} Q_{j,s} = \frac{\sum_{1}^{n-s+1} P_{j,s}}{\sum_{1}^{n-s+1} I_{j,s}}, Q = PTI\ ratio$$

$$\sigma(Q_{i,s} \mid I_i(s)) = \frac{\rho_s^I}{\sqrt{I_{i,s}}}$$

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Paid SD Parameters: PTI Ratio

$$(\hat{\rho}_s^P)^2 = \frac{1}{n-s} * \sum_{1}^{n-s+1} P_{j,s}(Q^{-1}{}_{j,s} - \hat{q}^{-1}{}_s)^2,$$

$$q^{-1}{}_s = \frac{1}{\sum_{1}^{n-s+1} I_{j,s}} * \sum_{1}^{n-s+1} P_{j,s} Q^{-1}{}_{j,s} = \frac{\sum_{1}^{n-s+1} P_{j,s}}{\sum_{1}^{n-s+1} I_{j,s}}, Q^{-1} = ITP\ ratio$$

$$\sigma(Q_{i,s} \mid P_i(s)) = \frac{\rho_s^I}{\sqrt{P_{i,s}}}$$

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Steps

- Calculate ATAs
- Calculate ITP and PTI
- Calculate standard deviations
- Calculate standardized residuals
- Calculate correlations
- Calculate adjusted factors
- Use new factors to add a diagonal
- Use to calculate new ITP and PTI and repeat

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Set Up Steps in Excel

Incurred to Paid
Sqrd Deviations ITP Ratios

q'	1.878	1.178	1.078	1.058	1.054	1.042
n-s	6	5	4	3	2	1
Var	223.294	24.900	4.694	2.620	3.208	0.056
rho(P,s)	14.943	4.990	2.167	1.619	1.791	0.236
Year	1	2	3	4	5	6
1	0.032	0.000	0.000	0.000	0.000	0.000
2	0.063	0.017	0.004	0.003	0.002	0.000
3	0.032	0.000	0.001	0.000	0.000	
4	0.120	0.003	0.000	0.000		
5	0.139	0.013	0.001			
6	0.002	0.006				
7	0.336					

Sqrd Deviations of Paid ATA
Paid ATA

n-s	5	4	3	2	1	
Var	181.1	13.4	0.2	0.0	0.2	
Sigma	13.456	3.666	0.482	0.210	0.479	
Year	1	2	3	4	5	6
1	0.483	0.002	0.000	0.000	0.000	0.000
2	0.035	0.000	0.000	0.000	0.000	
3	0.051	0.000	0.000	0.000		
4	0.015	0.002	0.000			
5	0.172	0.010				
6	0.118					

Incurred ATA
Sqrd Dev ATA factors

f	1.652	1.019	1.000	1.011	0.990	0.996
n-s	5	4	3	2	1	
Var	94.622	6.474	1.008	0.014	0.740	
Sigma	9.727	2.544	1.004	0.120	0.860	
Year	1	2	3	4	5	
1	0.249	0.000	0.000	0.000	0.000	
2	0.072	0.003	0.000	0.000	0.000	
3	0.023	0.004	0.000	0.000		
4	0.002	0.000	0.000			
5	0.007	0.001				
6	0.000					

See downloadable file

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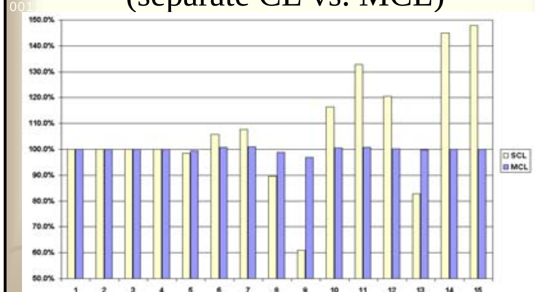
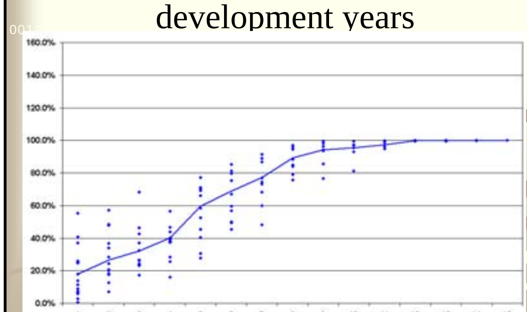
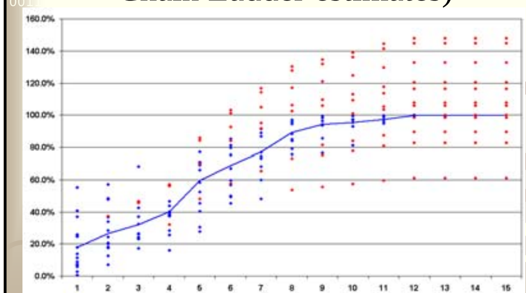
Initial example: ult. P/I ratios (separate CL vs. MCL)



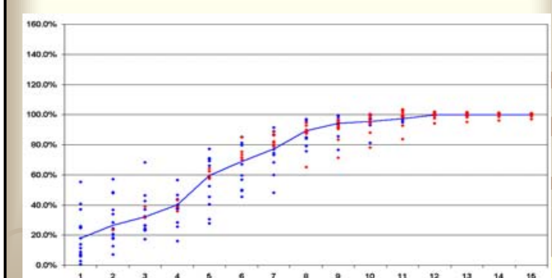
Triangle of P/I ratios vs. development years



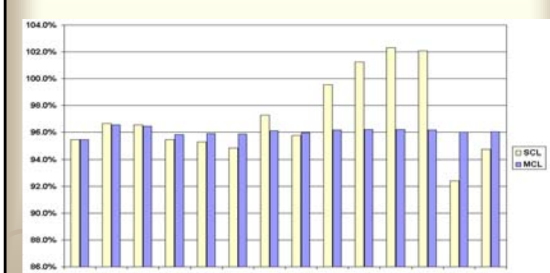
P/I quadrangle (with separate Chain Ladder estimates)

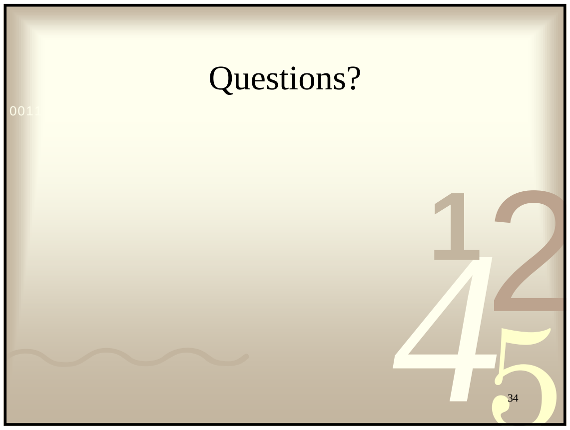


P/I quadrangle (with Munich Chain Ladder)



Another example: ultimate P/I ratios (SCL vs. MCL)




Questions?