# CAS Spring Meeting - Session C22 Technical Provisions in Solvency II What EU Insurers Could Do with Schedule P

ANA /

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# **Outline of Presentation**

- Technical Provisions in Solvency II
- As described in the EU Framework Directive
- American NAIC Schedule P Loss Triangles
- How to use data in Schedule P
- Calculate best estimate
- Calculate risk margin (Market Value Margin)



# Background

- Solvency II adopted by European Parliaments
- Effective October 31, 2012
- Objectives include:
- Increased focus on effective risk management, control and governance
- Market consistent valuation of assets & liabilities
- Increased disclosure and transparency



# **Focus of Presentation Technical Provisions**



Conceptually in line with IFRS



#### Principles Underlying Technical Provisions Described in Articles 76-83 of EU Framework Directive



#### **EUROPEAN UNION**

#### THE EUROPEAN PARLIAMENT

#### THE COUNCIL

Brussels, 19 October 2009 (OR. en)

2007/0143 (COD)

PE-CONS 3643/1/09 REV 1

SURE 15 ECOFIN 349 CODEC 693

LEGISLATIVE ACTS AND OTHER INSTRUMENTS

Subject: DIRECTIVE OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II) (recast)



# **Clip from Article 77 Calculation of technical provisions**

- 1. The value of technical provisions shall be equal to the sum of a best estimate and a risk margin as set out in paragraphs 2 and 3.
- 2 The best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure.

The best estimate shall be calculated gross, without deduction of the amounts recoverable from reinsurance contracts and special purpose vehicles. Those amounts shall be calculated separately, in accordance with Article 81.



# Clip from Article 77 Calculation of technical provisions

 Insurance and reinsurance undertakings shall value the best estimate and the risk margin separately.

# 5. Where insurance and reinsurance undertakings value the best estimate and the risk margin separately, the risk margin shall be calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the lifetime thereof.

# Clip from Article 80 Segmentation

Insurance and reinsurance undertakings shall segment their insurance and reinsurance obligations into homogeneous risk groups, and as a minimum by lines of business, when calculating their technical provisions.



# **Schedule P Loss Triangles**

- Part of American NAIC Annual Statement
- All American insurers must submit to regulators
- Data is available to the public
  - At a price (academic discounts available)
  - <u>http://www.naic.org/store\_financial\_home.htm</u>
- Multiples lines of business
- Paid and incurred loss triangles

# Lines of Business – From Catalogue

#### UNDERWRITING AND INVESTMENT EXHIBIT PART 1 – PREMIUMS EARNED

		l Net Premiums Written per	2 Unearned Premiums Dec. 31 Prior Year- per Col. 3,	3 Unearned Premiums Dec. 31 Current Year- per Col. 5	4 Premiums Earned During Year
Line	of Business	Column 6, Part 1B	Last Year's Part 1	Part 1A	(Cols. 1 + 2 - 3)
Line Line Line Line Line Line Line Line	of Business  multiple peril s multiple peril aranty aranty se aranty practice—occurrence. practice—occurrence. int and health ent and health ent and health ent and health individual) ara and health individual ara and health individual) ara and health individual	Column 6, Part 1B			(Cols. 1+2-3)
of business . 35. TOTALS		er e			



# Earned Premium – From Catalogue

# Part 1Lines of Business A-T

ANNUAL STATEMENT FOR THE YEAR 2008 OF THE

#### SCHEDULE P – PART 1A – HOMEOWNERS/FARMOWNERS (\$000 OMITTED)

	Pr	emiums Ea	rned				Loss and Lo	ss Expense Pay	yments			12
	1	2	3			Defense	and Cost	Adjusting a	and Other	10	11	
				Loss Pay	ments	Containmer	at Payments	Paym	ents			
				4	5	6	7	8	9			Number of
Years in Which										Salvage		Claims Reported
Premiums Were	Direct			Direct		Direct		Direct		and	Total Net Paid	Direct
Earned and Losses	and		Net	and		and		and		Subrogation	(Cols. 4 – 5 +	and
Were Incurred	Assumed	Ceded	(Cols. 1-2)	Assumed	Ceded	Assumed	Ceded	Assumed	Ceded	Received	6 - 7 + 8 - 9	Assumed
1. Prior	XXX	XXX	XXX									XXX
2. 1999												
3. 2000												
4. 2001												
5. 2002												
6. 2003												
7. 2004												
8. 2005												
9. 2006												
10. 2007												
11. 2008												
12. Totals	XXX	XXX	XXX									XXX



# Loss Triangles – From Catalogue

- Part 2 Incurred Net Losses
- Part 3 Cumulative Paid Losses

	INC	URRED NET	LOSSES AND	DEFENSE AND	COST CONTAI	INMENT EXPE	NSES REPORTE	ED AT YEAR EN	ND (\$000 OMIT)	TED)	DEVELO	PMENT
	1	2	3	4	5	6	7	8	9	10	11	12
Years in Which											One	Two
Losses Were Incurred	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	Year	Year
1. Prior												
2. 1999												
3. 2000	XXX											
4. 2001	XXX	XXX										
5. 2002	XXX	XXX	XXX									
<ol> <li>2003</li> </ol>	XXX	XXX	XXX	XXX								
7. 2004	XXX	XXX	XXX	XXX	XXX							
8. 2005	XXX	XXX	XXX	XXX	XXX	XXX						
9. 2006	XXX	XXX	XXX	XXX	XXX	XXX	XXX					
10. 2007	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX				XXX
11. 2008	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX		XXX	XXX
12. Totals												

#### SCHEDULE P - PART 2A - HOMEOWNERS/FARMOWNERS

SCHEDULE P - PART 2B - PRIVATE PASSENGER AUTO LIABILITY/MEDICAL

1.	Prior										 	
2	1000										 	
3	2000	XXX			S. ma	Dada	-				 	
4	2001	XXX	XXX		(0)[2][1]	Keike	2(216)(		DIMINV/			
5	2002	XXX	XXX	XXX		19469	0		Range			
6	2003	XXX	XXX	XXX	XXX							
7	2004	XXX	XXX	XXX	XXX	XXX					 	
8	2005	VVV	VVV	VVV	XXX	VYY	VVV				 	
0.	2005	- AAA VVV	- AAA VVV	vvv	- AAA VVV	NVV VVV	AAA VVV	vvv			 	
9.	2000	XXX	XXX	AAA VVV	XXX	AAA VVV	XXX	XXX	VVV		 	VVV
10.	2007	AAA NYYY	AAA VVV	AAA VVV	AAA WWW	AAA VVV	AAA VVV	AAA VVV	AAA VVV	UUU	 VVV	AAA NYVY
- 11.	2008	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX	XXX

## From Data Reported in Schedule P Assemble a Triangle of Incremental Paid Losses

- Include Earned Premium
- This data is for a real insurer
- Commercial Auto Liability

AY	Premium	Lag 1	Lag <sub>2</sub>	Lag ₃	Lag 4	Lag 5	Lag <sub>6</sub>	Lag 7	Lag <sub>8</sub>	و Lag	Lag <sub>10</sub>
1	29,701	5,234	5,172	3,708	1,783	923	537	175	145	8	0
2	27,526	5,234	<mark>5,68</mark> 3	4,392	2,134	1,377	673	155	81	47	-
3	30,750	5,702	5,865	7,966	2,472	NA	143	152	73	-	-
4	35,814	6,349	4,611	3,959	2,522	1,924	622	206	-	-	-
5	42,277	8,377	6,890	4,055	3,795	1,292	1,422	-	-	-	-
6	50,088	9,291	13,836	12,441	4,086	2,293	-	-	-	-	-
7	56,921	12,029	12,462	8,369	7,034	-	-	-		- -	-
8	61,406	13,119	12,618	9,117	-	-	- Ar		-	-	-
9	67,983	15,860	14,893	_	_	_	_	-	_	-	-
10	73,359	16,498	-	-	-	_	-	-	-	-	-



# **Using Schedule P Data**

- We of course have the reported earned premium and incremental paid loss
- But we also have similar data for every American insurer for each line of business
- A great source of "prior information" that can be used in a Bayesian analysis
- Use Schedule P data for large insurers to create 5000 "benchmark scenarios" for each line of insurance



# Loss Model

Expected Loss

$$\mu_{AY,Lag} = Premium_{AY} \cdot ELR_{AY} \cdot Dev_{Lag} \cdot t^{AY+Lag-1}$$

Variance of Loss

$$Var\left[X_{AY,Lag}\right] = \mu_{AY,Lag} \cdot \tau_{Lag} \cdot (1+1/\alpha) + c \cdot \mu_{AY,Lag}^{2}$$
$$\tau_{Lag} = Sev \cdot \left(1 - \left(1 - \frac{Lag}{10}\right)^{3}\right) \text{ for } Lag = 1,2...,10.$$

• {*ELR*<sub>AY</sub>},{*Dev*<sub>Lag</sub>}, *t*, *c*, and *Sev* are unknown parameters,



# Tweedie Model of Losses in Each (*AY,Lag*) Cell

$$\mu_{AY,Lag} = Premium_{AY} \cdot ELR_{AY} \cdot Dev_{Lag} \cdot t^{AY+Lag-1}, p = \frac{\alpha+2}{\alpha+1}$$

$$\phi_{AY,Lag} \cdot \mu^{\rho}_{AY,Lag} = \mu_{AY,Lag} \cdot \tau_{Lag} \cdot (1 + 1 / \alpha) + c \cdot \mu^{2}_{AY,Lag}$$

- Pick a parameter set {*ELR<sub>AY</sub>*}, {*Dev<sub>Lag</sub>*},*t*,*c*, *Sev*
- Translate parameters into Tweedie parameters
  - $\mu_{AY,Lag}$  , p and  $\phi_{AY,Lag}$
- Calculate sample parameter sets from posterior distribution with Metropolis-Hastings Algorithm
  - See Session C3 Tomorrow
  - Keep 500 Parameter Sets



### Sample from Metropolis-Hastings Algorithm Applied to {Dev<sub>Lag</sub>} and {ELR<sub>AY</sub>} parameters

$ELR_1$	$ELR_2$	$ELR_3$	$ELR_4$	$ELR_5$	$ELR_6$	$ELR_7$	$ELR_8$	$ELR_9$	$ELR_{10}$
0.91503	0.66796	0.62778	0.58480	0.56635	0.67332	0.56119	0.68528	0.69505	0.70776
0.91503	0.66796	0.62778	0.58480	0.56635	0.67332	0.56119	0.68528	0.69505	0.70776
0.91503	0.66796	0.62778	0.58480	0.56635	0.67332	0.56119	0.68528	0.69505	0.70776
0.91503	0.66796	0.62778	0.58480	0.56635	0.67332	0.56119	0.68528	0.69505	0.70776
0.91503	0.66796	0.62778	0.58480	0.56635	0.67332	0.56119	0.68528	0.69505	0.70776
0.91503	0.66796	0.62778	0.58480	0.56635	0.67332	0.56119	0.68528	0.69505	0.70776
0.91503	0.66796	0.62778	0.58480	0.56635	0.67332	0.56119	0.68528	0.69505	0.70776
0.91503	0.66796	0.62778	0.58480	0.56635	0.67332	0.56119	0.68528	0.69505	0.70776
0.86193	0.63186	0.67501	0.57013	0.60554	0.64775	0.61769	0.74869	0.68954	0.68855
0.85805	0.62464	0.68672	0.55612	0.58922	0.63364	0.65857	0.70962	0.67289	0.64800
Dev <sub>1</sub>	Dev <sub>2</sub>	Dev <sub>3</sub>	$Dev_4$	$Dev_5$	Dev <sub>6</sub>	$Dev_7$	Dev <sub>8</sub>	Dev <sub>9</sub>	$Dev_{10}$
<i>Dev</i> <sub>1</sub> 0.16546	<i>Dev</i> <sub>2</sub> 0.25163	<i>Dev</i> <sub>3</sub> 0.22465	<i>Dev</i> <sub>4</sub> 0.16499	<i>Dev</i> <sub>5</sub> 0.10414	Dev <sub>6</sub> 0.05589	Dev <sub>7</sub> 0.02427	Dev <sub>8</sub> 0.00762	<i>Dev</i> <sub>9</sub> 0.00131	Dev <sub>10</sub> 0.00005
Dev <sub>1</sub> 0.16546 0.16546	Dev <sub>2</sub> 0.25163 0.25163	Dev <sub>3</sub> 0.22465 0.22465	Dev <sub>4</sub> 0.16499 0.16499	Dev <sub>5</sub> 0.10414 0.10414	<i>Dev</i> <sub>6</sub> 0.05589 0.05589	Dev <sub>7</sub> 0.02427 0.02427	Dev <sub>8</sub> 0.00762 0.00762	Dev <sub>9</sub> 0.00131 0.00131	Dev <sub>10</sub> 0.00005 0.00005
Dev <sub>1</sub> 0.16546 0.16546 0.16321	Dev <sub>2</sub> 0.25163 0.25163 0.24844	Dev <sub>3</sub> 0.22465 0.22465 0.22338	Dev <sub>4</sub> 0.16499 0.16499 0.16574	Dev <sub>5</sub> 0.10414 0.10414 0.10598	Dev <sub>6</sub> 0.05589 0.05589 0.05781	Dev <sub>7</sub> 0.02427 0.02427 0.02564	Dev <sub>8</sub> 0.00762 0.00762 0.00827	Dev <sub>9</sub> 0.00131 0.00131 0.00148	Dev <sub>10</sub> 0.00005 0.00005 0.00006
Dev <sub>1</sub> 0.16546 0.16546 0.16321 0.16613	Dev <sub>2</sub> 0.25163 0.25163 0.24844 0.24962	Dev <sub>3</sub> 0.22465 0.22465 0.22338 0.22293	Dev <sub>4</sub> 0.16499 0.16499 0.16574 0.16463	Dev <sub>5</sub> 0.10414 0.10414 0.10598 0.10487	Dev <sub>6</sub> 0.05589 0.05589 0.05781 0.05701	Dev <sub>7</sub> 0.02427 0.02427 0.02564 0.02520	Dev <sub>8</sub> 0.00762 0.00762 0.00827 0.00811	Dev <sub>9</sub> 0.00131 0.00131 0.00148 0.00144	Dev <sub>10</sub> 0.00005 0.00005 0.00006 0.00006
Dev <sub>1</sub> 0.16546 0.16546 0.16321 0.16613 0.16613	Dev <sub>2</sub> 0.25163 0.25163 0.24844 0.24962 0.24962	Dev <sub>3</sub> 0.22465 0.22338 0.22293 0.22293	Dev <sub>4</sub> 0.16499 0.16499 0.16574 0.16463 0.16463	Dev <sub>5</sub> 0.10414 0.10414 0.10598 0.10487 0.10487	Dev <sub>6</sub> 0.05589 0.05589 0.05781 0.05701 0.05701	Dev <sub>7</sub> 0.02427 0.02427 0.02564 0.02520 0.02520	Dev <sub>8</sub> 0.00762 0.00762 0.00827 0.00811 0.00811	Dev <sub>9</sub> 0.00131 0.00131 0.00148 0.00144 0.00144	Dev <sub>10</sub> 0.00005 0.00005 0.00006 0.00006
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Dev <sub>1</sub> 0.16546 0.16546 0.16321 0.16613 0.16613 0.16613	Dev <sub>2</sub> 0.25163 0.25163 0.24844 0.24962 0.24962 0.24962 0.24962	Dev <sub>3</sub> 0.22465 0.22338 0.22293 0.22293 0.22293 0.22293	$Dev_4$ 0.16499 0.16499 0.16574 0.16463 0.16463 0.16463 0.16463	Dev <sub>5</sub> 0.10414 0.10414 0.10598 0.10487 0.10487 0.10487 0.10487	Dev <sub>6</sub> 0.05589 0.05589 0.05781 0.05701 0.05701 0.05701	Dev <sub>7</sub> 0.02427 0.02427 0.02564 0.02520 0.02520 0.02520 0.02520	Dev <sub>8</sub> 0.00762 0.00762 0.00827 0.00811 0.00811 0.00811	Dev9 0.00131 0.00131 0.00148 0.00144 0.00144 0.00144	<i>Dev</i> <sub>10</sub> 0.00005 0.00006 0.00006 0.00006 0.00006 0.00006
Dev <sub>1</sub> 0.16546 0.16546 0.16321 0.16613 0.16613 0.16613 0.16613	Dev <sub>2</sub> 0.25163 0.25163 0.24844 0.24962 0.24962 0.24962 0.24962 0.24962	Dev <sub>3</sub> 0.22465 0.22338 0.22293 0.22293 0.22293 0.22293 0.22293 0.22293	$Dev_4$ 0.16499 0.16499 0.16574 0.16463 0.16463 0.16463 0.16463 0.16463	Dev <sub>5</sub> 0.10414 0.10414 0.10598 0.10487 0.10487 0.10487 0.10487 0.10487	Dev <sub>6</sub> 0.05589 0.05589 0.05781 0.05701 0.05701 0.05701 0.05701	Dev <sub>7</sub> 0.02427 0.02427 0.02564 0.02520 0.02520 0.02520 0.02520 0.02520	Dev <sub>8</sub> 0.00762 0.00827 0.00811 0.00811 0.00811 0.00811	Dev9 0.00131 0.00131 0.00148 0.00144 0.00144 0.00144 0.00144	<i>Dev</i> <sub>10</sub> 0.00005 0.00006 0.00006 0.00006 0.00006 0.00006
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# **Graphical Representation of Metropolis-Hastings Sample**



ELR Paths

**Dev Paths** 



Note that the posteriors are tighter, showing how the data narrows the range of results.

"Information Reduces **Uncertainty**"



# Graphical Representation of Metropolis-Hastings Sample

Prior Distribution of 'c' Parameter



Note that the posteriors are tighter, showing how the data narrows the range of results.

> "Information Reduces Uncertainty"

# Graphical Representation of Metropolis-Hastings Sample

Prior Distribution 't' Parameter



Posterior Distribution of 't' Parameter For Insurer #1

6

8

0

Frequency 0 60



Note that the posteriors are tighter, showing how the data narrows the range of results.

> "Information Reduces Uncertainty"



# Graphical Representation of Metropolis-Hastings Sample



Prior Distribution of 'sev' Parameter





Note that the posteriors are tighter, showing how the data narrows the range of results.

"Information Reduces Uncertainty"



### **Calculating the Best Estimate** From Article 77 – Framework Directive

- The best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure.
- Expected loss for *n*<sup>th</sup> Metropolis-Hastings scenario

$$\mu_{n,AY,Lag} = Premium_{AY} \cdot ELR_{AY} \cdot Dev_{Lag} \cdot t^{AY+Lag-1}$$

Best Estimate i = 4%

$$\sum_{AY=2}^{10} \sum_{Lag=12-AY}^{10} \left( \frac{1}{500} \sum_{n=1}^{500} \mu_{n,AY,Lag} \right) \cdot \frac{1}{\left(1+i\right)^{AY+Lag-11.5}} = 91,220$$



# **Technical Provisions, Risk Margins and the IAA**

- International Association of Insurance Supervisors (IAIS) requested help from the International Actuarial Association (IAA) to work on the issues of risk based capital and risk margins for loss reserves.
- Available on IIA Website http://www.actuaries.org/LIBRARY/Papers/IAA Measurement of Liabilities 2009-public.pdf
- Refer to risk margin as Market Value Margin (MVM)



# **IAIS – Properties of Risk Margins**

- 1. The less that is known about the current estimate and its trend; the higher the risk margins should be.
- 2. Risks with low frequency and high severity will have higher risk margins than risks with high frequency and low severity.
- 3. For similar risks, contracts that persist over a longer timeframe will have higher risk margins than those of shorter duration.
- 4. Risks with a wide probability distribution will have higher risk margins than those risks with a narrower distribution.
- 5. To the extent that emerging experience reduces uncertainty, risk margins will decrease, and vice versa.



# Possibilities

- Undiscounted reserves
- Only satisfy Property 3
- Percentile method
- Does not satisfy Property 3

# Cost of Capital Method

- Satisfies all properties

# • So – What is the Cost of Capital Method?



# What is Capital?

- Sufficient for time horizon of one year
- Controversial Many prefer a longer time horizon
- Capital = TVaR@99% Expected Loss
- Calculate for each future payment year
  - i.e. sub diagonal
- Remember the time value of money
- FFT methods on Tweedie distributions allow for reliable calculation of TVaR



# Statistics of Interest for Risk Margin Distribution of Reserve Outcomes

# **Incremental Paid Losses**

AY	Premium	Lag <sub>1</sub>	Lag <sub>2</sub>	Lag <sub>3</sub>	Lag <sub>4</sub>	Lag <sub>5</sub>	Lag <sub>6</sub>	Lag <sub>7</sub>	Lag <sub>8</sub>	Lag <sub>9</sub>	Lag <sub>10</sub>
1	29,701	5,234	5,172	3,708	1,783	923	537	175	145	8	0
2	27,526	5,234	5,683	4,392	2,134	1,377	673	155	81	47	<i>X</i> <sub>2,10</sub>
3	30,750	5,702	5,865	7,966	2,472	NA	A 143	152	73	<b>X</b> <sub>3,9</sub>	<i>X</i> <sub>3,10</sub>
4	35,814	6,349	4,611	3,959	2,522	1,924	622	206	<i>X</i> <sub>4,8</sub>	<i>X</i> <sub>4,9</sub>	<i>X</i> <sub>4,10</sub>
5	42,277	8,377	6,890	4,055	3,795	1,292	1,422	X <sub>5,7</sub>	X <sub>5,8</sub>	<b>X</b> <sub>5,9</sub>	<i>X</i> <sub>5,10</sub>
6	50,088	9,291	13,836	12,441	4,086	2,293	<b>X</b> <sub>6,6</sub>	X <sub>6,7</sub>	X <sub>6,8</sub>	<b>X</b> <sub>6,9</sub>	<i>X</i> <sub>6,10</sub>
7	56,921	12,029	12,462	8,369	7,034	X <sub>7,5</sub>	<b>X</b> <sub>7,6</sub>	X <sub>7,7</sub>	X <sub>7,8</sub>	<b>X</b> <sub>7,9</sub>	<i>X</i> <sub>7,10</sub>
8	61,406	13,119	12,618	9,117	<i>X</i> <sub>8,4</sub>	X <sub>8,5</sub>	<i>X</i> <sub>8,6</sub>	X <sub>8,7</sub>	X <sub>8,8</sub>	X <sub>8,9</sub>	X <sub>8,10</sub>
9	67,983	15,860	14,893	<b>X</b> <sub>9,3</sub>	<i>X</i> <sub>9,4</sub>	X <sub>9,5</sub>	X <sub>9,6</sub>	X <sub>9,7</sub>	X <sub>9,8</sub>	Х <sub>9,9</sub>	<b>X</b> <sub>9,10</sub>
10	73,359	16,498	<i>X</i> <sub>10,2</sub>	<i>X</i> <sub>10,3</sub>	<b>X</b> <sub>10,4</sub>	<i>X</i> <sub>10,5</sub>	<b>X</b> <sub>10,6</sub>	<i>X</i> <sub>10,7</sub>	<i>X</i> <sub>10,8</sub>	<b>X</b> <sub>10,9</sub>	<i>X</i> <sub>10,10</sub>

#### Predictive Distribution of Reserve Outcomes – 1 Year

$$R_1 = \sum_{AY=2}^{10} X_{AY,12-AY}$$

Predictive Distribution of Reserve Outcomes – 10 Year

$$R_{10} = \sum_{AY=2}^{10} \sum_{Lag=12-AY}^{10} X_{AY,Lag}$$



# Statistics of Interest for Risk Margins Predictive Distributions of Reserve Outcomes

# • Simulation

- Randomly select  $\{ELR_i\}, \{Dev_j\}, t, c, Sev$  from the posterior

- Simulate 
$$R_{10} = \sum_{AY=2}^{10} \sum_{Lag=12-AY}^{10} X_{AY,Lag}$$
 where  $X_{AY,Lag} \sim \text{Tweedie}$ 

- Simulate  $R_1$  Similarly
- Use the Fast Fourier Transform
- Faster, more accurate, but uses some math



# **Calculating Capital Needs in the Future** 1 Year Time Horizon - Discount @ 4%

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
t	$L_t^{Nom}$	$\Delta \mathcal{L}_t^{Nom}$	$L_t^{Disc}$	$TVaR_t^{Nom}$	$\Delta TVaR^{\mathit{Nom}}_t$	$TVaR_t^{Disc}$	$C_t$
0	40,375	13,882	37,526	52,875	15,933	48,415	<mark>10,889</mark>
1	26,493	12,004	24,870	36,942	15,641	34,103	<mark>9,233</mark>
2	14,490	6,867	13,624	21,301	8,603	19,516	<mark>5,893</mark>
3	7,622	3,661	7,165	12,698	4,741	11,524	<mark>4,358</mark>
4	3,962	1,919	3,719	7,957	2,606	7,150	<mark>3,432</mark>
5	2,042	766	1,910	5,352	834	4,779	<mark>2,869</mark>
6	1,276	484	1,205	4,517	230	4,119	<mark>2,914</mark>
7	792	341	760	4,287	190	4,050	<mark>3,290</mark>
8	451	451	442	4,097	4,097	4,017	<mark>3,575</mark>



# Calculating Capital Needs in the Future 10 Year Time Horizon - Discount @ 4%

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
t	$L_t^{Nom}$	$\Delta \mathcal{L}_t^{Nom}$	$L_t^{Disc}$	$TVaR_t^{Nom}$	$\Delta TVaR^{\mathit{Nom}}_t$	$TVaR_t^{Disc}$	$C_t$
0	97,503	40,375	91,220	128,894	48,491	118,529	<mark>27,309</mark>
1	57,128	26,493	53,695	80,403	31,742	73,819	<mark>20,124</mark>
2	30,635	14,490	28,824	48,661	17,133	44,401	<mark>15,576</mark>
3	16,145	7,622	15,201	31,528	9,412	28,705	<mark>13,504</mark>
4	8,523	3,962	8 <i>,</i> 035	22,116	6,225	20,255	<mark>12,219</mark>
5	4,561	2,042	4,317	15,891	4,321	14,717	<mark>10,400</mark>
6	2,519	1,276	2,407	11,570	3,673	10,899	<mark>8,493</mark>
7	1,243	792	1,202	7,898	3,801	7,590	<mark>6,388</mark>
8	451	451	442	4,097	4,097	4,017	<mark>3,575</mark>



# Explanation of C<sub>t</sub> Calculation (Documentation)

- (1) The time, t, after the liability is set.
- (2) The expected value of all future payments  $\lim_{t \to T} L_{r}^{Nom} = \sum_{AY=2+t}^{10} \sum_{Log=12+t-AY}^{10} \mu_{AY,Log}$ .
- $(3) \quad \Delta L_t^{Nom} = L_t^{Nom} L_{t+1}^{Nom}.$
- (4) The discounted liability  $_{k}L_{t}^{Disc} = \sum_{k=t}^{8} \frac{\Delta L_{k}^{Nom}}{(1+i)^{k-t+0.5}}$ .
- (5) The Tail-Value-at-Risk, i.e., the conditional expected value of the random loss,

 $\sum_{AY=2+t}^{10} \sum_{Lag=12+t-AY}^{10} X_{AY,Lag} \lim_{k \to \infty} \text{ that the loss exceeds the 99th percentile.}$ 

- (6)  $\Delta TVaR_t^{Nom} = TVaR_t^{Nom} TVaR_{t+1}^{Nom}$ .
- (7) The discounted TVaR<sub>t</sub><sup>Disc</sup> =  $\sum_{k=t}^{8} \frac{\Delta TVaR_{k}^{Nom}}{(1+i)^{k-t+0.5}}$ .

(8) The needed capital at time t is expected to  $\underbrace{be}_{t} C_{t} = TVaR_{t}^{Disc} - L_{t}^{Disc}$ .



# **Risk Margin Version 1 – Capital Cash Flow (CCF)**

- C<sub>t</sub> = required capital at time t for one year.
- TVaR@99% Expected Loss (discounted)
- i = risk-free rate of return
- r = risky rate of return due to insurer's investors
- $MVM_{CCF} = C_0 PV(Released Capital @ rate r)$

$$MVM_{CCF} = C_0 - \sum_{t=0}^{\infty} \frac{C_t \cdot (1+i) - C_{t+1}}{(1+r)^{t+1}}$$
$$MVM_{CCF} = (r-i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^{t+1}}$$

After some algebra



# Versions 2 and 3

 Capital Cash Flow (CCF)

$$MVM_{CCF} = (r-i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^{t+1}}$$

- Swiss Solvency Test (SST)
- Starts at t = 1. Ignores capital raised in first year.
- Discounts at rate *i* instead of rate r.
- Solvency II/QIS4 (SII)
- Starts at t = 0

$$MVM_{SST} = (r-i) \cdot \sum_{t=1}^{\infty} \frac{C_t}{(1+i)^{t+1}}$$

$$MVM_{SII} = (r-i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1+i)^{t+1}}$$

# All three versions satisfy the IAIS criteria.



**Rationale Behind MVM**<sub>SST</sub>

$$MVM_{SST} = (r-i) \cdot \sum_{t=1}^{\infty} \frac{C_t}{(1+i)^{t+1}}$$

"The risk margin can be expressed as the expected present value of the cost of capital necessary to buffer the nonhedgeable risk of insurance liabilities during the entire lifetime of the insurance liabilities."



Rationale Behind MVM<sub>SII</sub>

$$MVM_{SII} = (r-i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1+i)^{t+1}}$$

Both Solvency II and SST require capital to cover risk over a one year time horizon. SST says that you don't need a risk margin to cover the first year. Solvency II says you do.



# The Results

<i>r</i> = 10%	i =4%		Best Estimate = 91,220							
Time										
Horizon	MVM <sub>CCF</sub>	%	MVM <sub>SST</sub>	%	MVM <sub>QIS4</sub>	%				
1	1,994	2.2%	1,854	2.0%	2,411	2.6%				
10	5,082	5.6%	4,736	5.2%	6,129	6.7%				



# Summary

- Used Bayesian analysis and likelihood of a triangle of data based on the Tweedie model to calculate posterior probabilities of scenarios
- Used posterior probability of scenarios to calculate
- Current Estimate
- Risk margin

