Robust Loss Development Using MCMC

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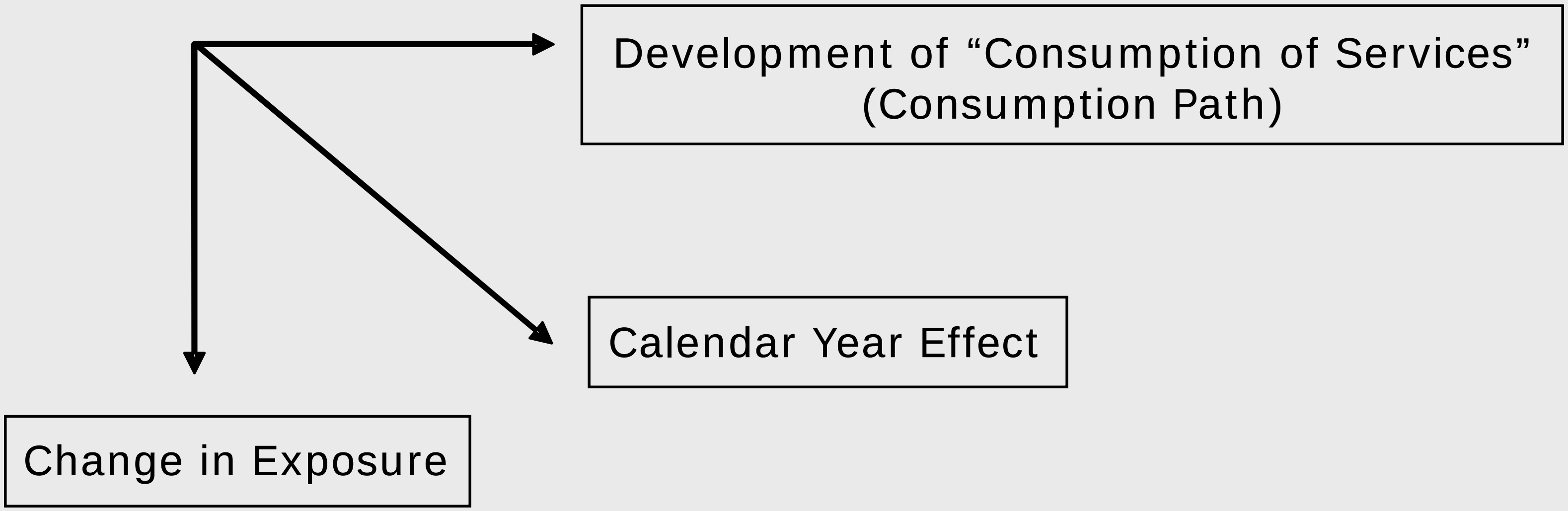
Overview

- Robust Loss Development Using MCMC
  - Bayesian time series model of loss development
  - Robust (skewed and heteroskedastic Student-*t*, model-averaging using RJMCMC, endogenous breakpoint in development pattern)
- The model has been tested for two years on workers' compensation indemnity and medical triangles of about 35 U.S. states; it has also been tested on pool reserving triangles
- The model has been bundled into the R package *lossDev* and is available on CRAN: http://cran.r-project.org/
- For an application of the model to large workers' compensation triangles see "The Workers Compensation Tail Revisited," which is available at ww.ncci.com/documents/CASJournal-Schmid.pdf

Time Series Methodology

- Times series models of loss development have been pioneered by Kremer [4], Verrall [5, 6], Zehnwirth [7], Barnett and Zehnwirth [1], and de Jong [2]
- These times series approaches attempt to model explicitly the process that generates the data
- Unlike models that mechanistically fit to data, these time series models can not only be judged *ex post* by how well they are able to replicate the data but can also be evaluated *ex ante* by the adequacy of the equations that describe the data-generating process
- Estimating the equations of the data-generating process facilitates learning about the nature of loss triangles

Triangle Dynamics



For this architecture of triangle analysis, see Barnett and Zehnwirth [1]

Main Features of the Statistical Model (1/3)

- The model is Bayesian and estimated using MCMC (Markov Chain Monte Carlo simulation)
  - The model is estimated using JAGS with own C++ routines for Reversible Jump MCMC and over-relaxed slice sampling added on
- The model fits to the (natural) logarithms of incremental payments
  - Negative payments or payments at zero amounts are coded as missing values
- The model uses a skewed Student-*t* likelihood (Kim and McCulloch [3])
  - The degrees of freedom of the *t*-distribution are endogenous
  - The scale parameter of the *t*-distribution is allowed to vary in development time

Main Features of the Statistical Model (2/3)

- The calendar year effect is modeled as a normal distribution around an expert prior for the rate of inflation
  - For General Liability triangles, a suitable expert prior for the calendar year effect may be the CPI (Consumer Price Index) rate of inflation
  - For Auto Bodily Injury Liability triangles, the Medical Care component of the CPI (M-CPI, for short) may serve as an expert prior
- A Bernoulli distribution (the parameter of which varies on a Gompertz curve in development time) accounts for the variation in the probability of observing a payment at zero amount

Main Features of the Statistical Model (3/3)

- A first-order autoregressive process of the error the calendar year effect is optional and used for both triangles studied here
- Similar, a first-order autoregressive process of the rate of exposure growth is optional and used for both triangles studied here
- There is a changepoint version of the model that allows for a structural break in the consumption path
  - An interval of exposure years has to be supplied—the model then determines the changepoint within this interval
  - The second of the two triangles studied here is estimated using the changepoint specification

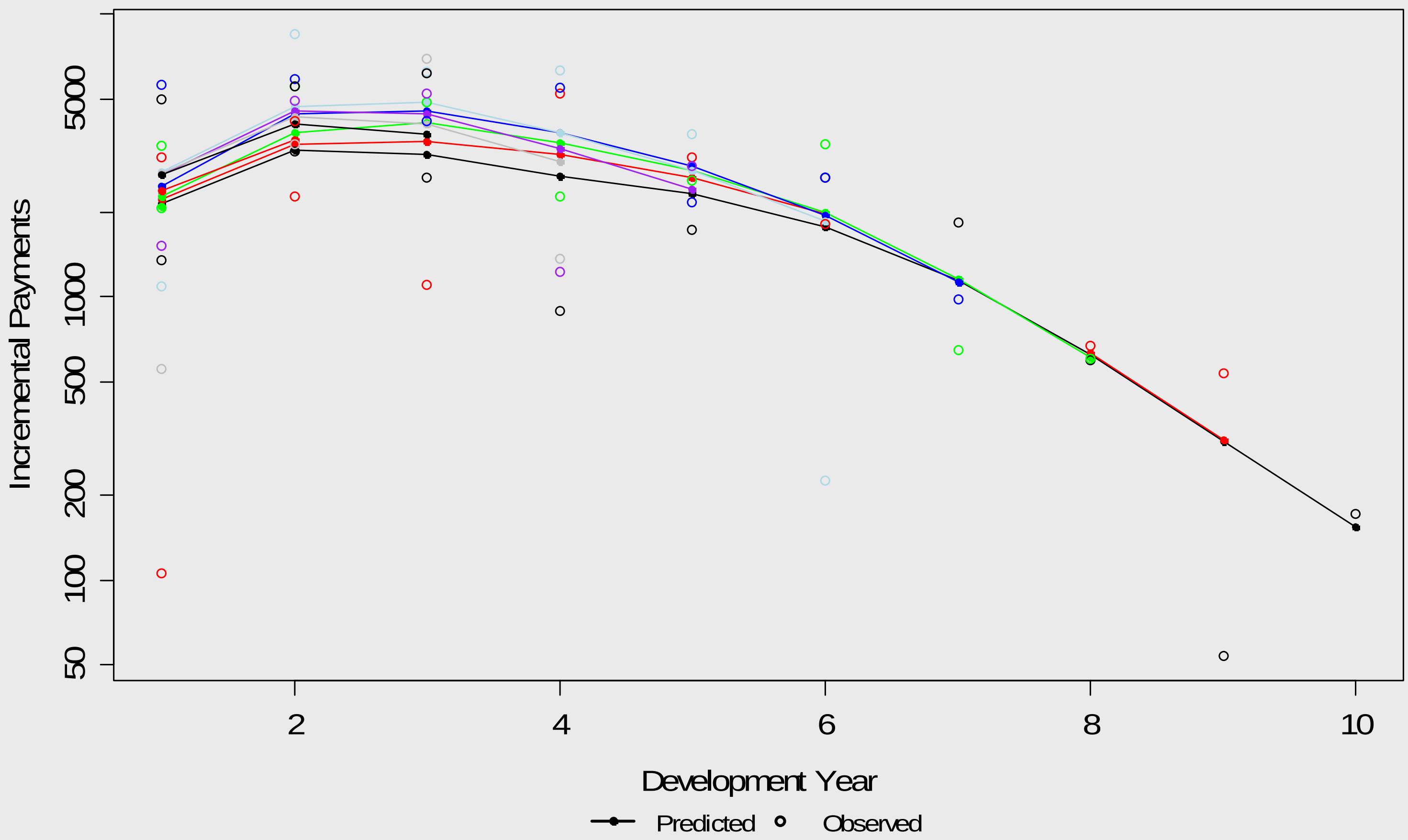
A General Liability Triangle

- Annual Incremental Incurred Loss Triangle of Automatic Facultative Business in General Liability (Excluding Asbestos & Environmental)

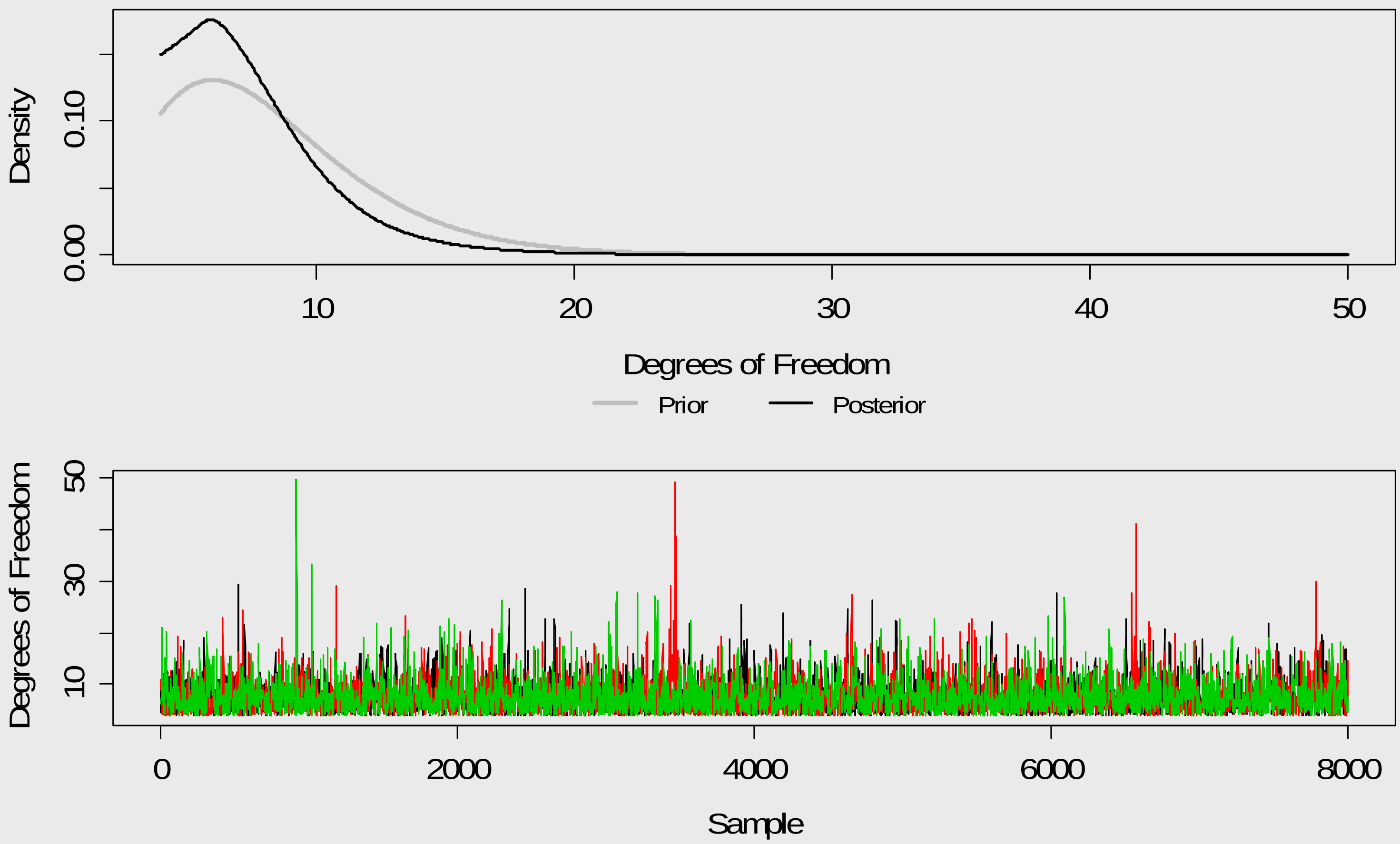
Year	1	2	3	4	5	6	7	8	9	10
1981	5,012	3,257	2,638	898	1,734	2,642	1,828	599	54	172
1982	106	4,179	1,111	5,270	3,116	1,817	-103	673	535	
1983	3,410	5,582	4,881	2,268	2,594	3,479	649	603		
1984	5,655	5,900	4,211	5,500	2,159	2,658	984			
1985	1,092	8,473	6,271	6,333	3,786	225				
1986	1,513	4,932	5,257	1,233	2,917					
1987	557	3,463	6,926	1,368						
1988	1,351	5,596	6,165							
1989	3,133	2,262								
1990	2,063									

Data source: Mack, Thomas, "Which Stochastic Model is Underlying the Chain Ladder Method," Casualty Actuarial Society *Forum*, Fall 1995, pp. 229-240, http://www.casact.org/pubs/forum/95fforum/95ff229.pdf

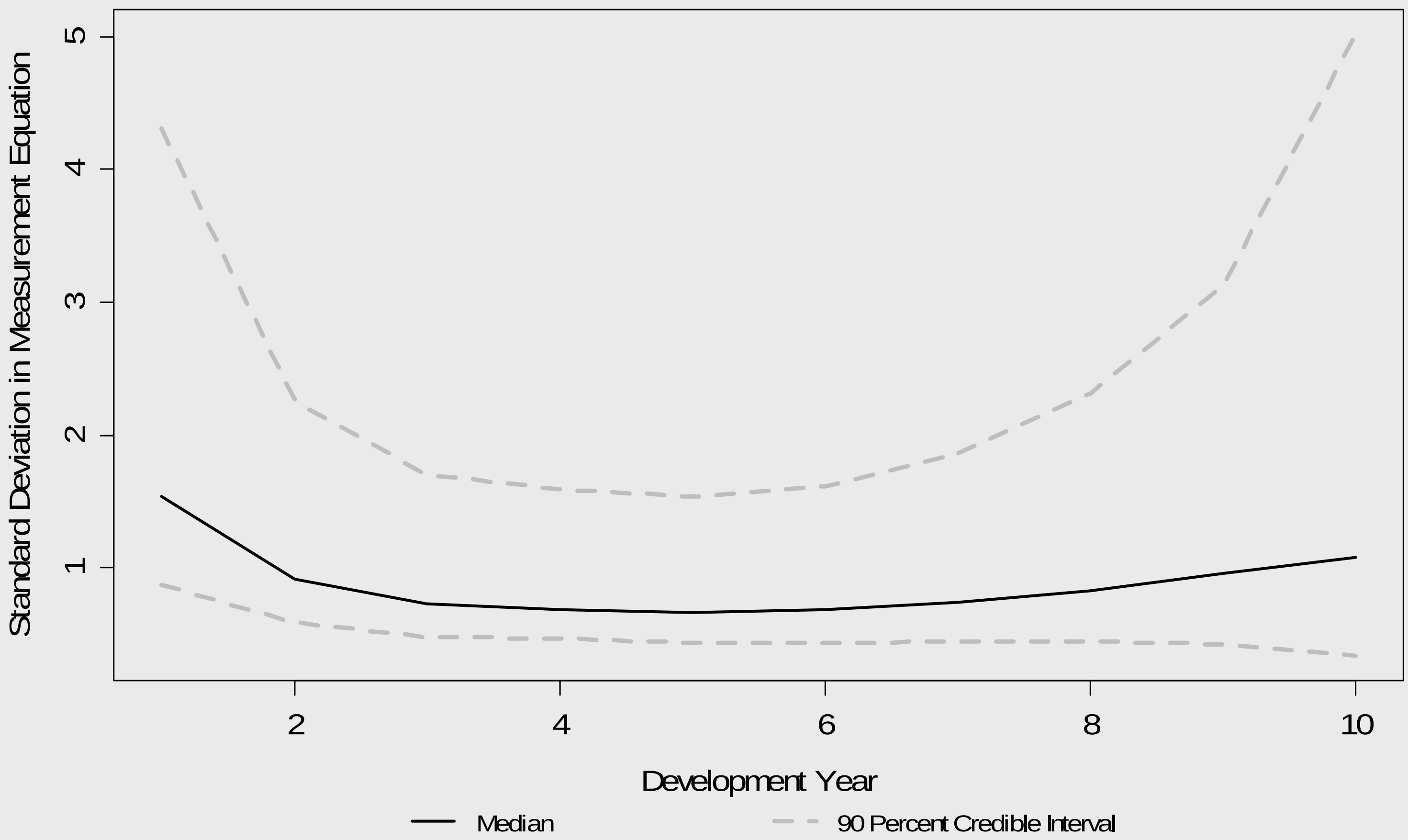
Log Incremental Payments



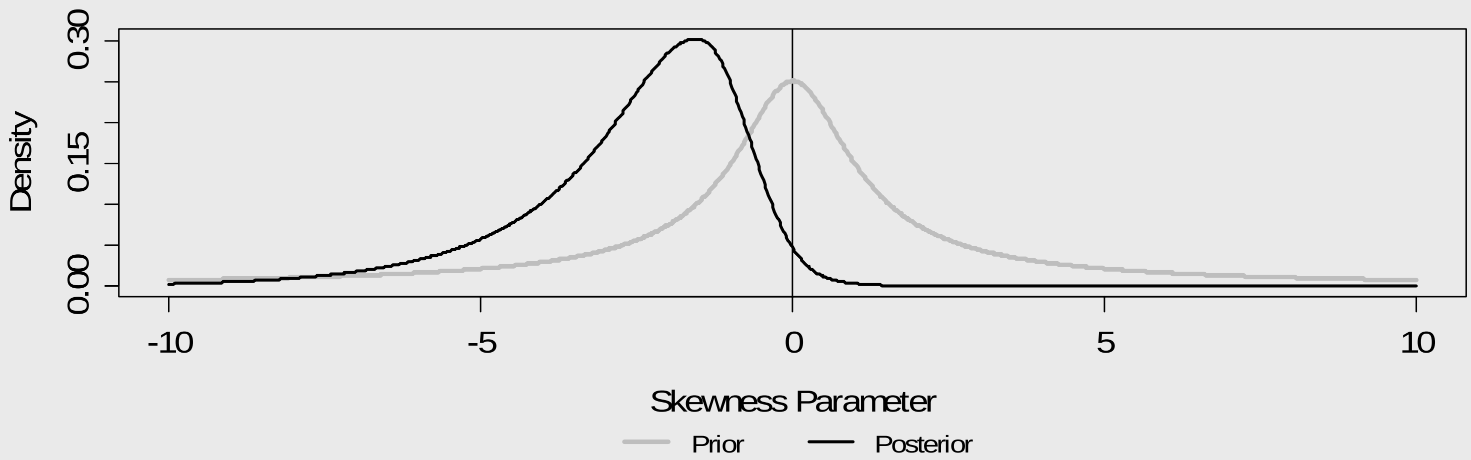
Degrees of Freedom

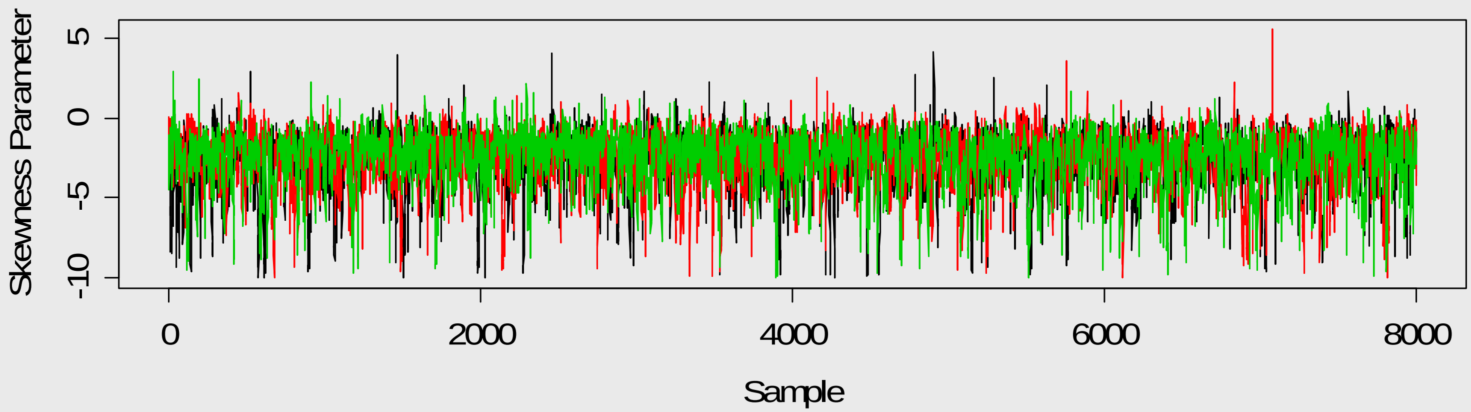


Standard Deviation

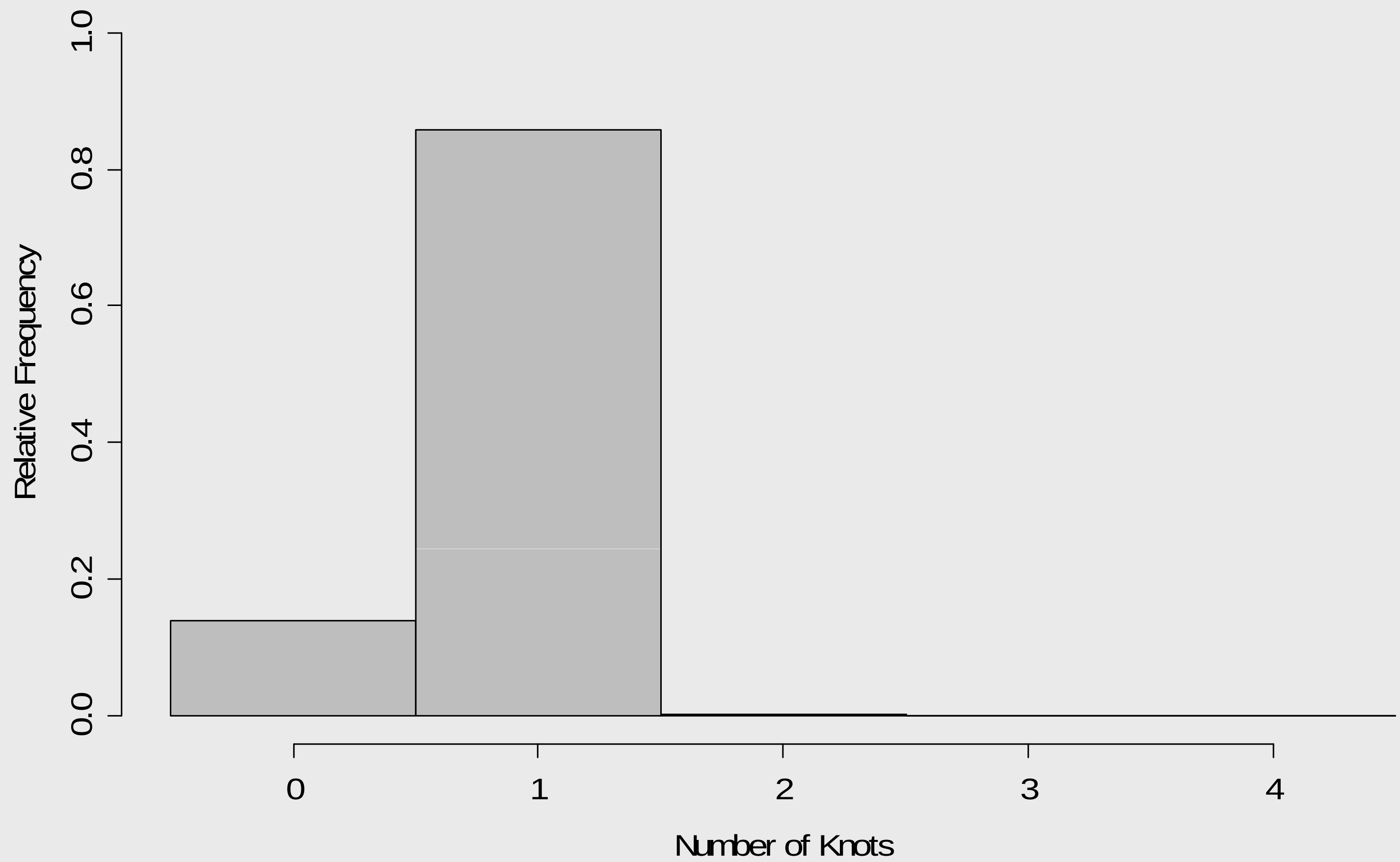


Skewness Parameter

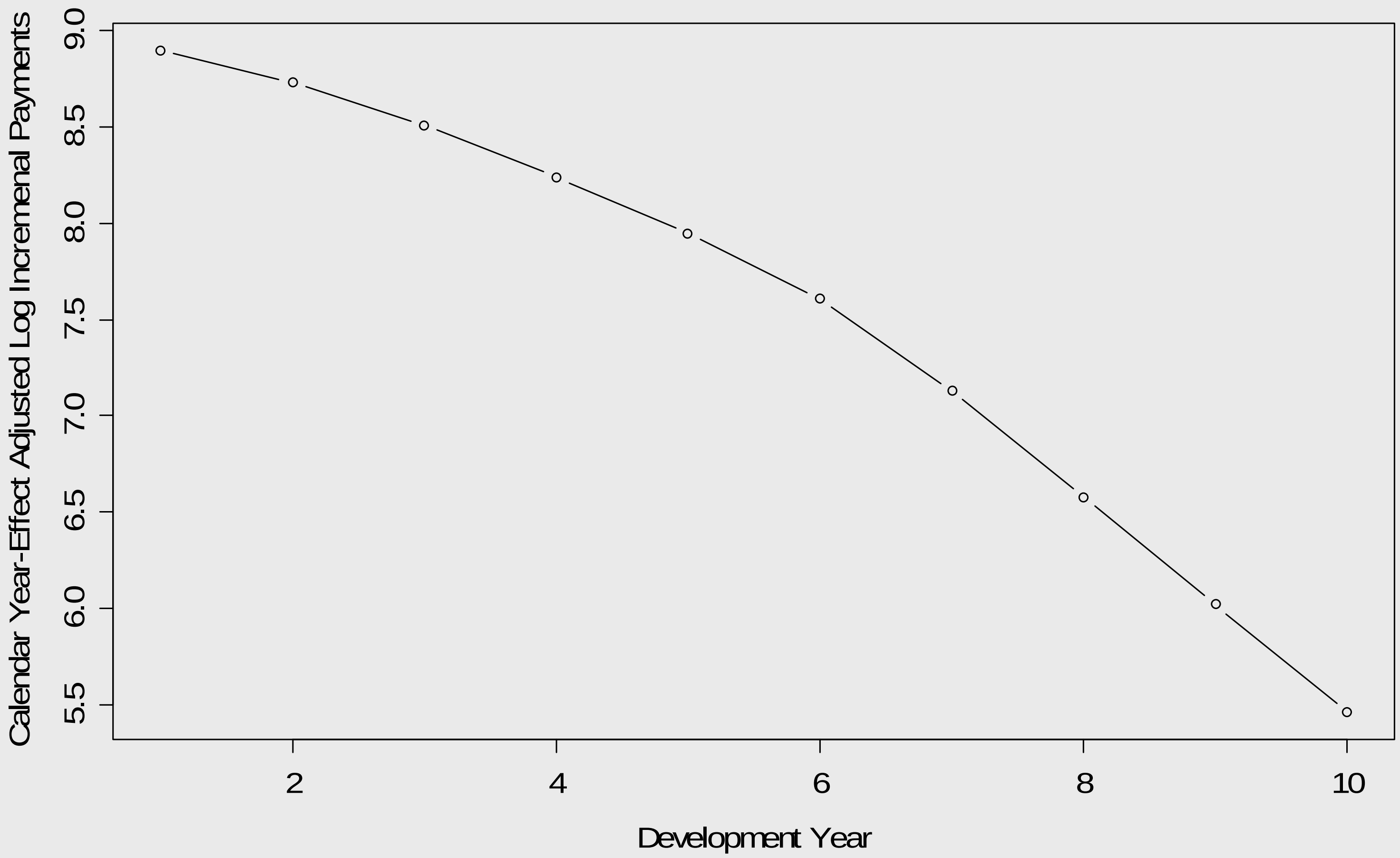




Number of Knots

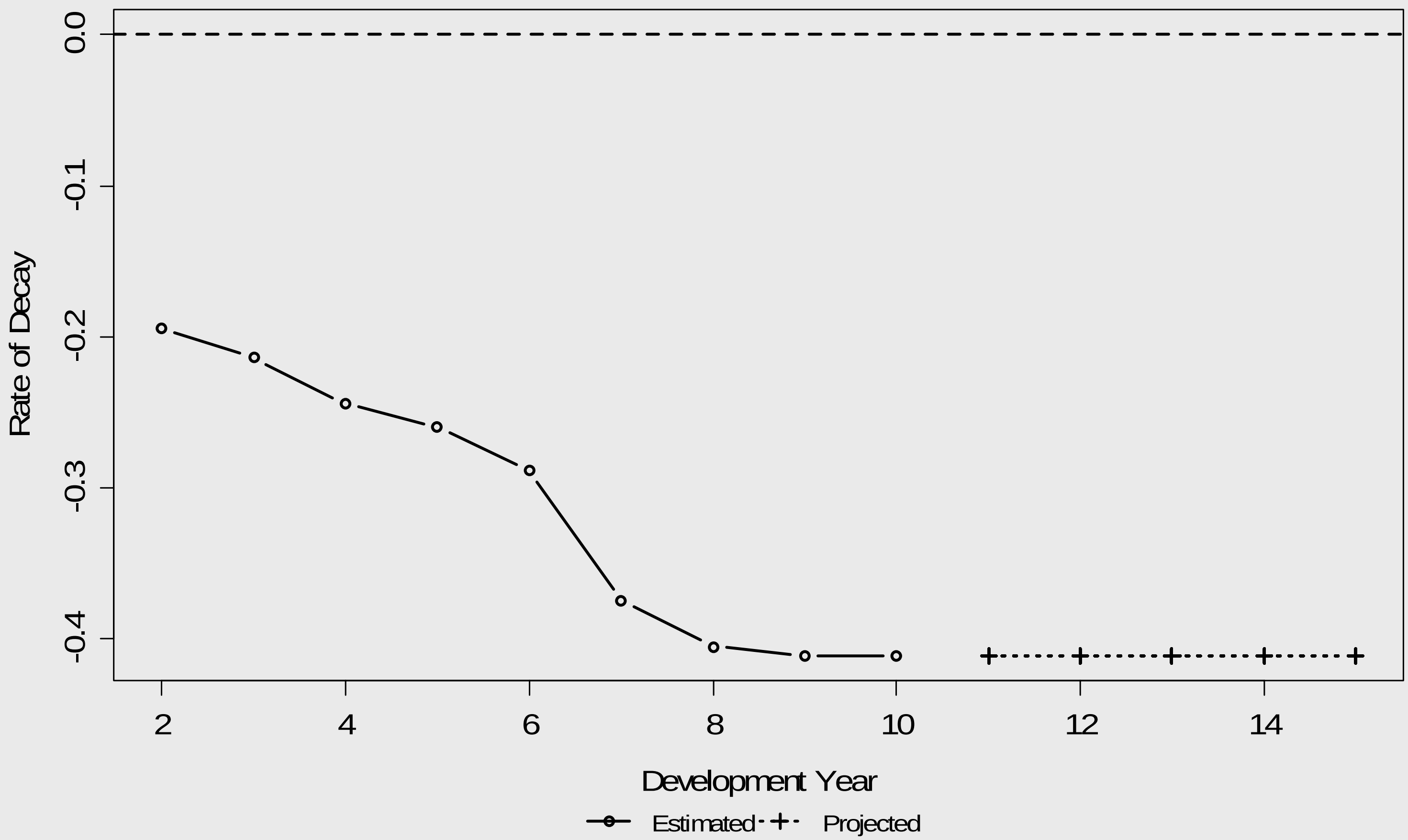


Consumption Path

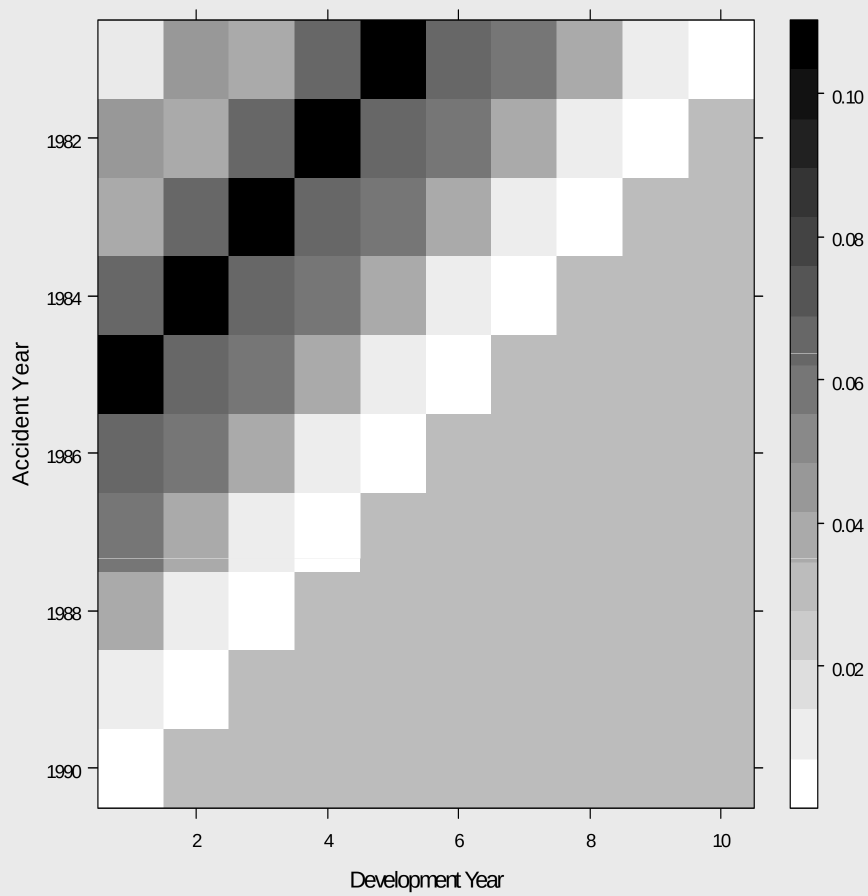


Rate of Decay

(Of Calendar Year Effect Adjusted Incremental Payments)

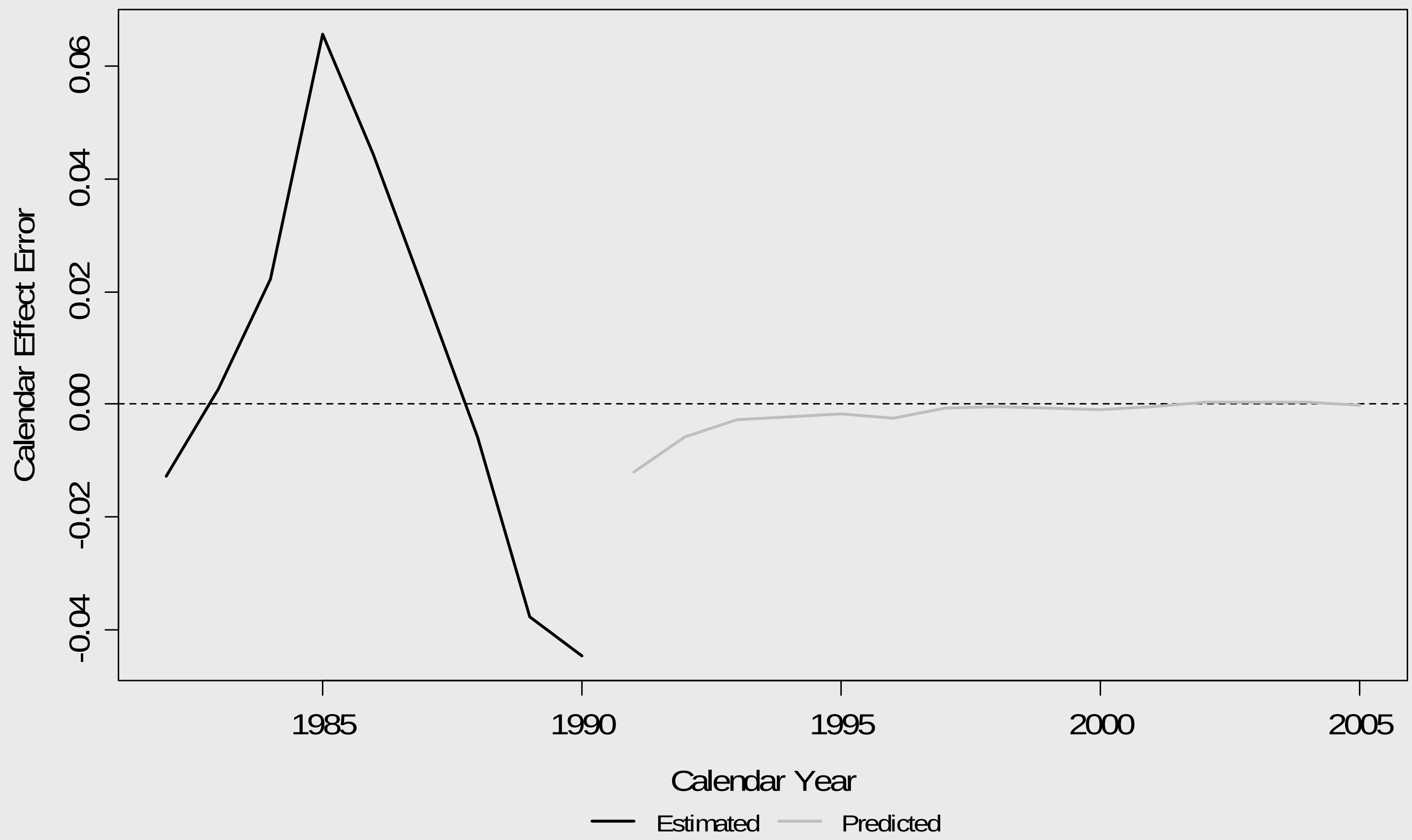


Calendar Year Effect

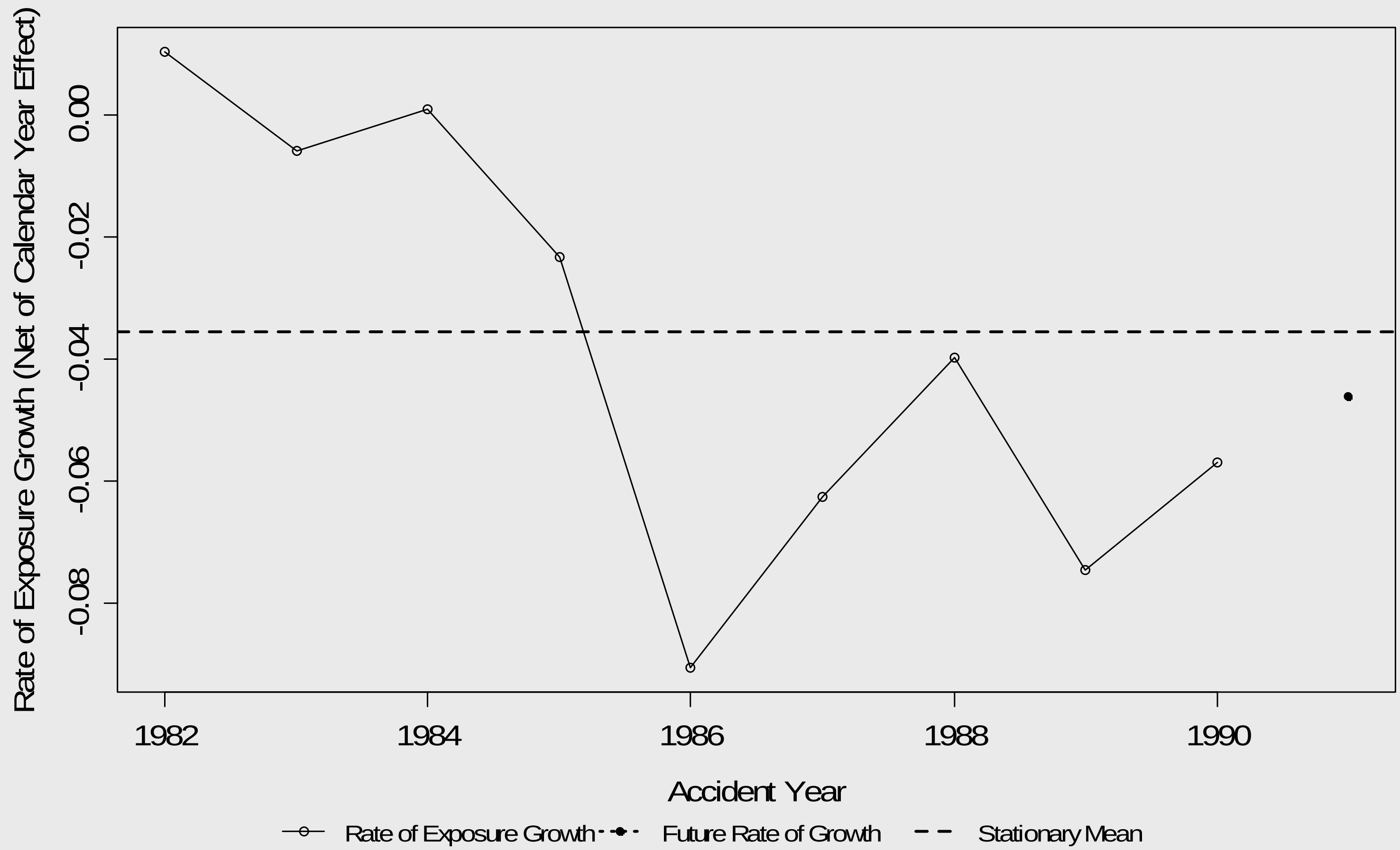


Autoregressive Error in Calendar Year Effect

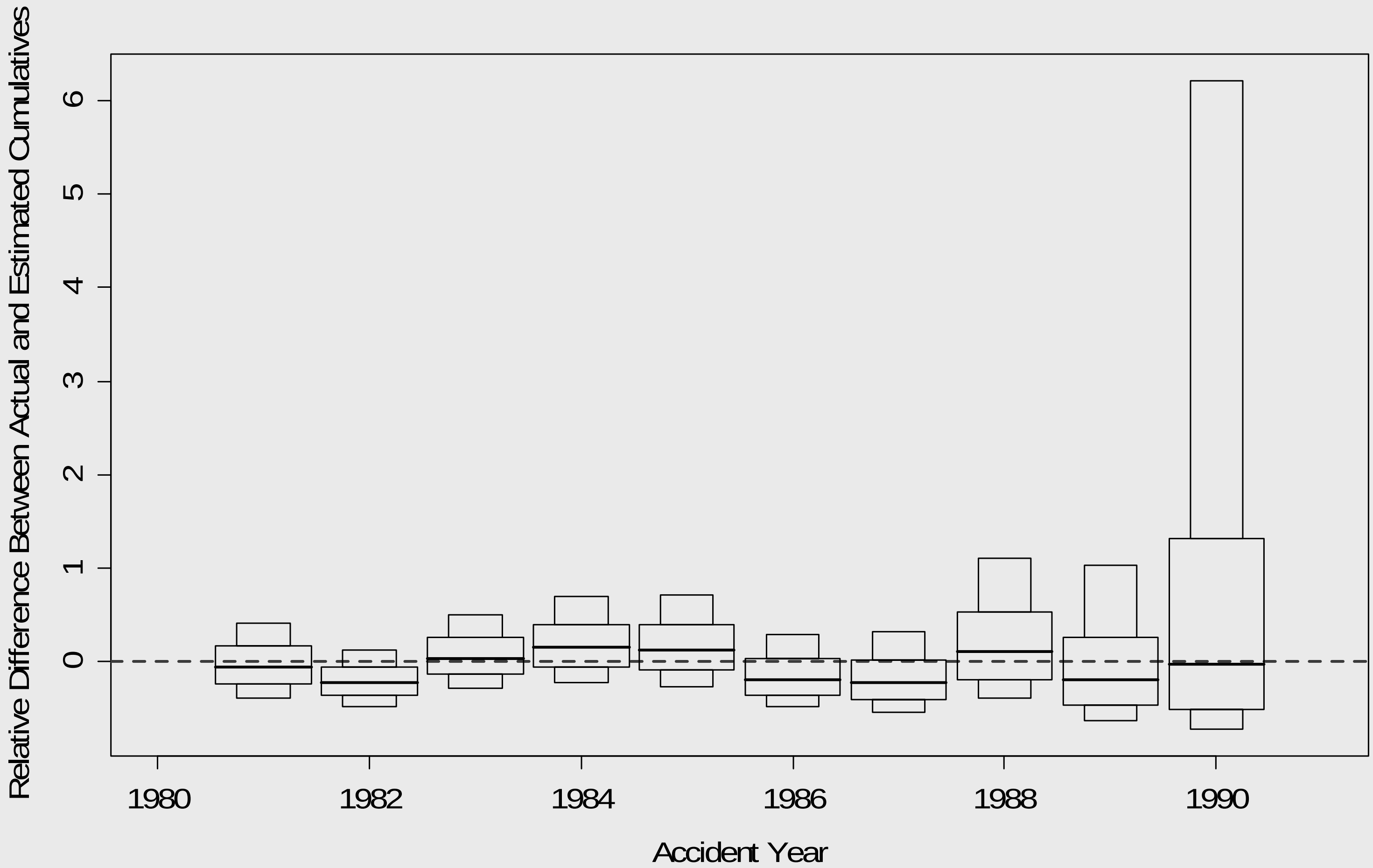
(In addition to the autoregressive process in the expert prior)



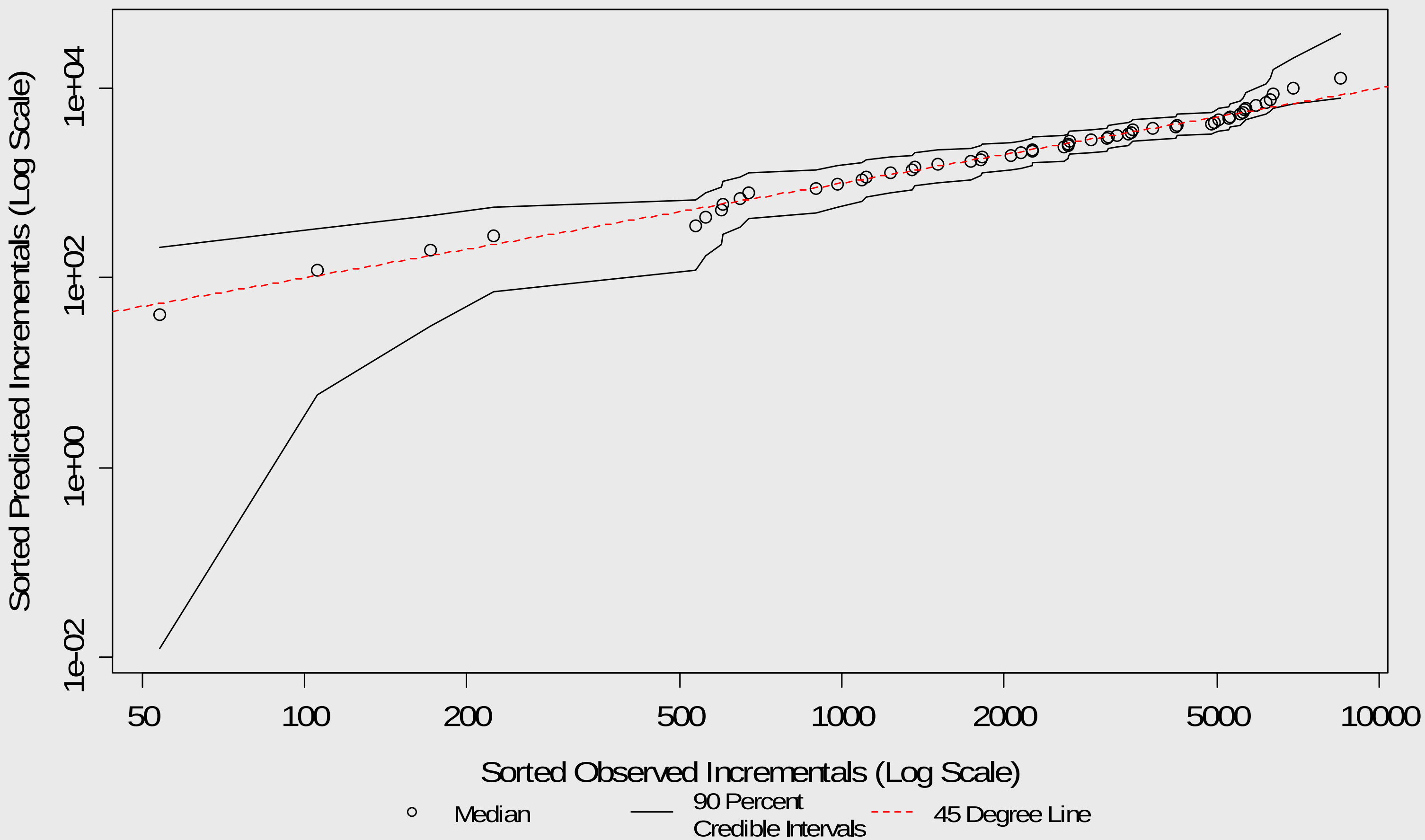
Exposure Growth



Cumulative Diagnostic Chart



Generic QQ-Plot



An Auto Bodily Injury Liability Triangle (1/2)

- Annual Cumulative Paid Loss Triangle of Private Passenger Auto Bodily Injury Liability

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1974	267	1,975	4,587	7,375	10,661	15,232	17,888	18,541	18,937	19,130	19,189	19,209	19,234	19,234	19,246	19,246	19,246	19,246
1975	310	2,809	5,686	9,386	14,884	20,654	22,017	22,529	22,772	22,821	23,042	23,060	23,127	23,127	23,127	23,127	23,159	
1976	370	2,744	7,281	13,287	19,773	23,888	25,174	25,819	26,049	26,180	26,268	26,364	26,371	26,379	26,397	26,397		
1977	577	3,877	9,612	16,962	23,764	26,712	28,393	29,656	29,839	29,944	29,997	29,999	29,999	30,049	30,049			
1978	509	4,518	12,067	21,218	27,194	29,617	30,854	31,240	31,598	31,889	32,002	31,947	31,965	31,986				
1979	630	5,763	16,372	24,105	29,091	32,531	33,878	34,185	34,290	34,420	34,479	34,498	34,524					
1980	1,078	8,066	17,518	26,091	31,807	33,883	34,820	35,482	35,607	35,937	35,957	35,962						
1981	1,646	9,378	18,034	26,652	31,253	33,376	34,287	34,985	35,122	35,161	35,172							
1982	1,754	11,256	20,624	27,857	31,360	33,331	34,061	34,227	34,317	34,378								
1983	1,997	10,628	21,015	29,014	33,788	36,329	37,446	37,571	37,681									
1984	2,164	11,538	21,549	29,167	34,440	36,528	36,950	37,099										
1985	1,922	10,939	21,357	28,488	32,982	35,330	36,059											
1986	1,962	13,053	27,869	38,560	44,461	45,988												
1987	2,329	18,086	38,099	51,953	58,029													
1988	3,343	24,806	52,054	66,203														
1989	3,847	34,171	59,232															
1990	6,090	33,392																
1991	5,451																	

Data source: Hayne, Roger M., "Measurement of Reserve Variability," Casualty Actuarial Society *Forum*, Fall 2003, pp. 141-172, http://www.casact.org/pubs/forum/03fforum/03ff141.pdf

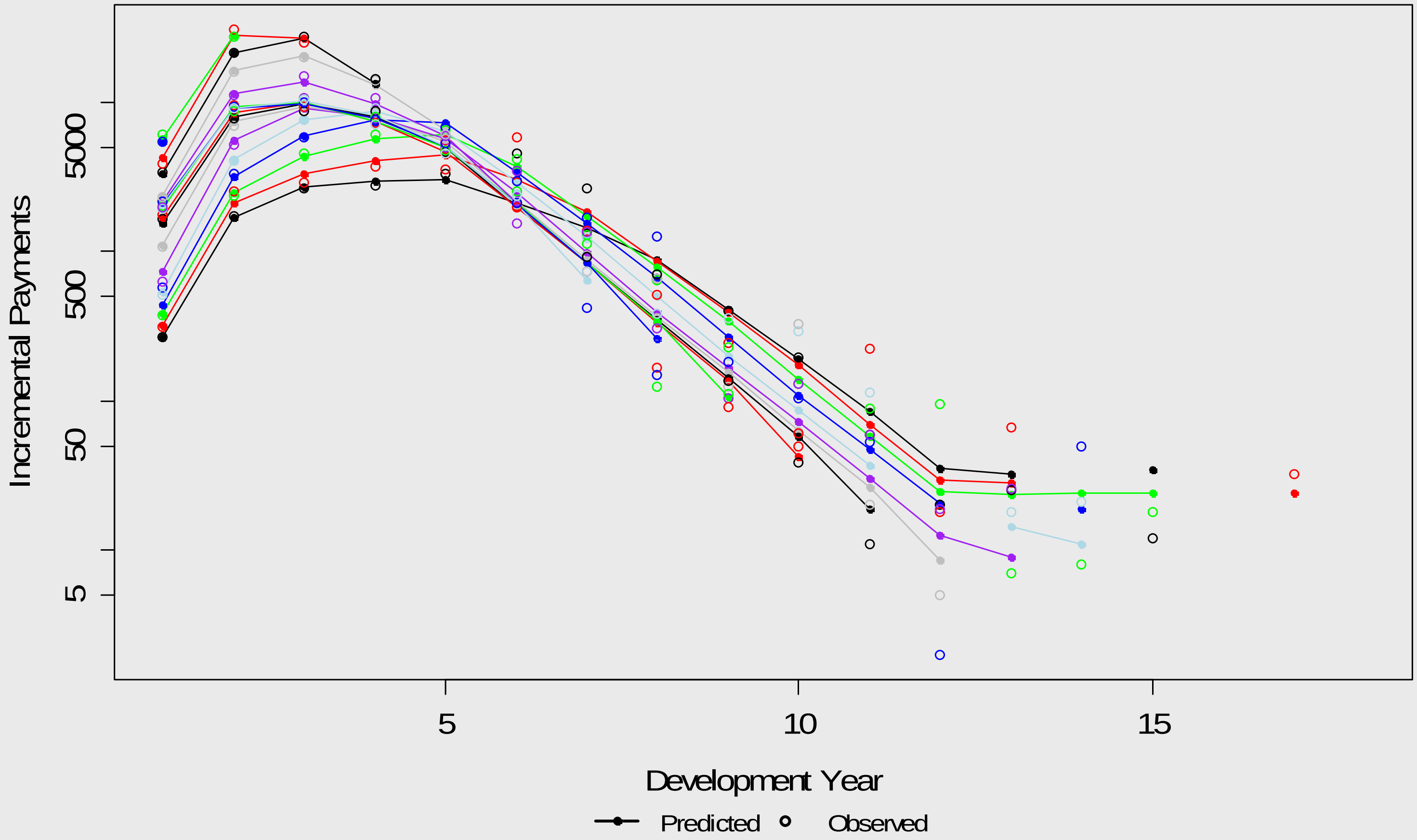
An Auto Bodily Injury Liability Triangle (2/2)

- The triangle exhibits payments at zero amounts in high development years
- Potentially, there is a structural break in the consumption path (along with a calendar year effect because the event affected open claims at any maturity)
  - Roger Hayne provided the following background information to the triangle:

    In late December 1986 there was a judicial decision limiting a judge's power to dismiss a case as a matter of law in certain non-trivial circumstances (thus not requiring a trial of fact). For quite some time prior to this a judge could review a case and determine if injuries were threshold-piercing as a matter of law and not as a matter of fact to be decided by the trier of fact (jury or judge).

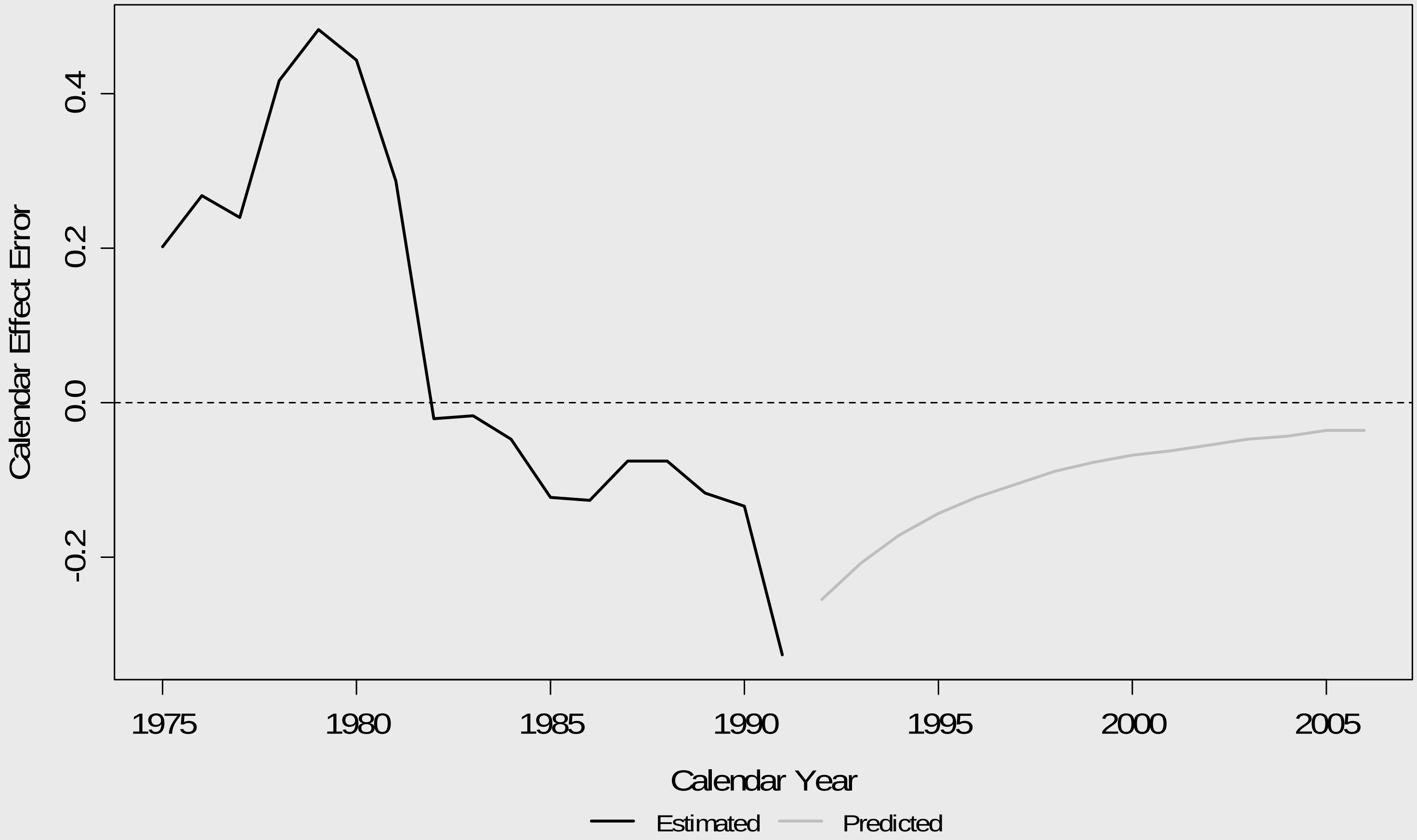
Data source: Hayne, Roger M., "Measurement of Reserve Variability," Casualty Actuarial Society *Forum*, Fall 2003, pp. 141-172, http://www.casact.org/pubs/forum/03fforum/03ff141.pdf

Log Incremental Payments

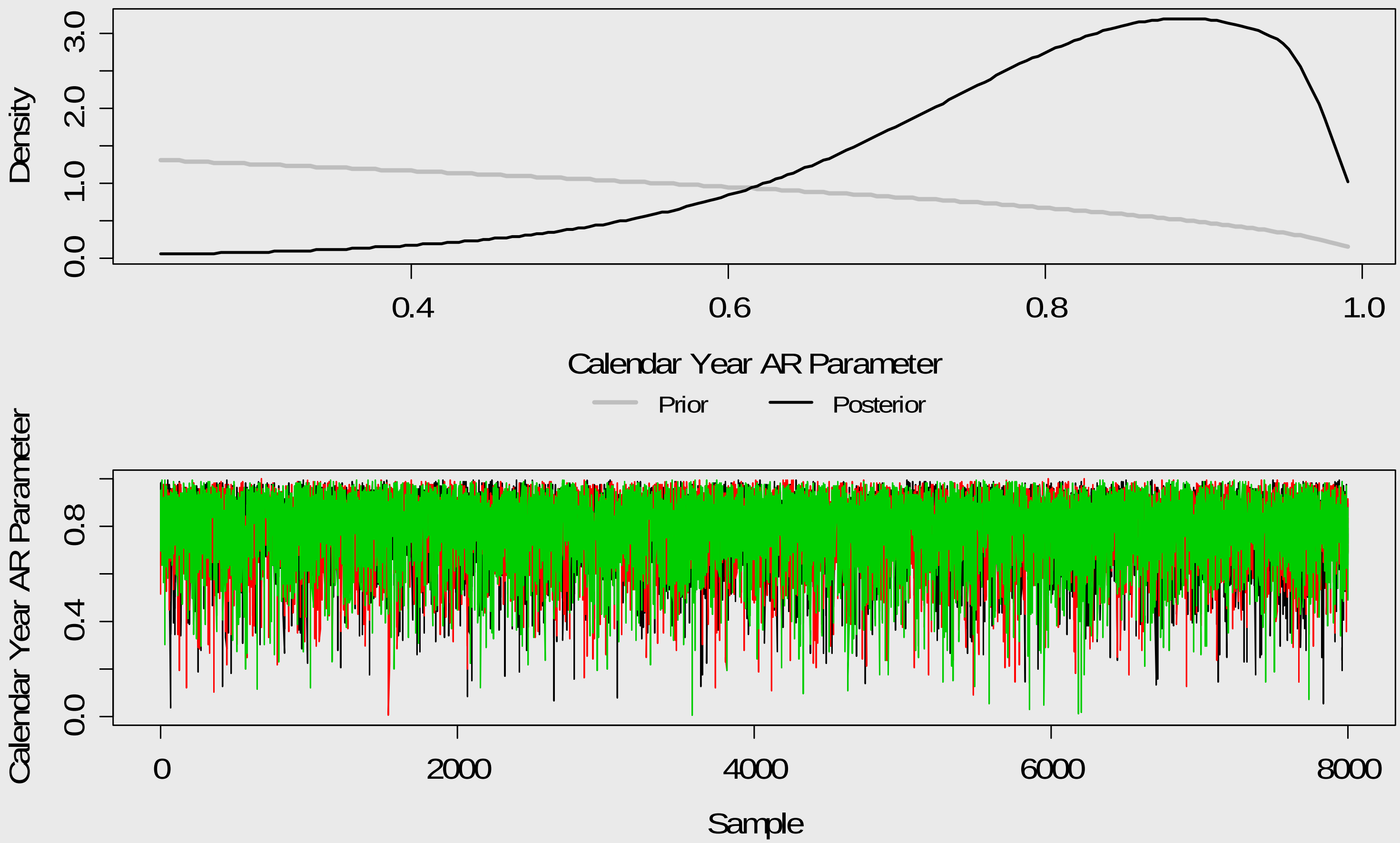


Autoregressive Error in Calendar Year Effect

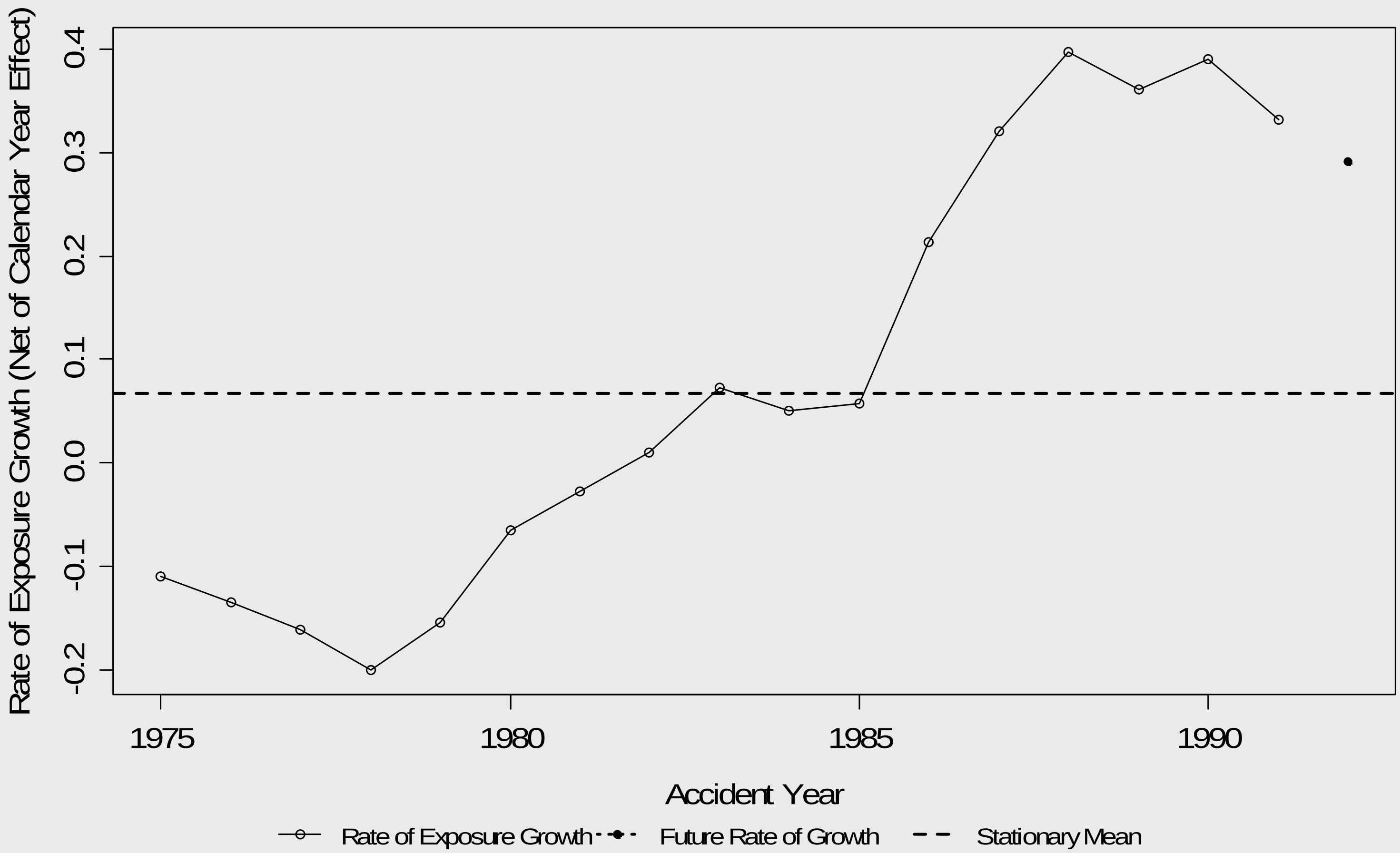
(In addition to the autoregressive process in the expert prior)



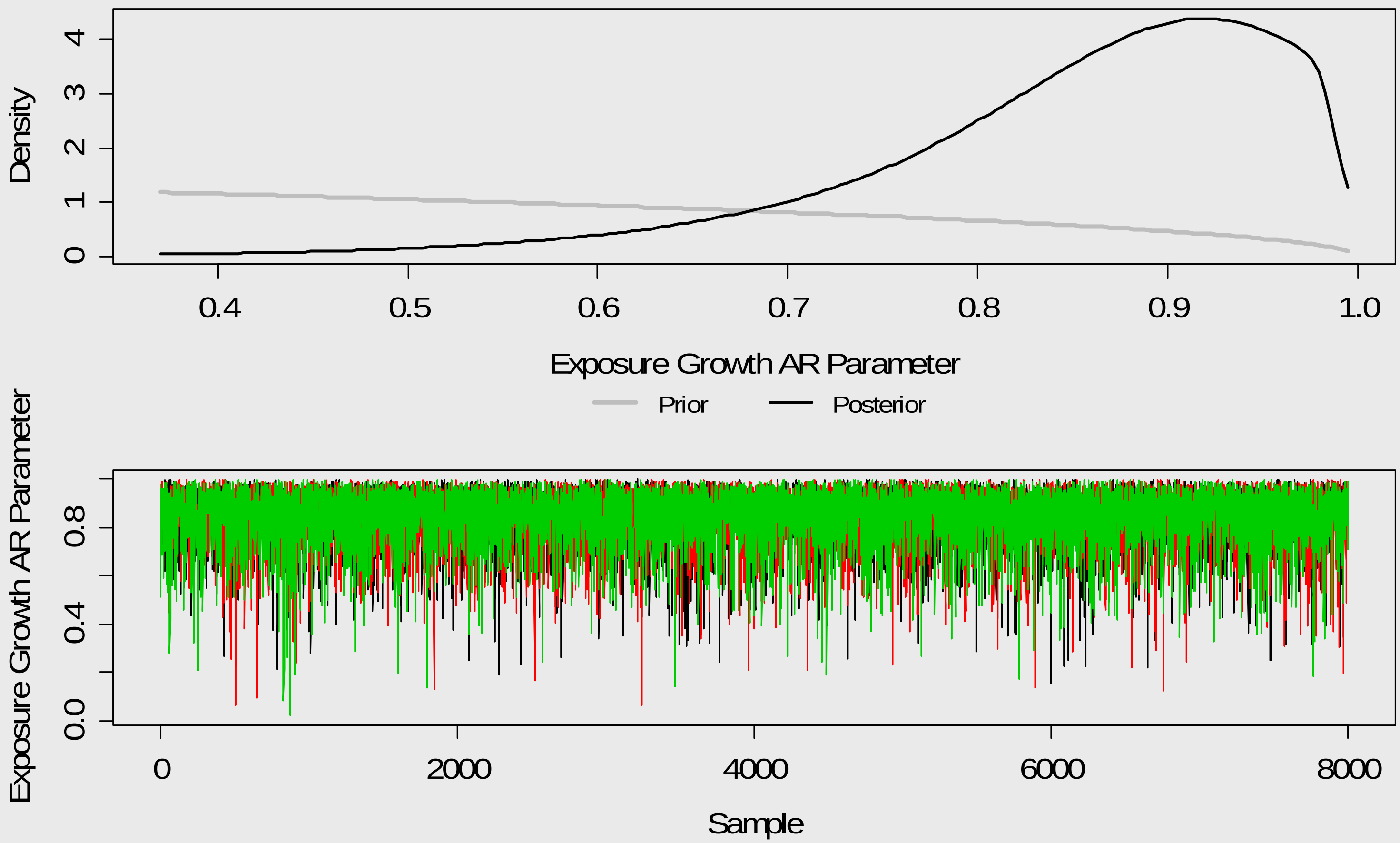
Autoregressive Coefficient (Calendar Year Effect)



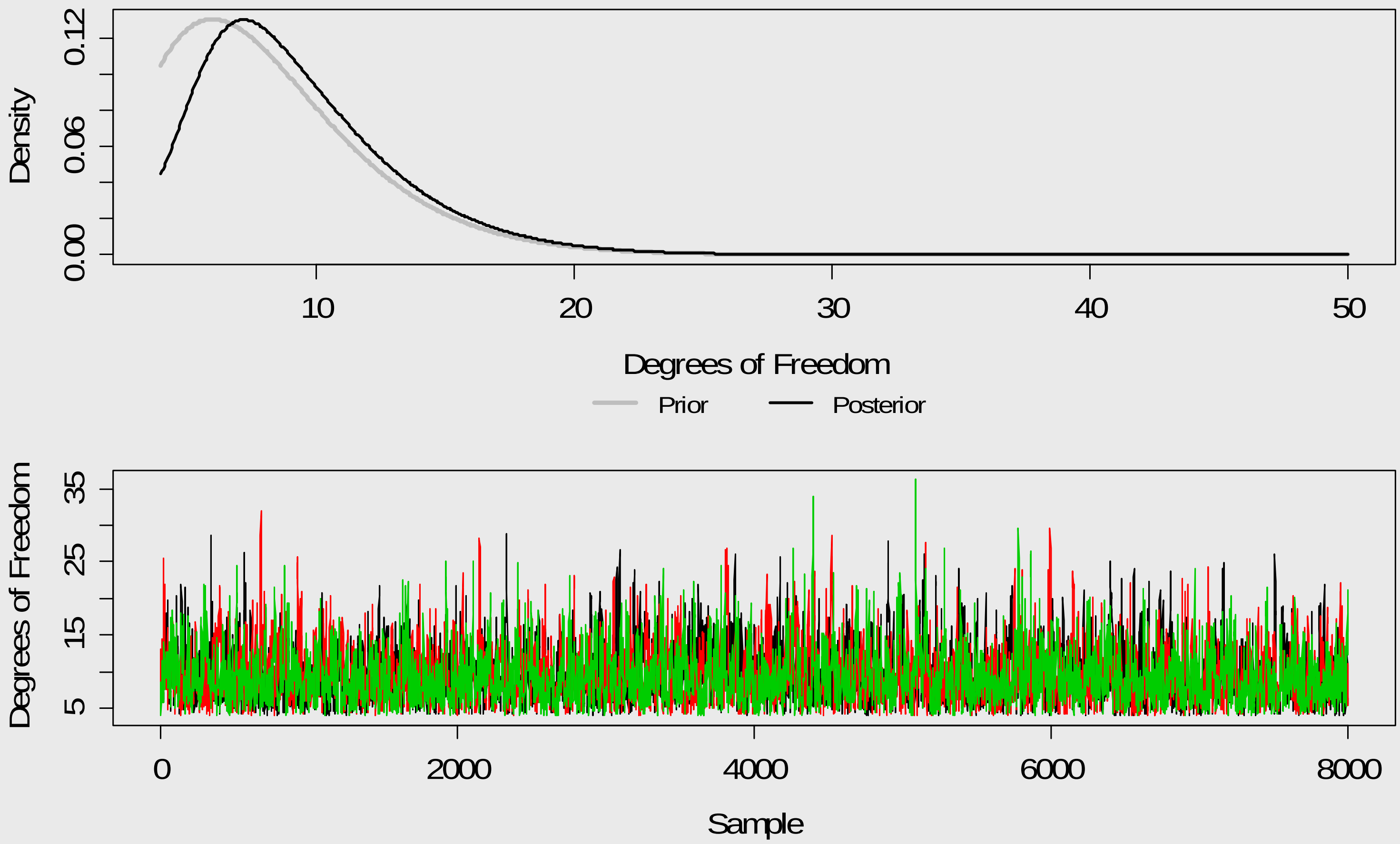
Exposure Growth



Autoregressive Coefficient (Exposure Growth)



Degrees of Freedom



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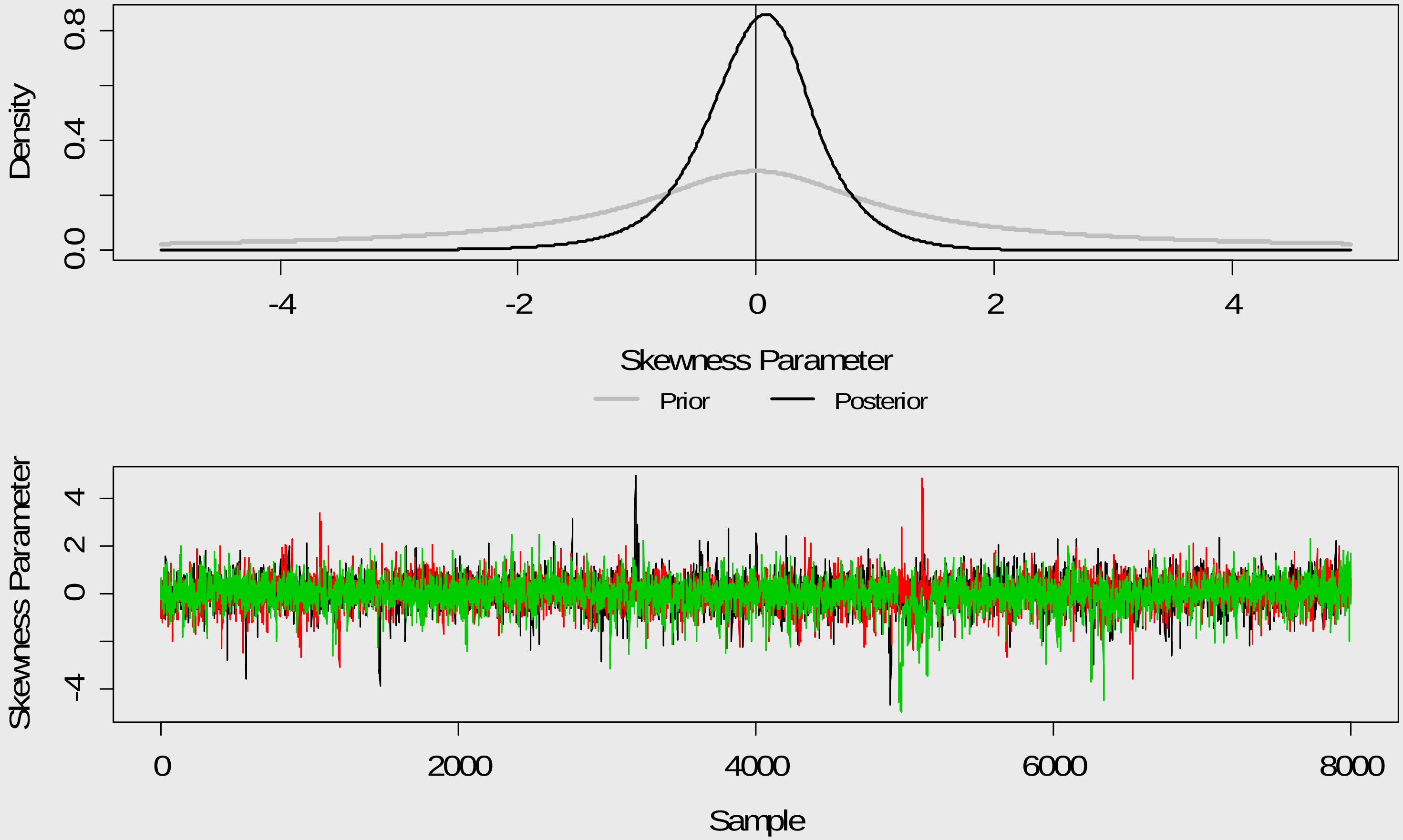
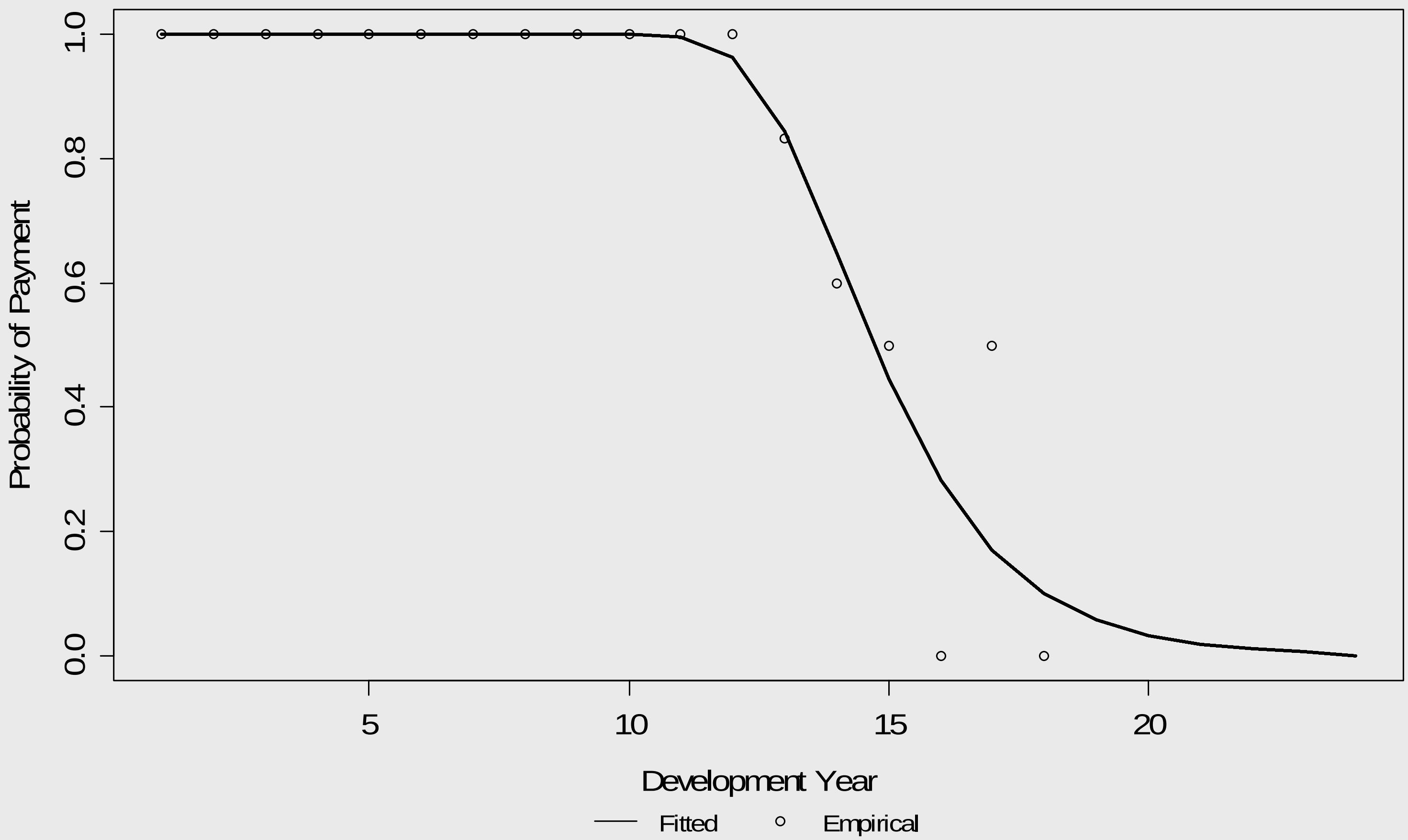
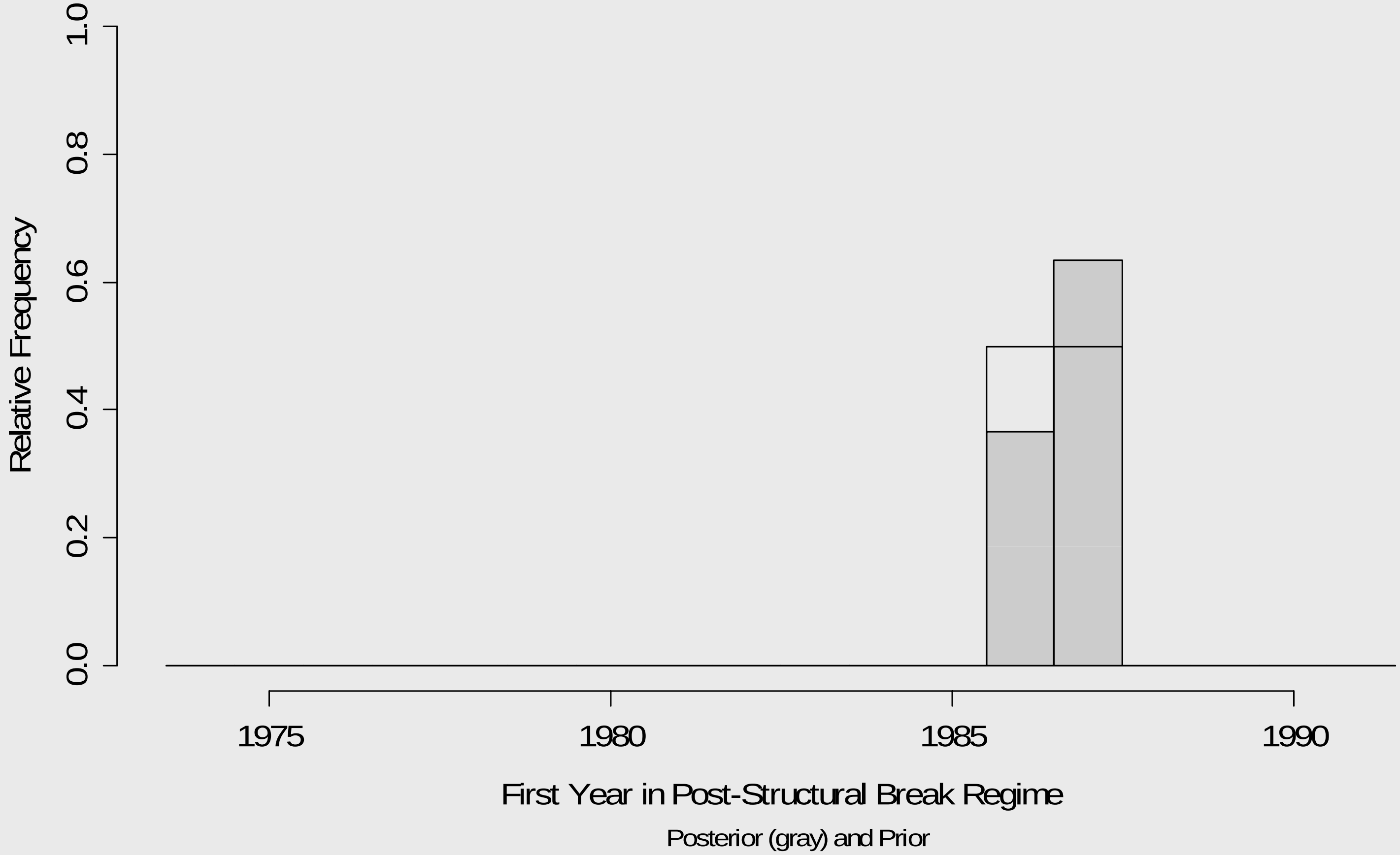


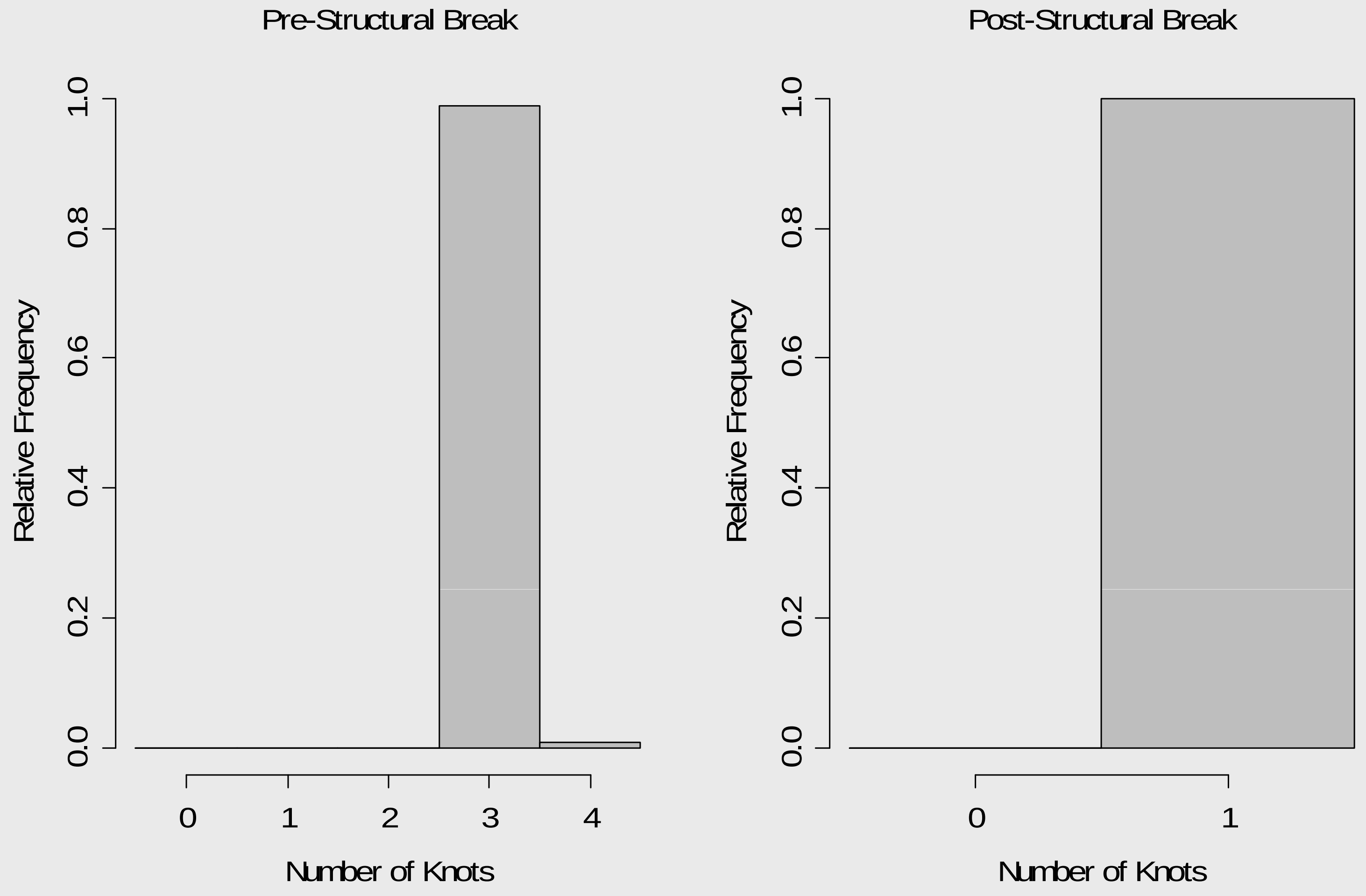
Probability of Not Observing a Payment at the Zero Amount



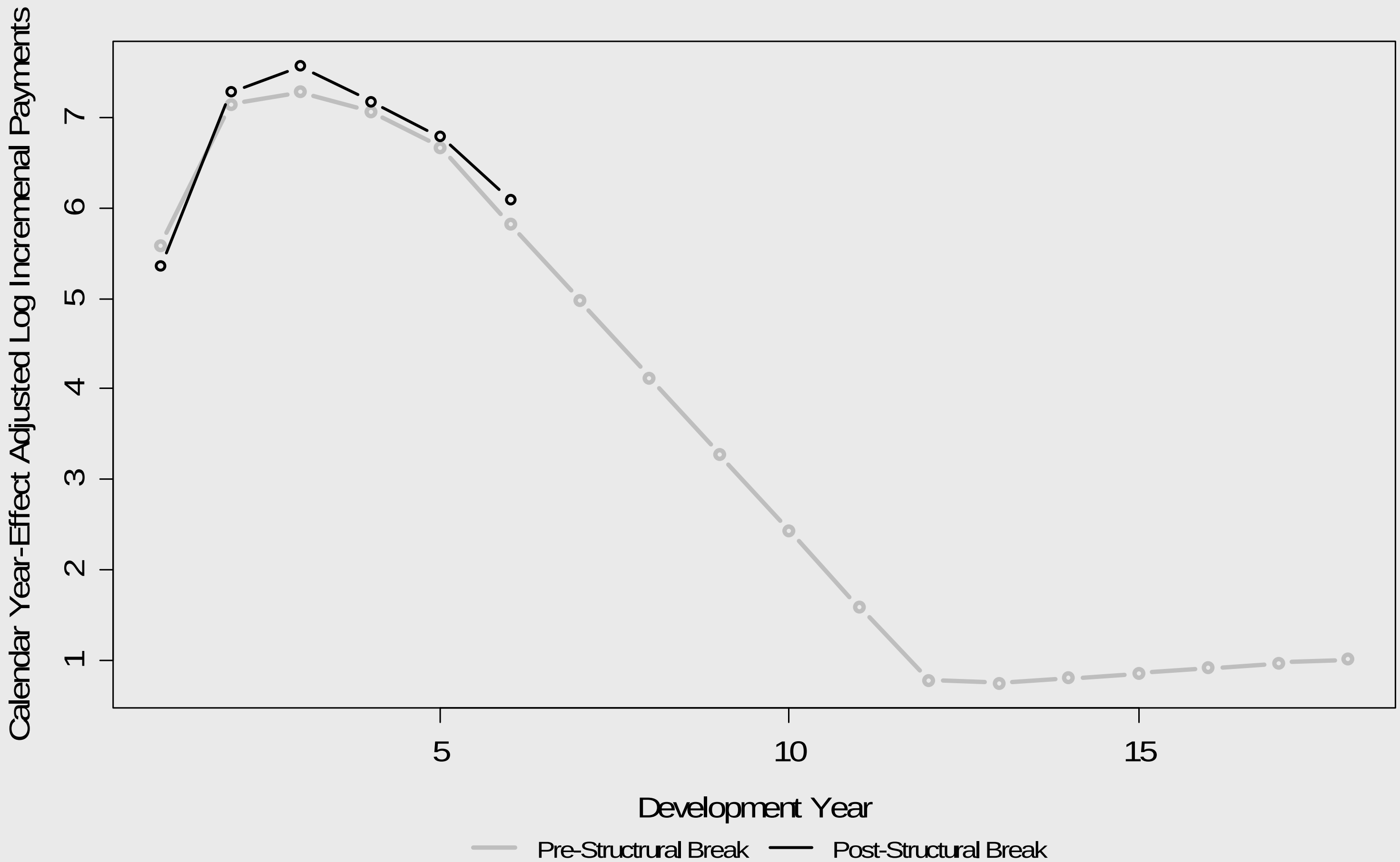
Distribution of Changepoint



Number of Knots

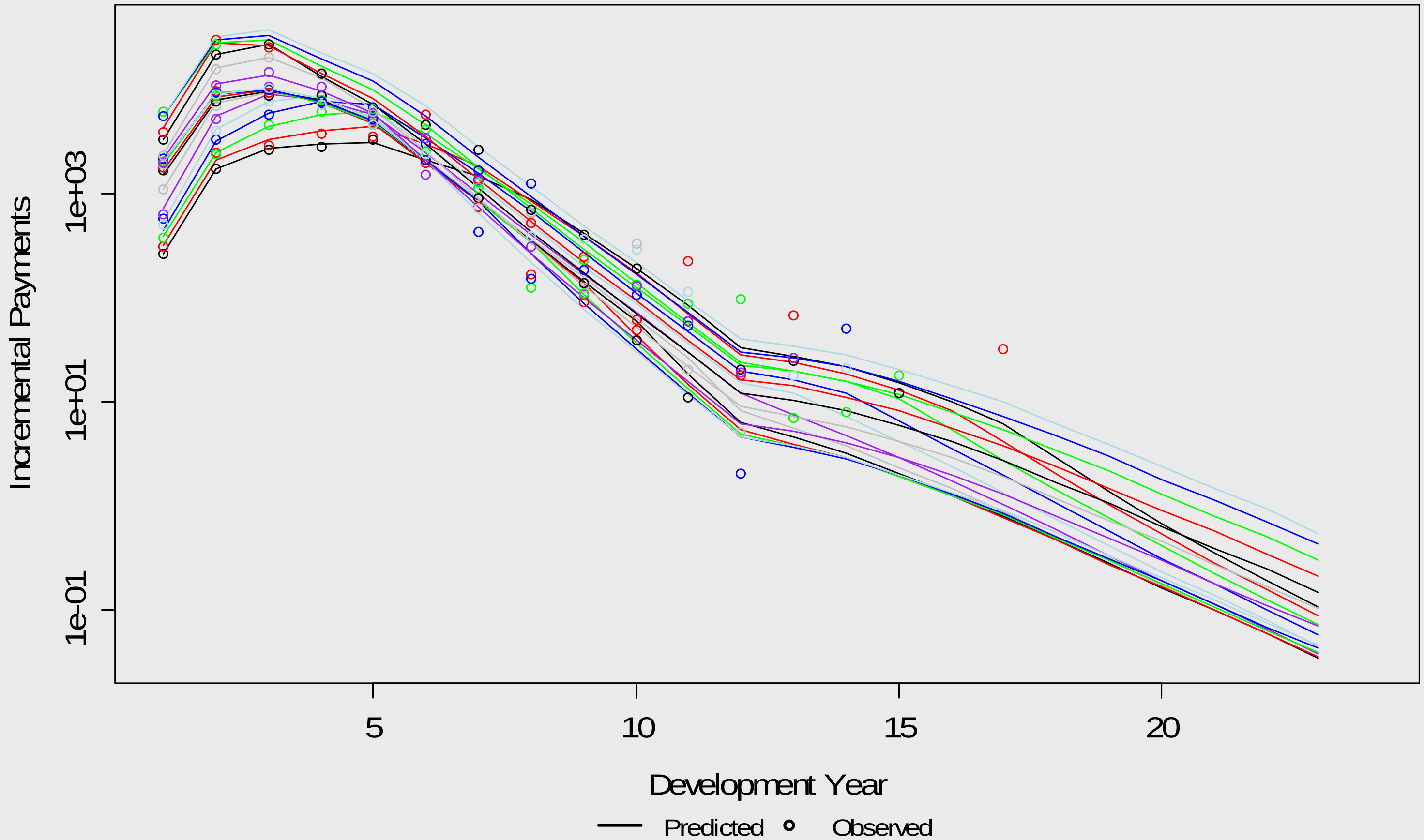


Consumption Path



Log Incremental Payments

Adjusted for “Probability of Payment”



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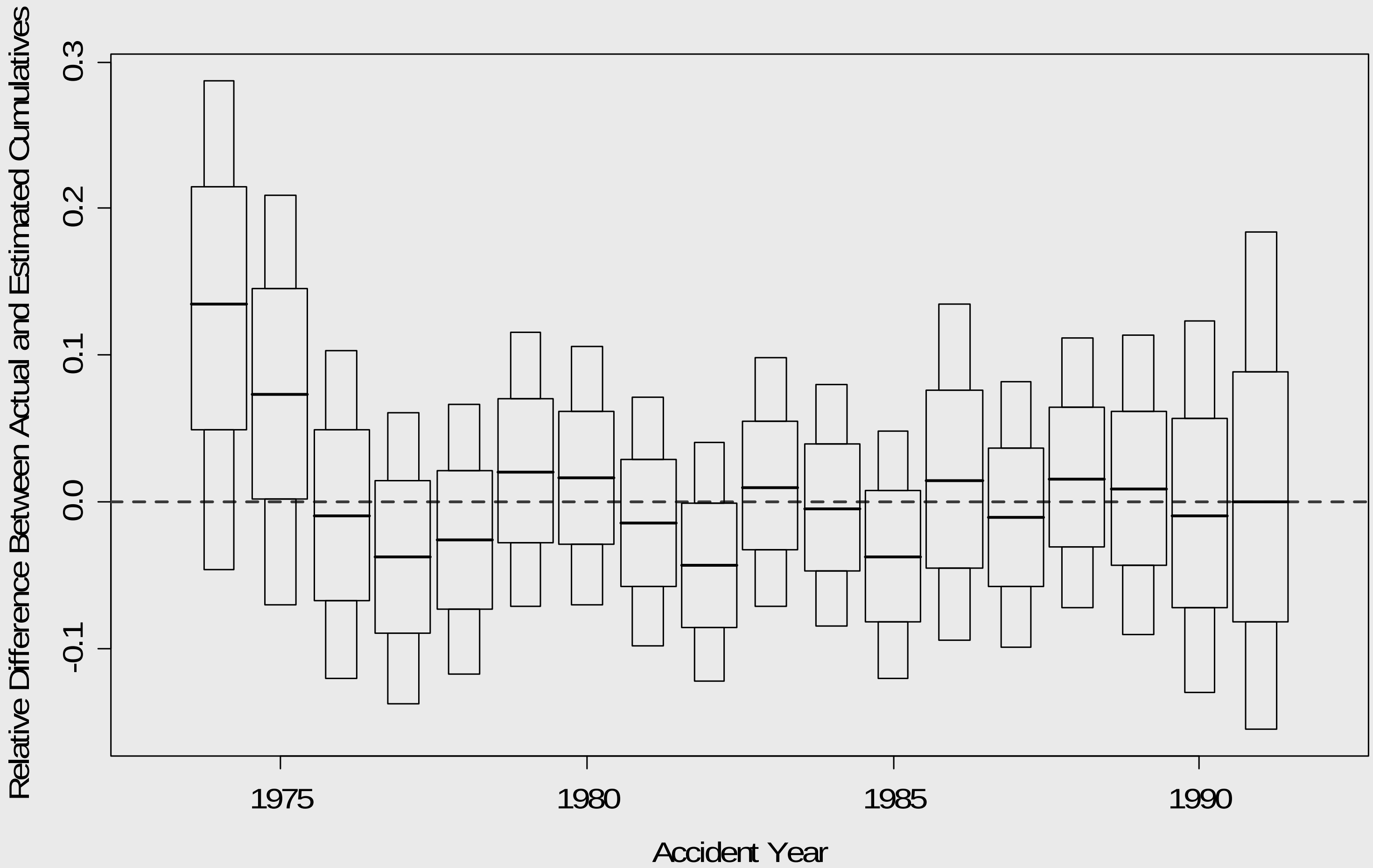

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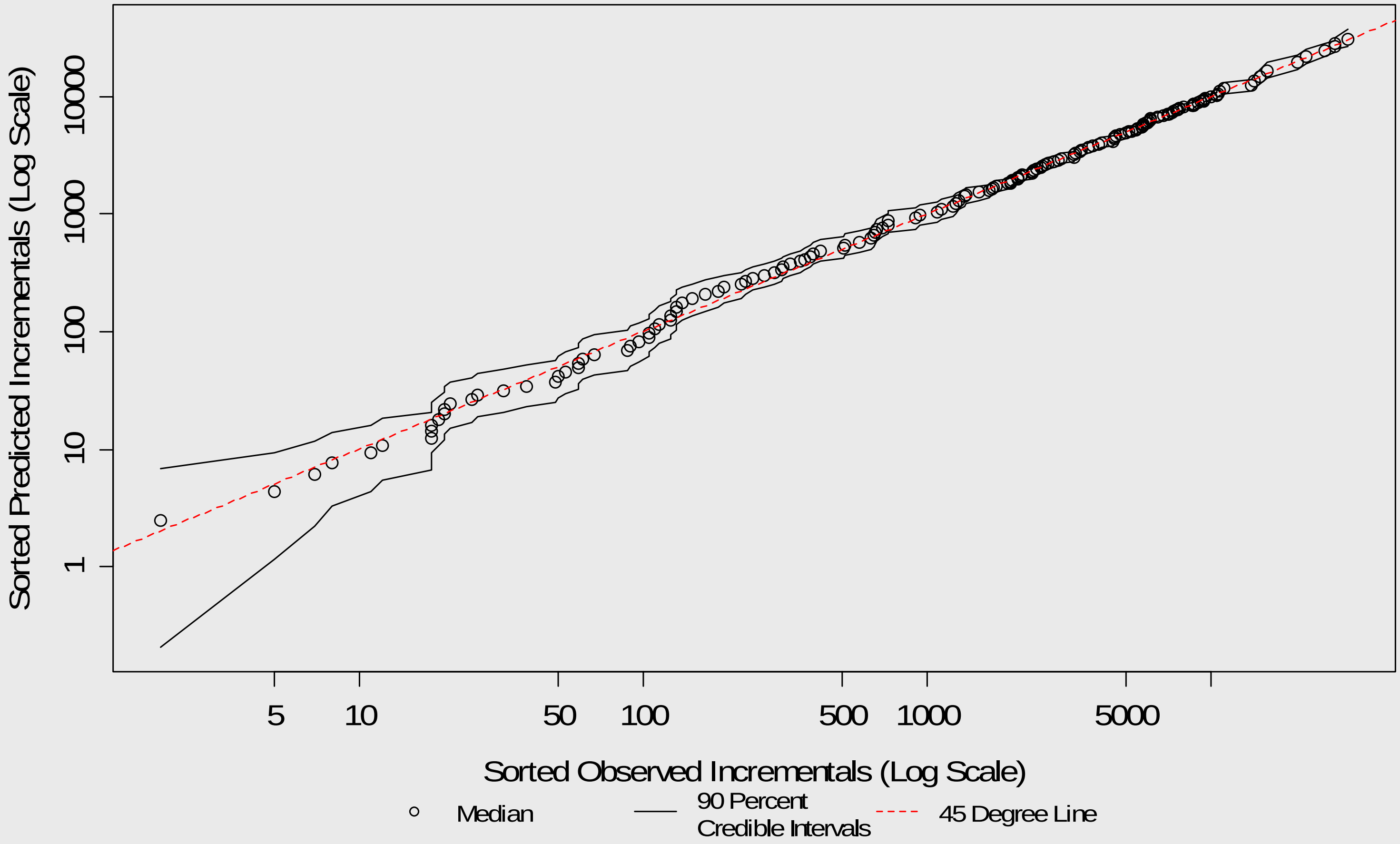
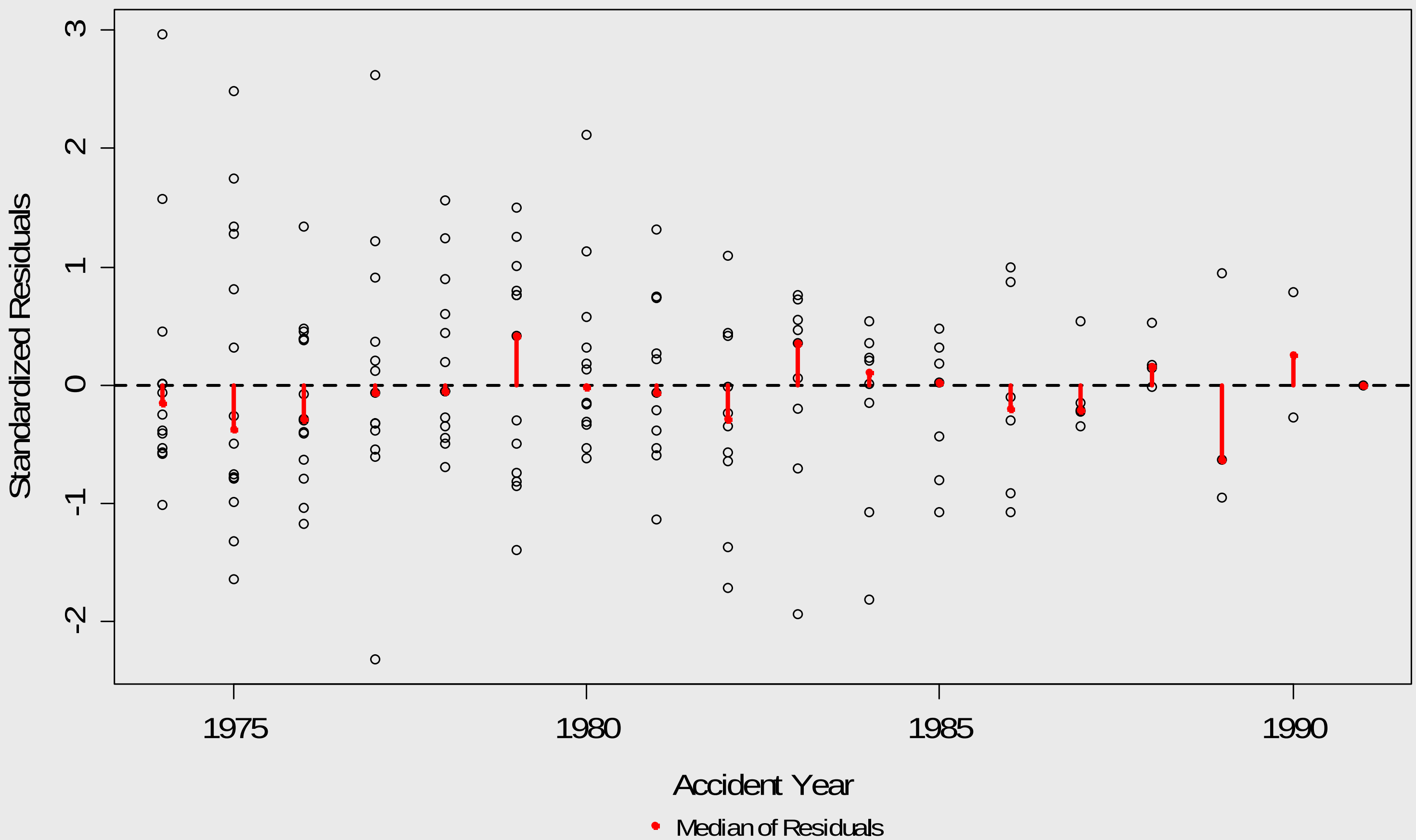


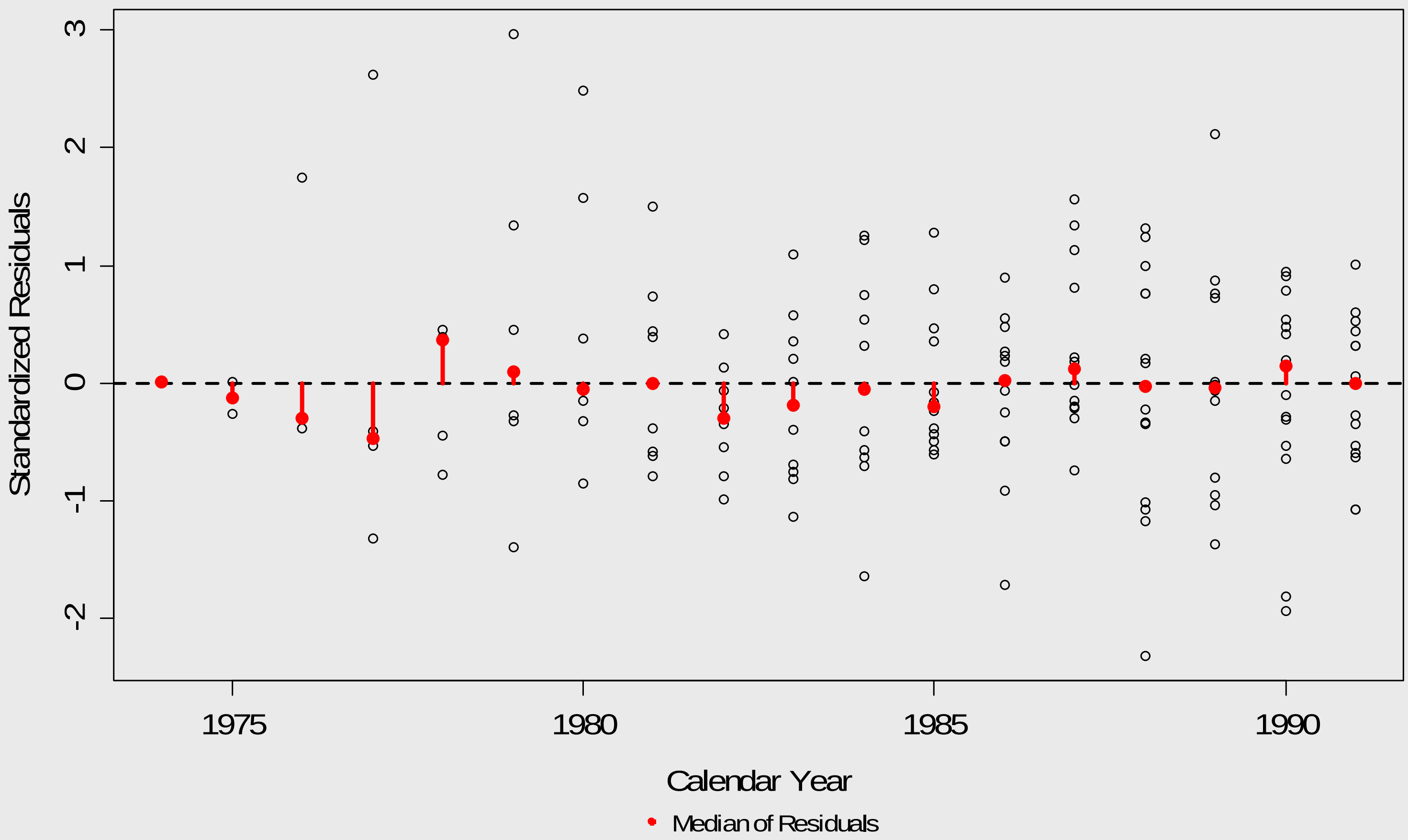
Standardized (Skewed) Residuals

Accident Year



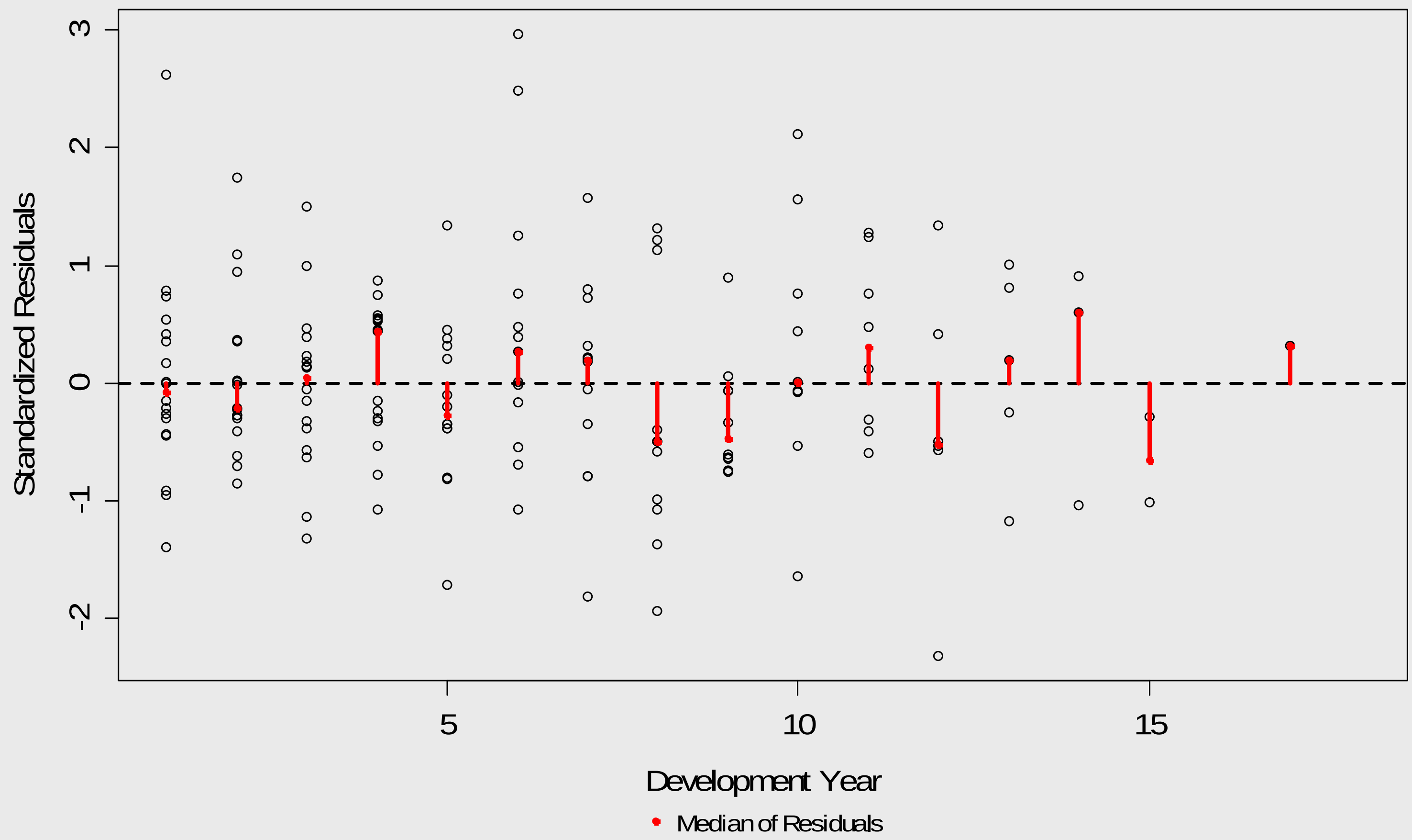
Standardized (Skewed) Residuals

Calendar Year



Standardized (Skewed) Residuals

Development Year



Conclusion

- The presented model of loss development is robust to heavy tails, skewness, and heteroskedasticity
- The model employs RJMCMC to determine the optimal trajectory of the calendar year effect adjusted and exposure adjusted incremental payments
- Further, in its changepoint version, the model is capable of accounting for a structural break, the location of which does not have to be exactly known but is determined by the model within a supplied time interval
- It is straightforward to turn the model into an overdispersed Poisson claim count model

References

[1] Barnett, Glen, and Ben Zehnwirth, "Best Estimates for Reserves," Casualty Actuarial Society *Proceedings*, Vol. 37, No. 167, 2000, pp. 245-321, http://www.casact.org/pubs/proceed/proceed00/00245.pdf.

[2] De Jong, Piet, "Forecasting Runoff Triangles," *North American Actuarial Journal*, Vol. 10, pp. 28-38, http://www.soa.org/library/journals/north-american-actuarial-journal/2006/april/naaj0602_2.pdf.

[3] Kim, Young Il, and J. Huston McCulloch, "The Skew-Student Distribution with Application to U.S. Stock Market Returns and the Equity Premium," Working Paper, Ohio State University, Department of Economics, October 2007, http://web.econ.ohio-state.edu/~ykim/Skew%20t_Equity%20premium.pdf

[4] Kremer, Erhard, "A Class of Autoregressive Models for Predicting the Final Claim Amount," *Insurance Mathematics and Economics*, Vol. 3, 1984, pp. 111-119.

[5] Verrall, Richard, "Modelling Claims Run-Off Triangles with Two-Dimensional Time Series," *Scandinavian Actuarial Journal*, 1989, pp. 129-138.

[6] Verrall, Richard, "A State Space Representation of the Chain Ladder Linear Model," *Journal of the Institute of Actuaries*, Vol. 116, 1989, pp. 589–610.

[7] Zehnwirth, Ben, "Probabilistic Development Factor Models with Applications to Loss Reserve Variability, Prediction Intervals and Risk Based Capital," Casualty Actuarial Society *Forum*, Vol. 2, Spring 1994, pp. 447-605, http://www.casact.org/pubs/forum/94spforum/94spf447.pdf.

Technical Appendix:

The Skewed Student-*t* Likelihood (1/3)

$$y_{i,j} \sim \mathrm{N}\left(\mu_{i,j} + \frac{\alpha}{\tau_{i,j}}, \tau_{i,j}\right)$$

$$\alpha \sim \text{Student}(0, 0.5, 2.1)\mathrm{I}(-\iota, \iota),\ \ \iota > 0$$

- The Student-*t* distribution is implemented as a mean-variance mixture of normal distributions
- The mean of the conditionally Gaussian random variable moves in proportion to its stochastic latent variance $1/\tau_{i,j}$
- For a skewness parameter α of zero, the skewed Student-*t* collapses into a symmetric (conventional) Student-*t*
- The prior for the skewness parameter is a truncated central Student-*t* distribution with low degrees of freedom (2.1)

Indexes: *i* represents rows (exposure years); *j* represents columns (development years)
Note: The second parameter of the normal distribution indicates the precision, which is defined as the reciprocal value of the variance.

Technical Appendix:
The Skewed Student-*t* Likelihood (2/3)

$$\tau_{i,j} = \frac{\omega_{i,j}}{\nu \cdot \sigma_j^2}$$

$$\omega_{i,j} \sim \chi^2_{(\nu)}$$

$$\nu \sim \chi^2_{(8)} \mathrm{I}(4,50)$$

- The mixing distribution for the latent variance is an inverse Chi-squared distribution
- The degrees of freedom are draw from a truncated Chi-squared distribution—the truncation at 4 (2) ensures a finite variance (when the skewness parameter is set to zero)
- The model is heteroskedastic by allowing the scale parameter σ_j to vary on the development time axis

Indexes: *i* represents rows (exposure years); *j* represents columns (development years)

Technical Appendix:

The Skewed Student-*t* Likelihood (3/3)

$$\log\left(\sigma_j{}^2\right) \sim \mathrm{N}\left(2\cdot\log\left(\sigma_{j-1}{}^2\right) - \log\left(\sigma_{j-2}{}^2\right), \upsilon\right), \; j \geq 3$$

$$\upsilon \sim \mathrm{Ga}\left(10, 0.5\right)$$

$$\sigma_j \sim \mathrm{U}\left(0, 10\right), \; j = 1, 2$$

- The log of the squared scale parameter is smoothed using a second-order random walk
- The innovation variance of the random walk is an inverse gamma distribution
- The scale parameters of the first two development years have highly uninformed uniform priors

Index *j* represents columns (development years)

Technical Appendix:

The Log Incremental Payments

$$\mu_{i,j} = S_j + \sum_{k=2}^{i+j} \kappa_k + \sum_{k=1}^{i} \eta_k$$

$$\delta_j = S_j - S_{j-1} \; , \; j \geq 2$$

$$\mu_{i,j} = S_1 + \sum_{k=2}^{i+j} \kappa_k + \sum_{k=1}^{i} \eta_k + \sum_{k=1}^{j} \delta_k$$

- The log incremental payments (net of the shift due to skewness) is the sum of (1) the calendar-year effect adjusted log incremental payments in the first exposure year $S_1,\ldots,S_K$, (2) the cumulative calendar year effect, and (3) the cumulative exposure growth
- The rate of decay (on the logarithmic scale) is backed out of the log consumption path $S_1,\ldots,S_K$
- The third equation re-states the log incremental payments in (log) growth rates

Indexes: *i* represents rows (exposure years); *j* represents columns (development years)
K is the dimension of the triangle (number of rows, equal to number of columns)

Technical Appendix:
The Calendar Year Effect (1/2)

$$\kappa_{i+j} = \tilde{\kappa}_{i+j} + \mu_{\kappa,i+j}, \quad i+j \geq 3$$

$$\tilde{\kappa}_{i+j} \sim \mathrm{N}\left(0,\left(1-\rho_{\kappa}^{2}\right)\cdot\tau_{\kappa}\right), \quad i+j=3$$

$$\tilde{\kappa}_{i+j} \sim \mathrm{N}\left(\rho_{\kappa}\cdot\tilde{\kappa}_{i+j-1},\tau_{\kappa}\right), \quad i+j \geq 4$$

$$\tau_{\kappa} = 1/\sigma_{\kappa}^{2}$$

$$\sigma_{\kappa} \sim \mathrm{U}(0,10)$$

$$\rho_{\kappa} \sim \mathrm{Beta}(1,1.5)$$

- The calendar year effect is normally distributed around an expert prior
- $\mu_{\kappa,i+j}$ may be a weighted average of a constant nonzero rate of inflation, a zero rate of inflation, and a stochastic rate of inflation, as the stipulated cost of living adjustment may vary by type of indemnity claim
- The calendar year effect error follows an autoregressive process
- The beta prior for the autoregressive parameter is right-skewed, thus discouraging values close to the unit root

Indexes: Diagonals (which represent calendar years) are numbered *i+j,* where *i* represents rows (exposure years) and *j* represents columns (development years);
Note: The second parameter of the normal distribution indicates the precision, which is defined as the reciprocal value of the variance.

Technical Appendix:
The Calendar Year Effect (2/2)

$$\mu_{\kappa,i+j+1} = \mathrm{N}\left(a \cdot \mu_{\kappa,i+j} + b, \tau_{\varepsilon}\right), \quad 0 < a < 1$$

$$a \sim \mathrm{Beta}(1,1)$$

$$b \sim \mathrm{N}\left(0, 10^{-6}\right)$$

$$\tau_{\varepsilon} \sim \mathrm{Ga}(0.001, 0.001)$$

- When a stochastic rate of inflation is used as an expert prior (such as the CPI or M-CPI), then future values of this rate of inflation are simulated using a discrete Ornstein-Uhlenbeck (O-U) process
- The simulation of future rates of inflation using the O-U process implies a return of the inflation rate to its historic mean
- The discrete O-U process can be calibrated by estimating an autoregressive process

Index *m* represents calendar years
Note: The second parameter of the normal distribution indicates the precision, which is defined as the reciprocal value of the variance.

Technical Appendix:
Exposure Growth

$$\eta_i = \tilde{\eta}_i + \mu_\eta, \quad i \geq 2$$

$$\tilde{\eta}_i \sim \mathrm{N}\left(0, \left(1 - \rho_\eta^2\right) \cdot \tau_\eta\right), \quad i = 2$$

$$\tilde{\eta}_i \sim \mathrm{N}\left(\rho_\eta \cdot \tilde{\eta}_{i-1}, \tau_\eta\right), \quad i \geq 3$$

$$\mu_\eta \sim \mathrm{N}(0,1)$$

$$\tau_\eta = 1/\sigma_\eta^2$$

$$\sigma_\eta \sim \mathrm{U}(0,10)$$

$$\rho_\eta \sim \mathrm{Beta}(1,1.5)$$

- The rate of exposure growth is the growth in the volume of payments in a given exposure year, net of the calendar year effect
- The rate of exposure growth follows an autoregressive process
- The beta prior for the autoregressive parameter is right-skewed, thus discouraging values close to the unit root

Index *i* represents rows (exposure years)
Note: The second parameter of the normal distribution indicates the precision, which is defined as the reciprocal value of the variance.

Technical Appendix:

The Consumption Path (Without Break)

$$S_j = \beta_0 + \sum_{g=1}^{\xi} \beta_g \cdot \left(j - \theta_g\right) \cdot \mathrm{I}\left(j - \theta_g > 0\right)$$

$$\beta_g \sim \mathrm{N}\left(0, 0.0001\right) \; , \; g = 0 \ldots \xi$$

$$\vartheta_1 = 1$$

$$\frac{\vartheta_g - 1}{(b-1)} \sim \mathrm{Beta}\left(\gamma_1, \gamma_2\right) \; , \;\; g = 2 \ldots \xi$$

$$\theta_{1 \ldots \xi} = \mathrm{Sort}(\vartheta_{1 \ldots \xi})$$

$$\xi \sim \mathrm{Cat}\left(p_1 \ldots p_\lambda\right) \; , \;\; p_h = 1/\lambda \; , \;\; h = 1 \ldots \lambda$$

- The consumption path is modeled as a linear spline
- The location of the knots are endogenous, and so is their number
- The linear spline is estimated using RJMCMC (Reversible Jump MCMC), which facilitates Gibbs sampling when the dimension of the parameter space (e.g., locations of knots) changes
- It is recommended to use parameters for the beta distribution (which governs the location of the knots) that generate right skew, such as Beta(1,2) or, if the data are very noisy in the highest development years, Beta(1,3)

Index *j* represents columns (development years)
Note: The second parameter of the normal distribution indicates the precision, which is defined as the reciprocal value of the variance.

Technical Appendix:
The Changepoint

- As an option, the model allows for a structural break in the consumption path (a changepoint)
- The model estimates the probability distribution of the changepoint within a provided interval of exposure years
- Such an interval may be set at one year, but in general may comprise several years
- The changepoint *C* has a re-scaled and truncated beta prior (which is hump-shaped)

$$\tilde{C} \sim \mathrm{Beta}(2,2)$$

$$C = \mathrm{trunc}(\tilde{C} \cdot ((L+1)-M)) + M$$

where *M* is the first year in the interval of exposure years during which the structural break is allowed to occur, *L* is the last year, and *C* represents the first year in the new regime

- During the years of the changepoint interval, the consumption path is a mixture between the pre- and post-break regimes:

$$\mu_{i,j} = S_1 + \sum_{k=2}^{i+j} \kappa_k + \sum_{k=1}^{i} \eta_k + V_{i,j}$$

$$V_{i,j} = \mathrm{R}_{1,j} \cdot \mathrm{I}(C > \mathrm{i}) + \mathrm{R}_{2,j} \cdot \mathrm{I}(C \le \mathrm{i})$$

Indexes: *i* represents rows (exposure years); *j* represents columns (development years)
K is the dimension of the triangle (number of rows, equal to number of columns)

Technical Appendix:

The Consumption Path (With Break)

$$R_{r,j} = \beta_{r,0} + \sum_{g=1}^{\xi_r} \beta_{r,g} \cdot \left(j - \theta_{r,g}\right) \cdot \mathrm{I}\left(j - \theta_{r,g} > 0\right), \; r = 1,2$$

$$\beta_{r,g} \sim \mathrm{N}\left(0, 0.0001\right), \; g = 0 \ldots \xi_r, \; r = 1,2$$

$$\vartheta_{r,1} = 1, \; r = 1,2$$

$$\frac{\vartheta_{r,g} - 1}{(b_r - 1)} \sim \mathrm{Beta}\left(\gamma_{r,1}, \gamma_{r,2}\right), \; g = 2 \ldots \xi_r, \; r = 1,2$$

$$\vartheta_{r,1 \ldots \xi_r} = \mathrm{Sort}(\theta_{r,1 \ldots \xi_r}), \; r = 1,2$$

$$\xi_r \sim \mathrm{Cat}\left(p_1 \ldots p_{\lambda_r}\right), \; p_h = 1/\lambda_r, \; h = 1 \ldots \lambda_r, \; r = 1,2$$

- There are two consumption paths, one that applies to the pre-structural break regime, and one that applies to the post-structural break regime
- Thus, there are two beta distributions governing the locations of the knots for the two regimes
- It is recommended to use parameters for the beta distribution of the pre-structural break regime that similar to the ones in the version without a break: Beta(1,2) or, if the data are very noisy in the highest development years, Beta(1,3)
- For the post-structural break regime, if the consumption path is short, a uniform distribution, that is Beta(1,1), is recommended; otherwise, Beta(1,2)

Indexes: *i* represents rows (exposure years); *j* represents columns (development years)
Note: The second parameter of the normal distribution indicates the precision, which is defined as the reciprocal value of the variance.

Technical Appendix:

Probability of Payment (1/2)

- The model fits to logarithmic incremental payments—because the logarithm of zero (or less) is undefined, incremental payments at amounts of zero (or at negative amounts) are treated as missing values
- In order to account for payments at zero amounts, a Bernoulli process is estimated is estimated alongside the model to determine the probability that there occurs a payment at a zero amount
- The estimated incremental payments of the model are multiplied by one minus the parameter of this Bernoulli process, iteration by iteration
- Observations of payments at zero amounts may be sparse (if not non-existent). Further, although the proportions by column of payments at zero amounts may increase as development time progresses, the total number of observations by column invariably approaches unity before the triangle runs out of reported development years
- For this reason, when estimating the Bernoulli process, the prior of the parameter of this Bernoulli distribution (which varies by development year) is determined using a parametric specification

Technical Appendix:
Probability of Payment (2/2)

$$u_{i,j} \sim \mathrm{Bern}\left(p_j\right)$$

$$p_j = 1 - \exp\left(-\varphi_1 \cdot \exp\left(-\varphi_2 \cdot j\right)\right)$$

$$\log\left(\varphi_1\right) \sim \mathrm{N}\left(\log\left(\mu_{\varphi,1}\right) - \frac{1}{2 \cdot \tau_{\varphi_1}}, \tau_{\varphi_1}\right)$$

$$\log\left(\varphi_2\right) \sim \mathrm{N}\left(\log\left(\mu_{\varphi,2}\right) - \frac{1}{2 \cdot \tau_{\varphi_2}}, \tau_{\varphi_2}\right)$$

$$\tau_{\varphi_m} \sim \mathrm{Ga}\left(10, 0.1\right) \ , \quad m = 1,2$$

- The chosen functional form is a Gompertz curve, which affords the trajectory of the Bernoulli parameter a fair amount of flexibility
- The expected values $\mu_{\varphi,i+j}$ of the prior distributions of the Gompertz parameters are determined by means of nonlinear optimization prior to estimating the model
- $u_{i,j}$ is equal to 0 if there was a payment at a zero amount, equal to 1 if there was a payment at a positive amount, and a missing value otherwise

Indexes: *i* represents rows (exposure years); *j* represents columns (development years)
Note: The second parameter of the normal distribution indicates the precision, which is defined as the reciprocal value of the variance.