Extreme development techniques

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Overview

- Background and motivation
- Walkthrough of specific methods
  - Incremental paid/incurred loss development method
  - Case reserve run-off method
  - Recursive method
  - Curve fitting method

What are extreme development techniques?

Extreme development techniques are methods that may be necessary in the following situations:

- Claims and exposure data are limited to nearly non-existent
- Traditional development patterns are not available
- Data are so mature that ultimate loss estimates are "extremely" volatile

Some of these methods are extensions of traditional development methods, others are novel approaches to viewing loss development and projecting future claims.

When are extreme development techniques useful?

This session will discuss a number of examples of such extreme development methods and models that may be useful to actuaries who are modeling the following:

- Long-tailed lines of business
- Run-off portfolios
- Reinsurance liabilities

Techniques to be discussed today

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

Incremental loss development method

- When is this method appropriate?
  - When reliable data are only available from a certain point in time onward (e.g., after a systems conversion)
  - When the liabilities are very mature and paid-to-date or incurred-to-date measures are of limited value
- What data are needed?
  - Paid losses from a fixed point in time forward
  - Case reserve at date
  - Incurred losses from a fixed point in time forward

Step 1: calculation of change in paid losses

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

- Step 1: Calculate the change in paid loss based on the incremental paid triangle
  - Assumption: evaluated as of 31 December 2010
  - The following triangle is the incremental paid/loss triangle; we are going to calculate the incremental paid/loss development factors based on this triangle

Few more ages are not shown here due to limited room

Y¡}ªq‹›	I	J	K	L	M	N	O	P	Q	JO	JP	JQ	KH	KI	KJ
mGo q}y‹	IJ	JL	KN	LP	NH	OJ	PL	QN	IHP	KJL	KKN	KLP	KNH	KOJ	KPL
IQOO	E	E	E	E	E	E	MDHIMDMOO	JMDQMP	OHDOLJ	@PHHA	IDLQP	E	LIJ	IDKON	KQO
IQOP	E	E	E	E	E	PDMNHDJNM	KQDQLK	LQDLHM	JNDOIP	IDKIJ	IMP	KJM	IDHIN	E	PDKLI
IQOQ	E	E	E	E	JIDPJMDNQN	KQNDMON	LNKDLML	LIJDMKL	IOHDLOI	E	KMO	JKP	JNI	IDQHM	INL
IQPH	E	E	E	KHDPLNDHLH	JDIIQDMIQ	MQQDINO	LOHDPKJ	NHODPIN	JIHDOKJ	MQDJML	JHK	@NOA	MDMQJ	E	E
IQPI	E	E	JLDKPPDJMO	IDPMKDIPI	KMKDNOO	JMMDIQK	KLODLIM	JIJDPNL	INHDMMM	@JIMA	NDKPL	ILDHPP	KNDMNN	E	E
IQPJ	E	IKDHMMDNNO	NDNJQDLHO	JDMJQDPIJ	PIODHNL	JKIDHPM	INLDPJI	KJODJKH	IKQDOPQ	JPDKKI	KDKHO	IJDNIH	E	E	E
IQPK	LDLMKDQPH	IKDJJNDQNH	ODMIKDMLL	LDJPPDKOK	IDNKKDHJM	PNIDKJN	LNLDKMJ	PPKDIMI	PJKDLOQ	NDMMN	IDPNK	E	E	E	E
IQPL	JDMHMDLMN	MDOINDIJO	KDNHNDJKQ	IDKQMDQLI	KPIDQOP	JLJDKMN	KJNDLOK	JJIDPIH	KIKDOPJ	JDHKN	E	E	E	E	E

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Incremental paid/loss development factors

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

U/W Year	12 - 24	24 - 36	36 – 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	324 - 336	336 - 348	348 - 360	360 - 372	372 - 384	384 - 396
1977	–	–	–	–	–	–	0.005	2.725	0.135	(1.872)	0.000	–	3.337	0.289	0.000
1978	–	–	–	–	–	0.005	1.237	0.541	0.534	0.120	2.058	3.128	0.000	–	0.000
1979	–	–	–	–	0.018	1.169	0.890	0.413	(0.033)		0.667	1.097	7.300	0.086–	
1980	–	–	–	0.069	0.283	0.786	1.291	0.347	1.243	0.003	(0.330)	(83.653)	0.000	–	–
1981	–	–	0.076	0.191	0.722	1.361	0.613	0.754	0.688	(29.638)	2.207	2.595	–	–	–
1982	–	0.508	0.382	0.323	0.283	0.713	1.985	0.427	1.604	0.117	3.813	–	–	–	–
1983	2.970	0.568	0.571	0.381	0.527	0.539	1.902	0.932	0.591	0.284	–	–	–	–	–
1984	2.281	0.631	0.387	0.274	0.634	1.347	0.679	1.415	0.688	–	–	–	–	–	–
WtdAvg:	2.722	0.555	0.239	0.130	0.095	0.204	0.376	0.699	0.687	0.142	2.284	2.978	0.451	0.171	0.000
AllYrAvg	2.626	0.569	0.354	0.247	0.411	0.846	1.075	0.944	0.681	(5.164)	1.402	(19.208)	2.659	0.187	0.000
AllYrAvg x H/L	2.626	0.568	0.384	0.262	0.432	0.911	1.102	0.747	0.646	(0.408)	1.233	1.846	1.668	0.187	0.000
Selected	–	0.568	0.239	0.130	0.432	0.204	1.075	0.747	0.681	0.142	0.994	1.846	0.000	0.000	0.000
	12	24	36	48	60	72	84	96	108	324	336	348	360	372	384
Incremental Pattern	1.000	0.568	0.136	0.018	0.008	0.002	0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000
Accumulated Values	**1.000**	**1.568**	**1.704**	**1.721**	**1.729**	**1.731**	**1.732**	**1.733**	**1.734**	**1.743**	**1.743**	**1.743**	**1.743**	**1.743**	**1.743**

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Calculation of change in paid loss

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

(1) U/W year	(2) Start age	(3) End age	(4) Total paid *At start age*	(5) Total paid *At end age*	(6) Total change *From start age to end age*
1977	8	33	5,041,536	5,209,555	168,019
1978	7	32	8,600,208	8,887,859	287,651
1979	6	31	22,222,272	23,948,040	1,725,768
1980	5	30	32,965,559	36,346,372	3,380,813
1981	4	29	26,241,438	28,627,600	2,386,162
1982	3	28	19,685,073	25,309,734	5,624,661
1983	2	27	17,680,940	37,835,918	20,154,978
1984	1	26	2,505,456	15,371,996	12,866,540
Total			134,942,480	181,537,074	46,594,593

Calculation details (use U/W yr **1984** as an example and refer to **triangle on page 6**):

1. Total paid @ age 1 = **2,505,456** (sum up all the incremental paid loss before the "stair")
2. Total paid @ age 26 = 2,505,456+5,716,127+…+2,036 = **15,371,996** (sum up all the incremental paid loss for U/W yr 1984)
3. Total change = 15,371,996 - 2,505,456 = **12,866,540**

Step 2: Curve fitting

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

We fitted x and y values into different distributions (e.g., Weibull, Gompertz and Richards model) to get the coefficients.

Age (in months)	X = Age (in years)	Actual: Y = Accumulated incremental selections
12	1	1.000
24	2	1.568
36	3	1.704
48	4	1.721
60	5	1.729
72	6	1.731
84	7	1.732
96	8	1.733
108	9	1.734
120	10	1.735
132	11	1.736
144	12	1.736
156	13	1.738
168	14	1.738
180	15	1.739
192	16	1.740
204	17	1.740
216	18	1.741
228	19	1.741
240	20	1.742
252	21	1.742
264	22	1.742
276	23	1.742
288	24	1.742
300	25	1.742

This column is from the triangle on page 8

From curve fitting software

Weibull model: y=a-b*exp(-c*x^d)

Coefficient Data:

a =	1.741
b =	230.178
c =	5.371
d =	0.424
Standard error:	0.0059047
Correlation coefficient:	0.9991290

Gompertz relation: y=a*exp(-exp(b-cx))

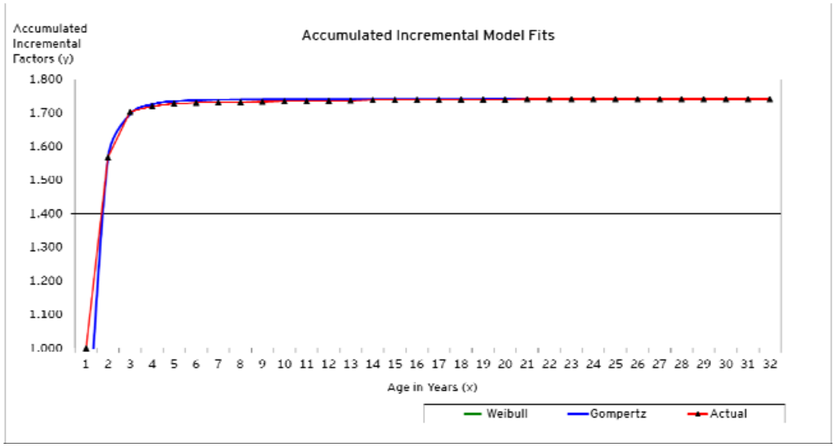
Coefficient data:

a =	1.740
b =	0.610
c =	1.439
Standard error:	0.0050646
Correlation coefficient:	0.9920541

Weibull: Y^ = a - b*exp(-c*X^d)	Gompertz: Y^ = a*exp(-exp(b-c*X))
0.671	1.125
1.570	1.569
1.696	1.698
1.726	1.730
1.735	1.738
1.738	1.739
1.739	1.740
1.740	1.740
1.740	1.740
1.740	1.740
1.740	1.740
1.741	1.740
1.741	1.740
1.741	1.740
1.741	1.740
1.741	1.740
1.741	1.740
1.741	1.740
1.741	1.740
1.741	1.740
1.741	1.740
1.741	1.740
1.741	1.740
1.741	1.740
1.741	1.740

Accumulated incremental paid ratio model selection

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method



Step 3: accumulated incremental ratios calculation

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

- Step 3: calculate accumulated incremental ratios implied after fitting and comparing different distributions that behave like (transformable to) cumulative distribution functions
  - Assumption: we use Weibull model as an example; in practice, other models can also be used

(1) U/W year	(2) Start age	(3) End age	Weibull (7) Accumulated incremental (at start)	Weibull (8) Accumulated incremental (at end)
1977	8	KL	1.740	1.741
1978	7	KK	1.739	1.741
1979	6	KJ	1.738	1.741
1980	5	KI	1.735	1.741
1981	4	KH	1.726	1.741
1982	3	JQ	1.696	1.741
1983	2	JP	1.570	1.741
1984	1	JO	0.671	1.741

From page 9

Weibull model: y=a-b*exp(-c*x^d)
coefficient data:

a =	1.741
b =	230.178
c =	5.371
d =	0.424

Weibull model: y = a – b * exp(-c* **x** ^d)

1.741- 230.178 * exp(-5.371 * **27**^0.424) = **1.741**

Step 4: incremental ratios calculation and reserve projection

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

- Step 4: calculate the incremental loss development ratio to ultimate development based on curve fit and estimate the total reserves.

(1)	(2)	(3)	(4)	(5)	(6)	Weibull		Ratio to total period change	Estimated total reserves
U/W year	Start age	End age	Total paid	Total paid	Total change	(7) Accumulated incremental (at start)	(8) Accumulated incremental (at end)	(9)	(10)
			At start age	*At end age*	*From start age to end age*			[(Ult)-(8)] / [(8)-(7)]	(6) * (9)
1977	8	33	5,041,536	5,209,555	168,019	1.740	1.741	0.000023	**4**
1978	7	32	8,600,208	8,887,859	287,651	1.739	1.741	0.000015	**4**
1979	6	31	22,222,272	23,948,040	1,725,768	1.738	1.741	0.000010	**17**
1980	5	30	32,965,559	36,346,372	3,380,813	1.735	1.741	0.000006	**19**
1981	4	29	26,241,438	28,627,600	2,386,162	1.726	1.741	0.000003	**7**
1982	3	28	19,685,073	25,309,734	5,624,661	1.696	1.741	0.000001	**8**
1983	2	27	17,680,940	37,835,918	20,154,978	1.570	1.741	0.000000	**10**
1984	1	26	2,505,456	15,371,996	12,866,540	0.671	1.741	0.000000	**1**
Total			134,942,480	181,537,074	46,594,593	Ultimate:	1.741		**70**

Ultimate value = 1.741
According to the Weibull model y = a – b * exp(-c* **x** ^d), when x → ∞, y → a=1.741

Incremental ratio for U/W Yr 1977: (1.741– 1.741) / (1.741– 1.740) = **0.000023**
Estimated unpaid reserve for U/W Yr 1977: 0.000023* $168,019= $4

Case reserve run-off method

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

- When is this method appropriate?
  - When there is a long history of incremental paid/incurred losses
  - When the incremental activity is more significant than in cases where incremental method may be more appropriate
- What data are needed?
  - Incremental paid/loss
  - Cumulative incurred loss

Step 1: data aggregation and preparation

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

► Step 1: construct case reserve run-off triangle

- ► Given incremental paid triangle and care reserve triangle

Case reserve triangle

U/W Year	Prior	17	18	19	20	21	22	23	24	25
1986	…	62,902	65,699	62,588	56,280	37,858	37,861	34,066	32,543	32,528
1987	…	60,184	48,214	30,148	15,021	11,400	8,150	5,203	4,886	–
1988	…	58,897	59,035	36,375	35,843	35,507	8,230	8,393	–	–
1989	…	32,175	38,316	35,614	37,146	23,874	25,179	–	–	–
1990	…	49,900	64,752	51,653	24,066	22,219	–	–	–	–

Incremental paid/loss triangle

U/W Year	Prior	17	18	19	20	21	22	23	24	25
1986	…	1,603	(3,604)	2,380	(134)	10,017	42	422	2,321	(4,124)
1987	…	15,151	16,063	4,059	3,548	5,737	(4,846)	3,157	140	–
1988	…	7,763	(6,911)	467	(2)	(1,152)	9,771	–	–	–
1989	…	1,426	502	(478)	(802)	12,191	–		–	–
1990	…	3,283	(119)	3,949	28,577	2,158	–	–	–	–

37,858+ [(-3604)+2380+(-134)+10017] =**46,516**

59,035 + [(-6911)]=**52,124**

Case reserve run-off triangle from the start age 17

U/W year	17	18	19	20	21	22	23	24	25
1986	62,902	62,095	61,364	54,921	46,516	46,562	43,189	43,987	39,848
1987	60,184	64,277	50,270	38,691	40,806	32,710	32,920	32,743	–
1988	58,897	52,124	29,931	29,396	27,908	10,403	10,566	–	–
1989	32,175	38,818	35,638	36,368	35,287	36,593	–	–	–
1990	49,900	64,633	55,483	56,473	56,784	–	–	–	–

Step 2: run-off factor calculation

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

► Step 2: calculate the run-off ATA and ATU factors

From prior slide

Case reserve run-off triangle from the start age 17

U/W year	17	18	19	20	21	22	23	24	25
1986	62,902	62,095	61,364	54,921	46,516	46,562	43,189	43,987	39,848
1987	60,184	64,277	50,270	38,691	40,806	32,710	32,920	32,743	–
1988	58,897	52,124	29,931	29,396	27,908	10,403	10,566	–	–
1989	32,175	38,818	35,638	36,368	35,287	36,593	–	–	–
1990	49,900	64,633	55,483	56,473	56,784	–	–	–	–

Case run-off ATA factor

U/W year	18/17	19/18	20/19	21/20	22/21	23/22	24/23	25/24	
1986	0.987	0.988	0.895	0.847	1.001	0.928	1.018	0.906	
1987	1.068	0.782	0.770	1.055	0.802	1.006	0.995	–	
1988	0.885	0.574	0.982	0.949	0.373	1.016	–	–	
1989	1.206	0.918	1.020	0.970	1.037	–	–	–	
1990	1.295	0.858	1.018	1.006	–	–	–	–	
Avg x hi/lo	1.056	0.963	0.946	0.961	0.942	1.012	0.993	1.034	
Wtd avg	1.089	1.068	0.934	0.967	0.938	1.034	0.996	1.082	
Selected	1.089	1.058	1.031	1.028	1.019	1.012	0.993	1.001	Tail
Implied ATU	1.547	1.420	1.342	1.302	1.266	1.241	1.227	1.235	1.233

Tail factor is usually selected based on industry factors

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Step 3: case to case: run-off ratio calculation

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

From slide 14: case reserve triangle

U/W year	17	18	19	20	21	22	23	24	25
1986	62,902	65,699	62,588	56,280	37,858	37,861	34,066	32,543	32,528
1987	60,184	48,214	30,148	15,021	11,400	8,150	5,203	4,886	–
1988	58,897	59,035	36,375	35,843	35,507	8,230	8,393	–	–
1989	32,175	38,316	35,614	37,146	23,874	25,179	–	–	–
1990	49,900	64,752	51,653	24,066	22,219	–	–	–	–

Case-to-case run-off ratio

U/W year	17	18	19	20	21	22	23	24	25
1986	1.000	1.058	1.020	1.025	0.814	0.813	0.789	0.740	0.816
1987	1.000	0.750	0.600	0.388	0.279	0.249	0.158	0.149	–
1988	1.000	1.133	1.215	1.219	1.272	0.791	0.794	–	–
1989	1.000	0.987	0.999	1.021	0.677	0.688	–	–	–
1990	1.000	1.002	0.931	0.426	0.391	–	–	–	–
Avg	1.000	0.947	0.872	0.759	0.647	0.577	0.539	0.473	0.508
Wtd Avg	1.000	0.916	0.808	0.706	0.600	0.515	0.479	0.431	0.422
Selection	1.000	0.916	0.872	0.759	0.600	0.546	0.479	0.448	0.422

Step 4: case to case: run-off ratio application and reserve projection

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

	Age in years	17	18	19	20	21	22	23	24	25
(1) (Slide 15)	Case-run-off factor	1.547	1.420	1.342	1.302	1.266	1.241	1.227	1.235	1.233
(2) (Slide 16)	Case to case-run-off ratio	1.000	0.916	0.872	0.759	0.600	0.546	0.479	0.448	0.422
((1)-1)/(2)	**Selected IBNR-to-case reserve tatio**	0.547	0.459	0.392	0.398	0.443	0.442	0.474	0.524	0.553

Age in years as of 31 December 2010	U/W year	Case ($)	IBNR-to-Case ratio	Estimated IBNR ($)
25	1986	32,528	0.553	IODQPL
24	1987	4,886	0.524	JDMNH
23	1988	8,393	0.474	KDQOM
22	1989	25,179	0.442	IIDILH
21	1990	22,219	0.443	QDPKO

Recursive method

- When is this method appropriate?
  - When only incremental loss data are available
  - When we assume the relationship of ΔP/ΔC is consistent as the exposure approaches ultimate
  - When only aggregate calendar year losses for all exposure years are available, particularly when all years are very mature
- What data are needed?
  - Incremental paid/loss
  - Change in case reserves

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Theory and calculation steps

- Calculate (incremental) paid to prior case ratio: "p"
- Calculate case to prior case ratio: "c"
- Assumptions:
  - These consumption ratios are consistent over time
  - Initial case reserve is $1

Time	Paid losses	Case
0		1
1	p	c
2	pc	cc
3	pcc	ccc
4	pccc	c^4
5	pc^4	c^5
6	pc^5	c^6
7	pc^6	c^7
8	pc^7	c^8
9	pc^8	c^9
10	pc^9	c^10
11	pc^10	c^11
12	pc^11	c^12
13		
14		

- Required reserves= sum(pmts) = p * (1+c+c^2+c^3+c^4+c^5+ ….)
- Since c < 1, (a requirement), sum(pmts) = p/(1-c) (based on geometric theory)
- c = Case$(k) / Case$ (k-1);
- p = Paid$ movement (k) / Case$ (k-1)
  = (CumPaid$(k) – CumPaid$(k-1)) / Case$ (k-1)
- Since c and p share the same denominator,
  sum(pmts) = p/(1-c)
  = Paid$ movement (k) / (Case$(k-1) – Case$(k))
  = [CumPaid$(k) – CumPaid$(k-1)] / [Case$(k-1)–Case$(k)]

sum(pmts) = p/(1-c) = ΔP/ΔC
This is the ΔP/ΔC ratio we need to select

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Few more things about this method

- **(ΔP/ΔC) x C = required reserves**
- If for every dollar of case reduction, there are Z (which is the selected ratio of ΔP/ΔC) dollars of paid losses, then the required reserves (case + IBNR) are (Z x C)
- ΔP/ΔC ratio: this ratio is a measurement of the interaction between paid and case movements. Paid losses almost always trigger case reserve changes
- We can interpret this as: future paid losses (to ultimate) will be related to case reserves in exactly the same ratio as ΔP/ΔC over the relevant period used
- This method does not require the availability of cumulative data. Thus if historical data are lost or missing, this method works. Since this is a calendar year method, it works well to combine exposure periods in order to stabilize the calculations

Numerical example

- Step 1: calculate and select the ratio of incremental payment relative to change in case reserves (ΔP/ΔC)

Calendar year	Company case reserves Beginning	Ending	Change case (-)	Company Incremental paid loss	ΔP/ΔC
	(1)	(2)	(3) = (1) - (2)	(4)	(5) = (4)/(3)
2000		2,674,000	–	–	–
2001	2,674,000	2,910,000	(236,000)	88,000	(0.37)
2002	2,910,000	2,798,000	112,000	(183,000)	(1.63)
2003	2,798,000	3,038,000	(240,000)	33,000	(0.14)
2004	3,038,000	1,887,000	1,151,000	722,000	0.63
2005	1,887,000	1,826,000	61,000	(21,000)	(0.34)
2006	1,826,000	1,323,000	503,000	557,000	1.11
2007	1,323,000	1,200,000	123,000	388,000	3.15
2008	1,200,000	1,315,000	(115,000)	43,000	(0.37)
2009	1,315,000	1,145,000	170,000	359,000	2.11
				Avg 3 yrs	1.63
				Avg 5 yrs	1.13
				Selected ΔP/ΔC ratio	1.63

Numerical example

- Step 2: calculate future payments and unpaid reserves
  - Assumption: the ratio ΔP/ΔC would be stable for a mature set of exposure

Calendar year	Case reserves at 12/31/XX	Selected ΔP/ΔC factor	Company incremental paid loss	Paid Since date	Required reserves estimates
	(1)	(2)	(3)	(4) in 2000 = (3) total (4) = (4) prior - (3)	(5)=(1)*(2)-(4)
2000	2,674,000	1.63	–	1,986,000	2,374,691
2001	2,910,000	1.63	88,000	1,898,000	2,847,554
2002	2,798,000	1.63	(183,000)	2,081,000	2,481,907
2003	3,038,000	1.63	33,000	2,048,000	2,906,293
2004	1,887,000	1.63	722,000	1,326,000	1,751,271
2005	1,826,000	1.63	(21,000)	1,347,000	1,630,794
2006	1,323,000	1.63	557,000	790,000	1,367,515
2007	1,200,000	1.63	388,000	402,000	1,554,929
2008	1,315,000	1.63	43,000	359,000	1,785,468
2009	1,145,000	1.63	359,000	–	1,867,237
Total			1,986,000	Selected reserve	**1,826,000**

Selected the 50th percentile value of estimated required reserves

Curve fitting

- When is this method appropriate?
  - In practice, this method has been used for major events and CAT losses where there is no “repetition” of development factors. The fits for the older events are the familiar S-shapes as well.
  - The aggregated cumulative calendar year incurred losses are fit using several models in order to determine total IBNR. The results of the fits are analyzed, focusing on the goodness of fit statistic (R^2), and the best fitting model is selected.
- What data are needed?
  - Cumulative incurred or paid losses

Step 1: data aggregation and preparation

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

- Step 1: transfer the dollar amount loss data to model data

Year	Cumulative incurred	Model data: x – Indexed year	Model data: y – Log_{10}(cumulative incurred)
2001	175,745,000	1	8.245
2002	175,774,000	2	8.245
2003	176,287,000	3	8.246
2004	176,085,000	4	8.246
2005	176,075,000	5	8.246
2006	176,052,000	6	8.246
2007	176,069,000	7	8.246
2008	175,810,000	8	8.245
2009	174,427,000	9	8.242
2010	174,244,000	10	8.241

Step 2: model selection and ultimate loss projection

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

- Step 2: fit the model data to several models and estimate coefficients and project ultimate losses and IBNR reserves
  - Many models can be curve fitting method candidates, such as Weibull, Gompertz, Logistic, MMF, Hyperbolic, and so on.
  - Here, we use the Gompertz model as an example to show the calculations
  - By fitting the model data from prior slide into the Gompertz model, we obtained the coefficients as follow:

Model: Gompertz relation: **y=a*exp(-exp(b-cx))**

Parameters estimated using curve fitting software
Assumes 20-year development pattern

a =	8.2441	s =	0.0024
b =	-4.3524	R^2 =	94%
c =	1.4516		

  - For presentation purposes, we assume that losses will be developed to ultimate in 10 year. This assumption should be modified based on actual circumstances

Ultimate loss projection

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

Year	x: Indexed year	y: Log_{10}(cumulative incurred)	Projected cumulative incurred
2001	1	8.219	
2002	2	8.238	
2003	3	8.243	
2004	4	8.244	
2005	5	8.244	
2006	6	8.244	
2007	7	8.244	
2008	8	8.244	
2009	9	8.244	
2010	10	8.244	
2011	11	8.244	175,445,257
2012	12	8.244	175,445,261
2013	13	8.244	175,445,262
2014	14	8.244	175,445,262
2015	15	8.244	175,445,262
2016	16	8.244	175,445,262
2017	17	8.244	175,445,262
2018	18	8.244	175,445,262
2019	19	8.244	175,445,262
2020	20	8.244	175,445,262

Model: Gompertz relation: y=a*exp(-exp(b-cx))

Parameters estimated using curve fitting software

Assumes 10-year development pattern

a =	8.2441	s =	0.0024
b =	-4.3524	R2 =	94%
c =	1.4516		

Use the coefficients in prior slide:
8.2441*exp(-exp(-4.3524-1.4516***20**)) = **8.244**

Projected ultimate loss:
10^8.244 = **175,445,262**

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Extreme development techniques

2011 Casualty Actuarial Society Spring Meeting

17 May 2011

Christopher Diamantoukos, Senior Actuarial Advisor, Ernst & Young LLP
Richard Plunkett, SVP and Actuary, Brandywine Group
Justin Brenden, Actuarial Advisor, Ernst & Young LLP

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