The Retrospective Testing of Stochastic Loss Reserve Models

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Background

- Risk based capital proposals, e.g. EU Solvency II and USA SMI rely on stochastic models.
  - VaR@99.5% and TVaR@99%
- There are many stochastic loss reserve models that claim to predict the distribution of ultimate losses.

Are any of these models right?

E-Forum Paper

Joint with Peng Shi – Northern Illinois University

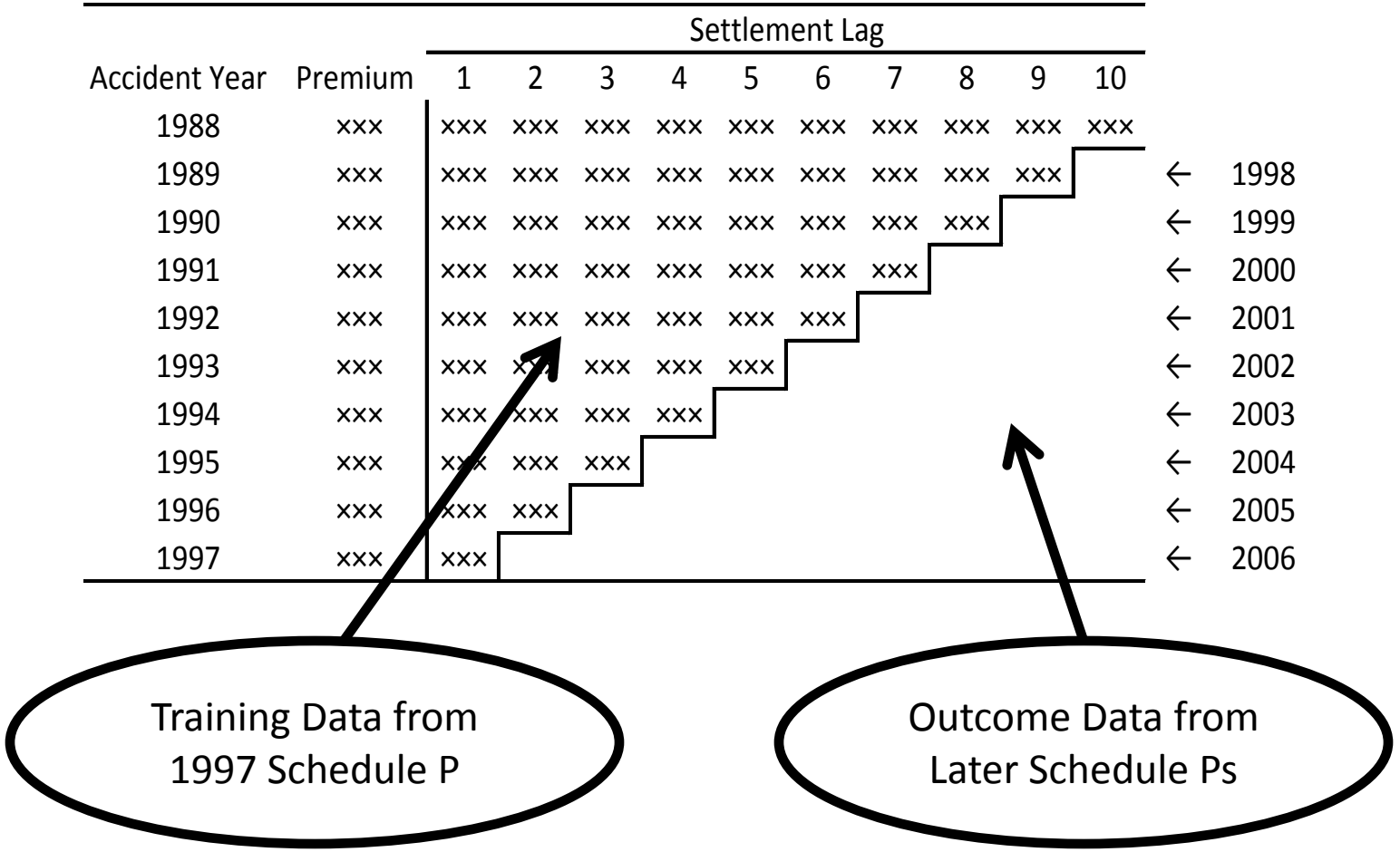
- Describes a database
  - Data from several American Insurers
  - Data for six lines of insurance
  - Paid and incurred loss triangles
  - Subsequent outcomes
  - Available online (Free)
- Predicts the distribution of outcomes of two models for several insurers for Commercial Auto Insurance
- Tests the predictions against subsequent reported outcomes.

The CAS Loss Reserve Database

- Schedule P (Data from Parts 1-4) for several US Insurers
  - Private Passenger Auto
  - Commercial Auto
  - Workers' Compensation
  - General Liability
  - Product Liability
  - Medical Malpractice (Claims Made)
- Available on CAS Website – New Version 9/1/2011

  http://www.casact.org/research/index.cfm?fa=loss_reserves_data

The CAS Loss Reserve Database

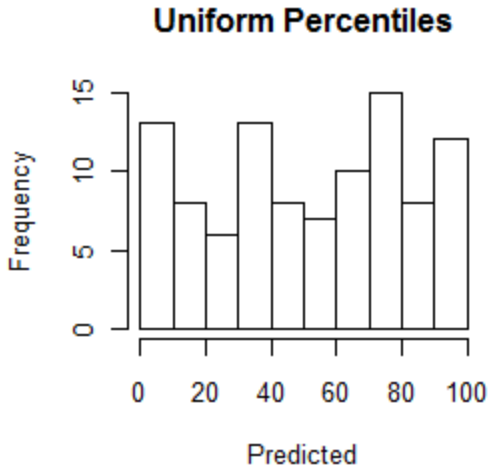


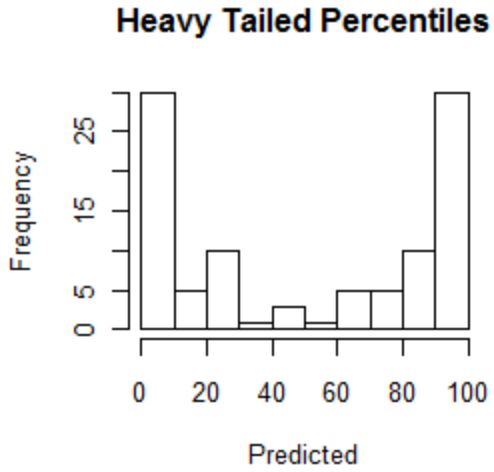
- Can we predict the distribution of outcomes? Or sums of outcomes?

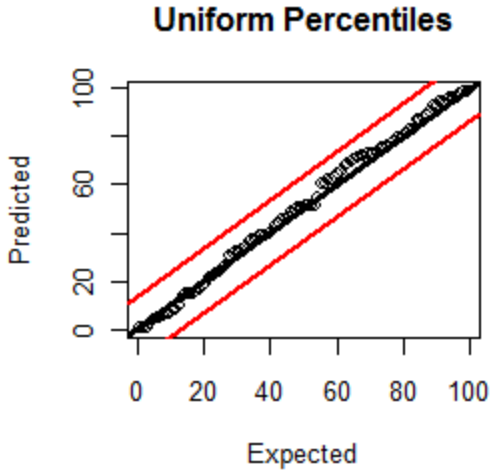
Criteria for a "Good" Stochastic Loss Reserve Model

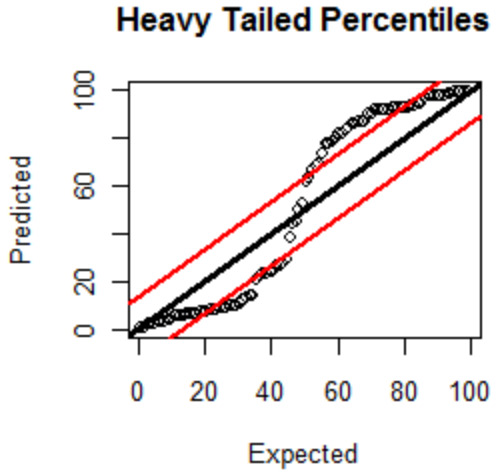
- Using the upper triangle "training" data, predict the distribution of the outcomes in the lower triangle
  - Can be observations from individual (AY, Lag) cells or sums of observations in different (AY,Lag) cells.
- Using the predictive distributions, find the percentiles of the outcome data.
- The percentiles should be uniformly distributed.
  - Histograms
  - Test with PP Plots/KS tests
    - Plot Expected vs Predicted Percentiles

Illustrative Tests of Uniformity





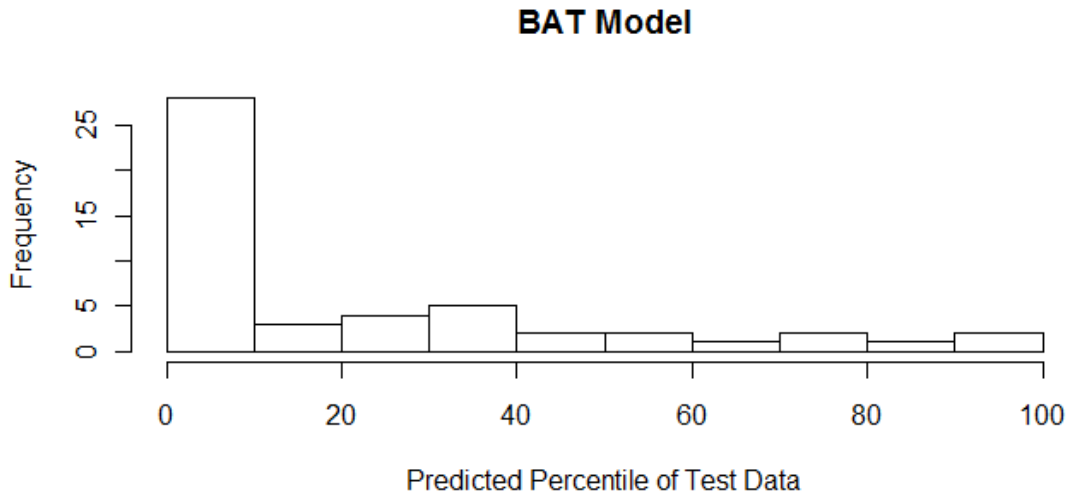


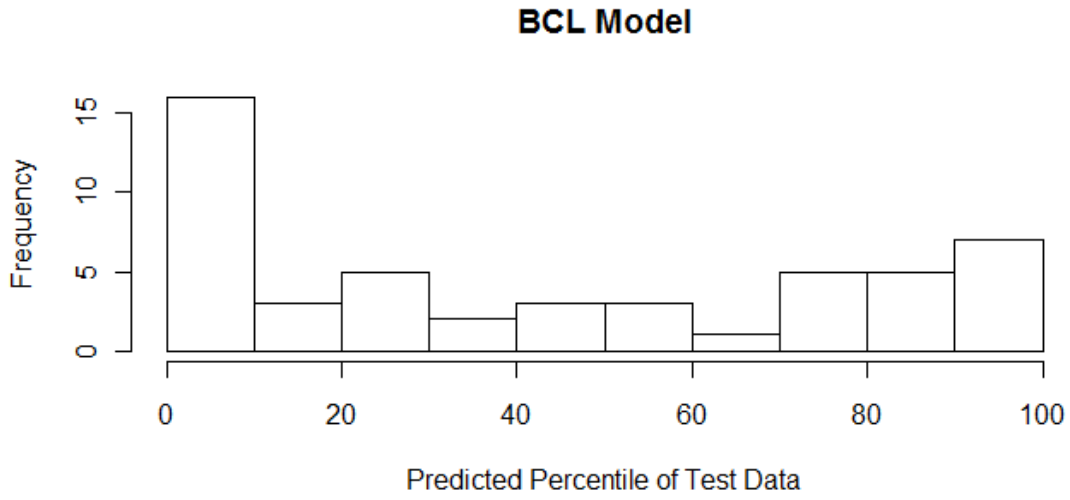


Examples of Tests in Meyers Shi Paper

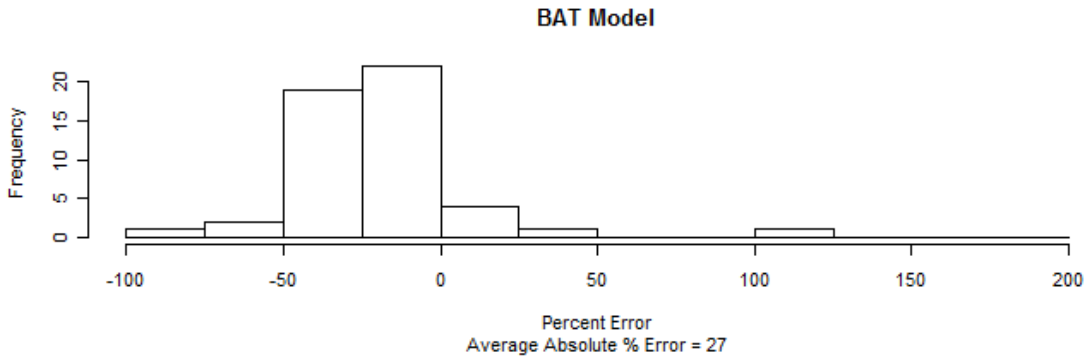
- Commercial Auto
- 50 Insurers – “Selected” going concern insurers
- Tested two stochastic loss reserve models
  - Bootstrap chain ladder (BCL) model
    - Used the “ChainLadder” package in R
    - Overdispersed Poisson for process risk.
  - Bayesian Autoregressive Tweedie (BAT) model
    - Described in the paper

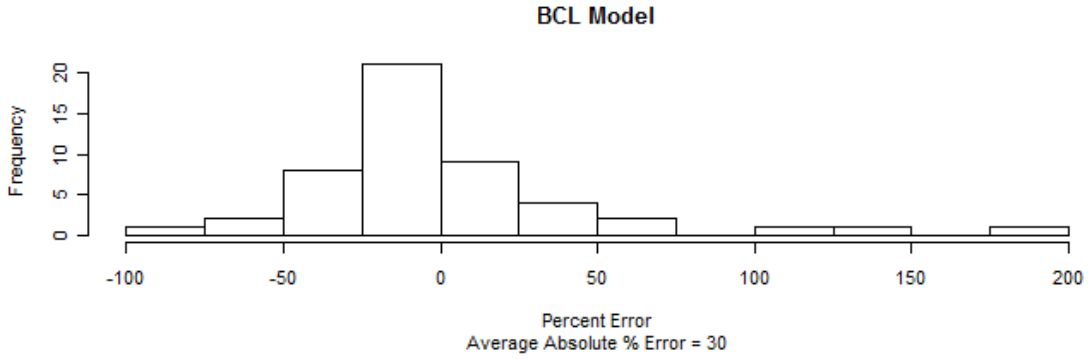
Predicted Percentiles of Outcomes in Meyers Shi

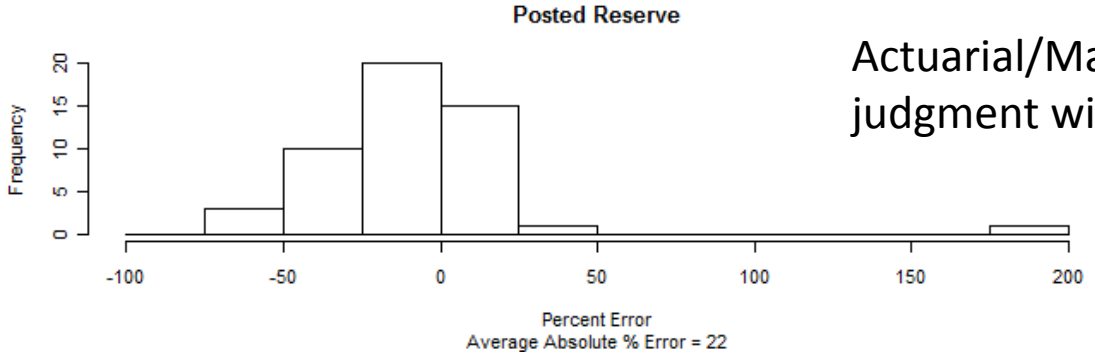




BAT, BCL and Posted Reserve % Error





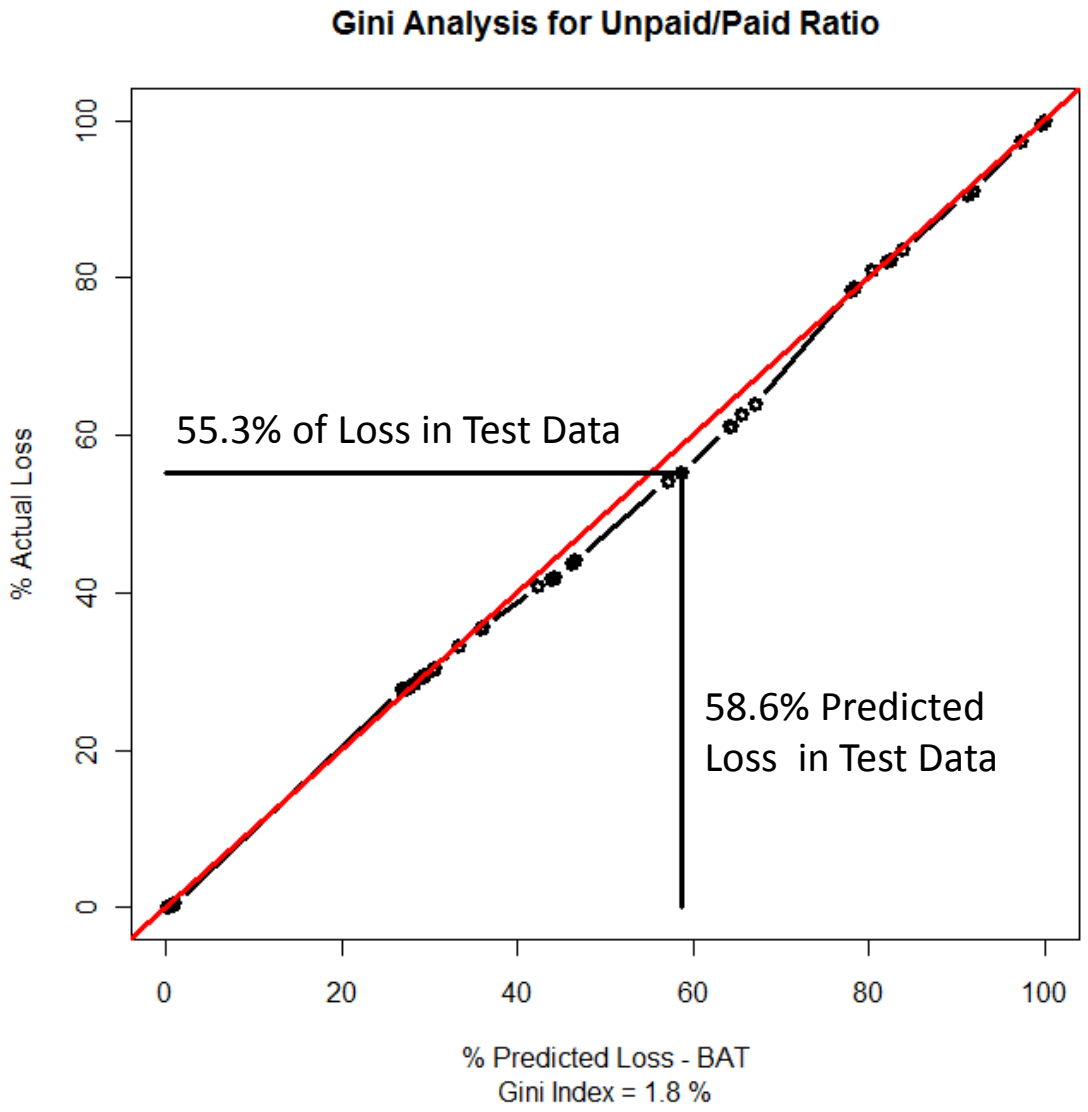


Actuarial/Management judgment wins!

Finding the Right Model

- These models used only paid data. Could we do a better job by including incurred loss data?
- BAT used earned premium data. Does this help or hinder the prediction?
- Is there other external data available?
- Work with other lines of insurance.

A Hint – Use Unpaid Loss Information



Implications of Using Incurred Claims Data

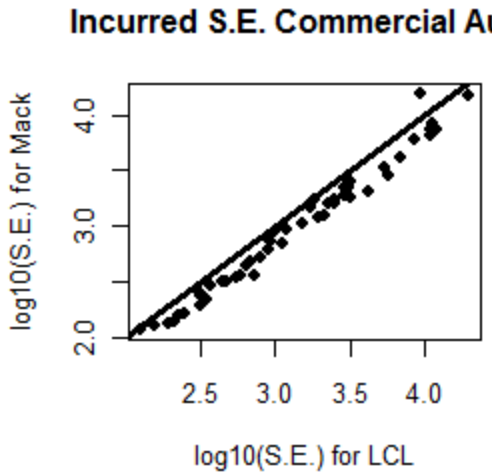
- I ruled out incremental claims models.
  - Frequent negative changes with incurred data
- Chose Mack chain ladder model as a base for comparison.
- Also looked at both paid and incurred cumulative data.

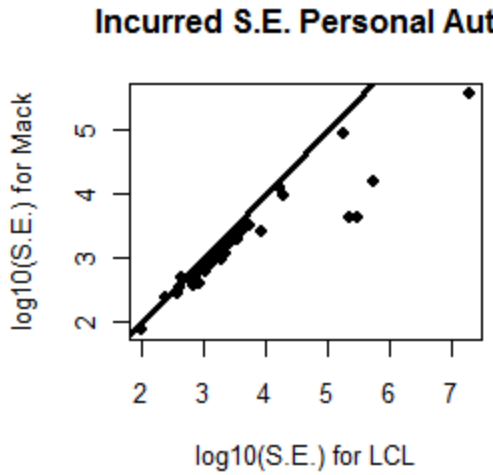
The Leveled Chain Ladder Model

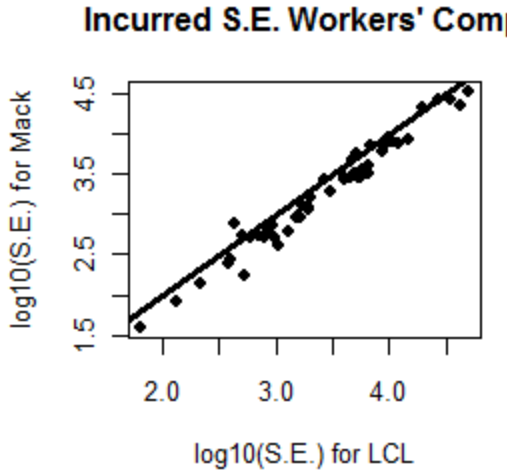
- New Model (?) – Leveled Chain Ladder
  - Chain ladder applies age-to-age factors to the latest reported (paid or incurred) loss.
  - “Replace” the latest reported loss with a “level” parameter.
- Reflect the uncertainty in the level parameter in the predictive distribution of outcomes.
- Used Bayesian MCMC software, JAGS, to quantify uncertainty in parameter estimates.
- Details in CLRS call paper.

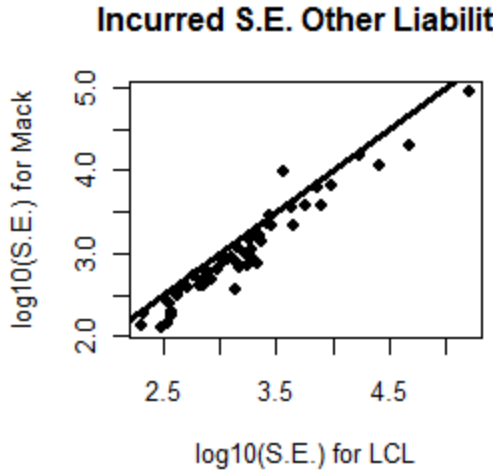
Motivation for LCL

Increase Estimates of Variability Over Mack







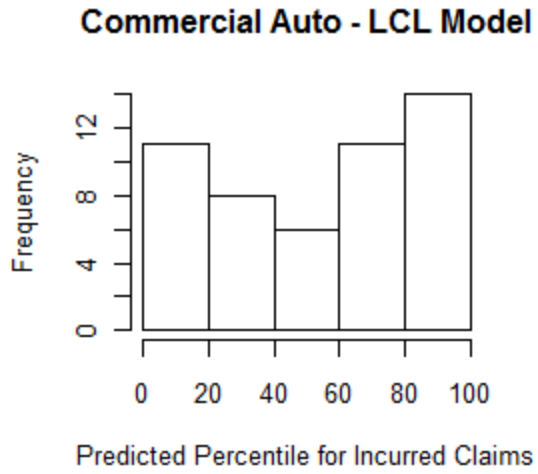


Design of Retrospective Test For 50 Insurers in CA, PA, WC and OL

- Estimate the predictive distribution of the reported claims at development year 10 for each insurer using both models.

$$\sum_{w=2}^{10} C_{w,10}$$

- Calculate the percentile of the reported sum for each insurer using both model.
- Test the uniformity of the calculated percentiles for both models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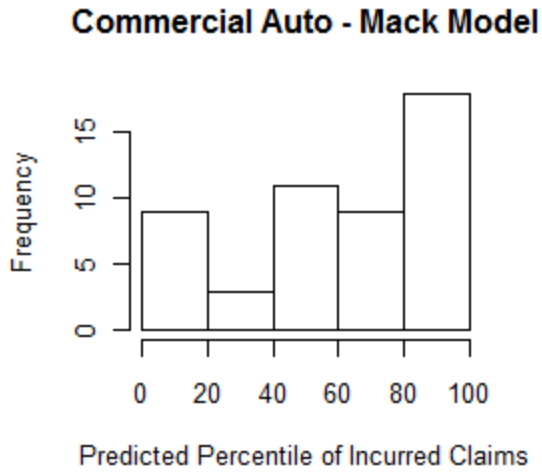
Predicted Percentile for Incurred Claims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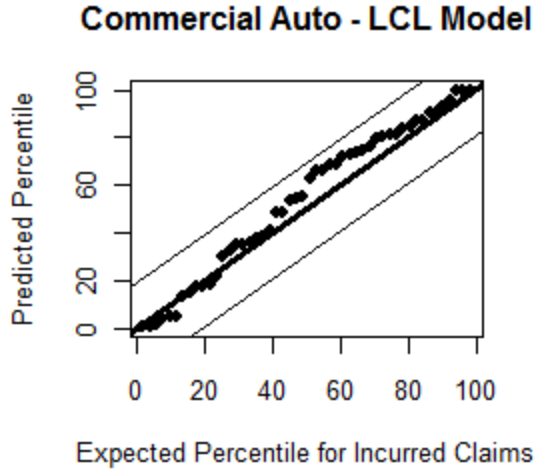
Predicted Percentile of Incurred Claims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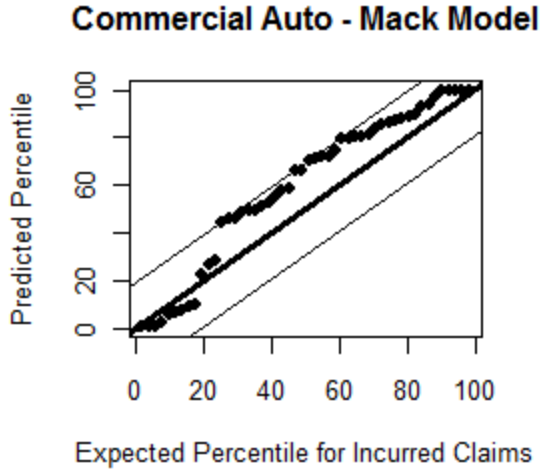
Expected Percentile for Incurred Claims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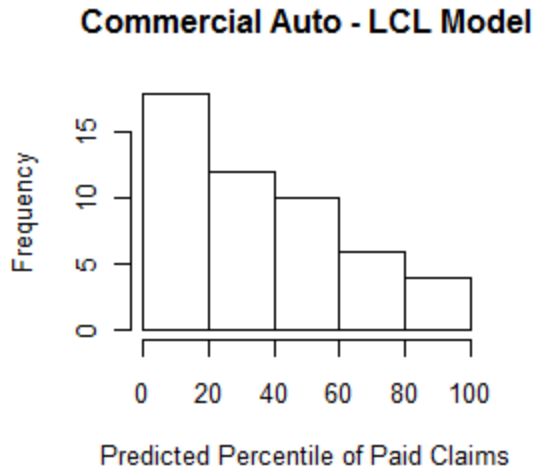
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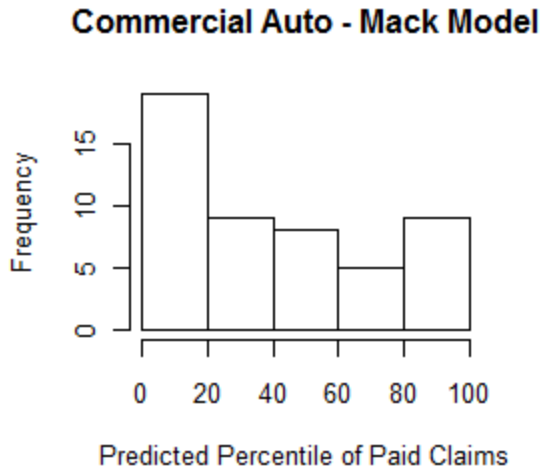
Predicted Percentile of Paid Claims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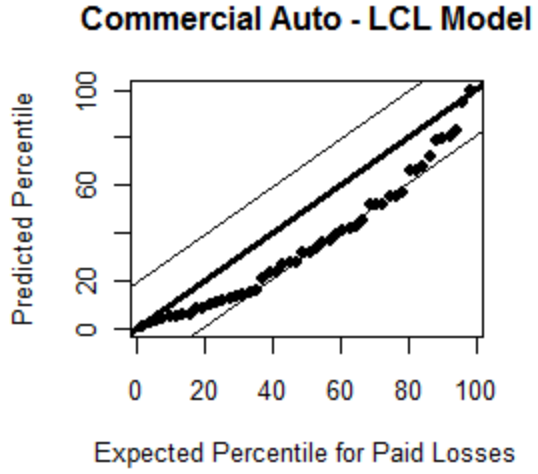
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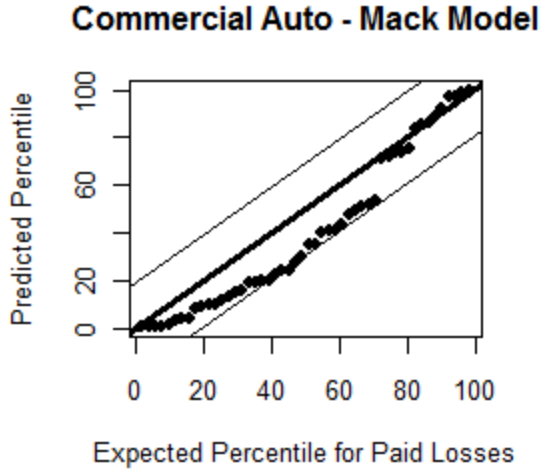
Expected Percentile for Paid Losses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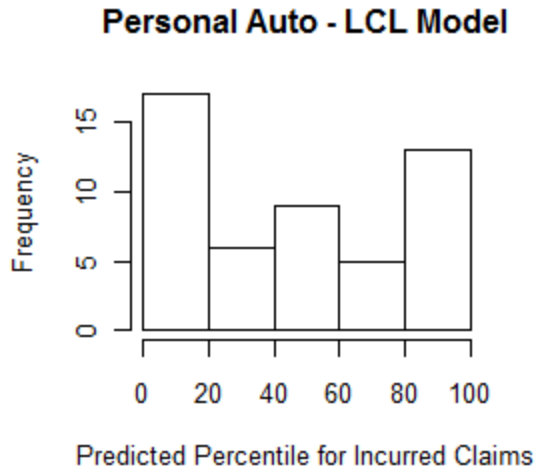
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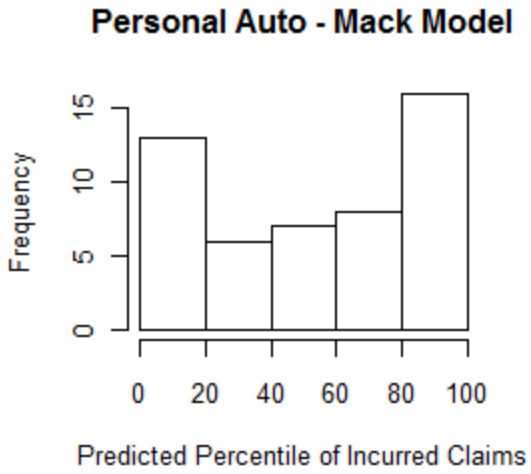
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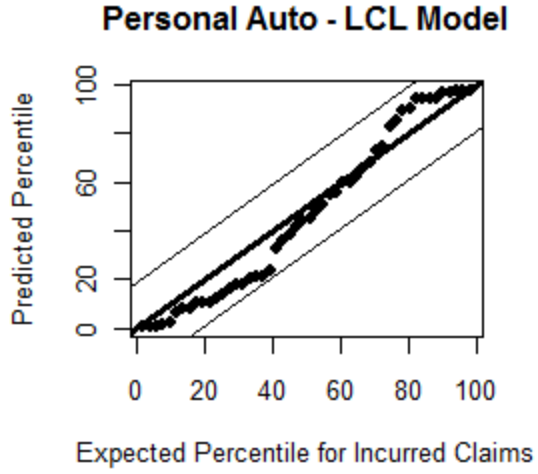
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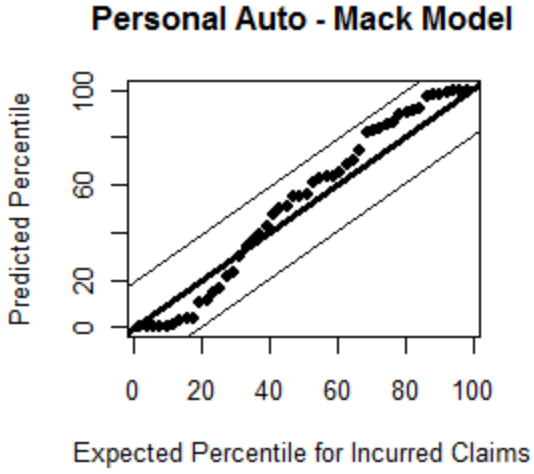
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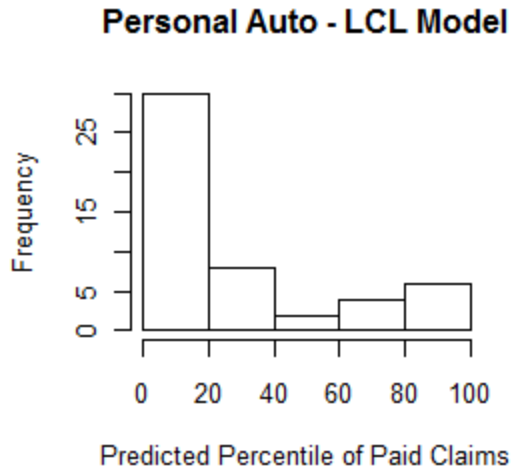
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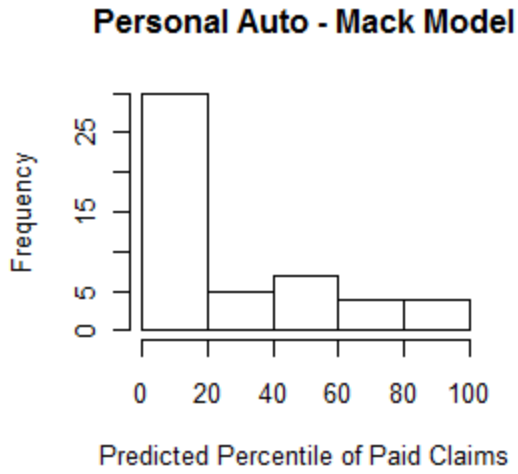
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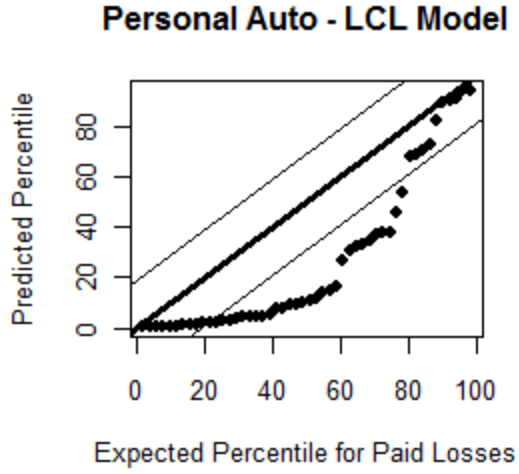
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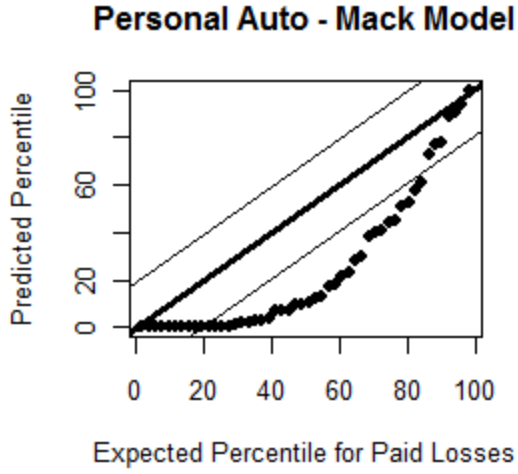
Expected Percentile for Paid Losses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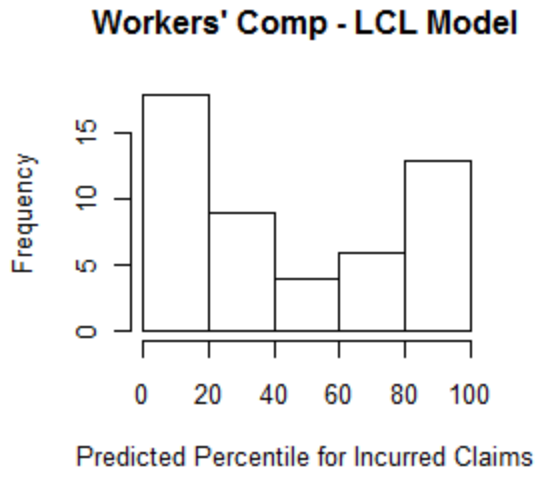
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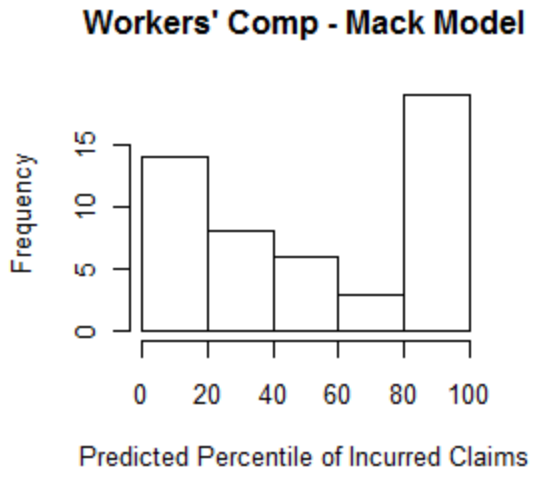
Predicted Percentile for Incurred Claims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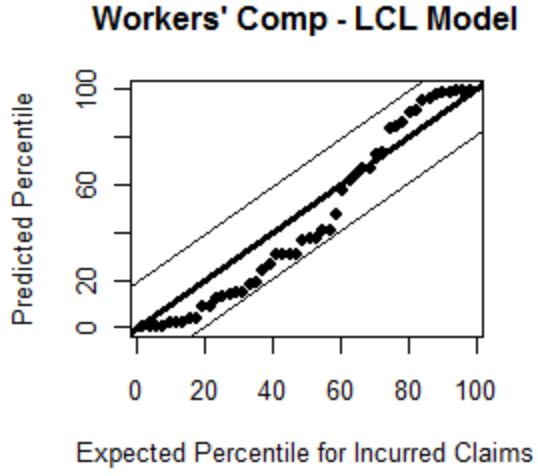
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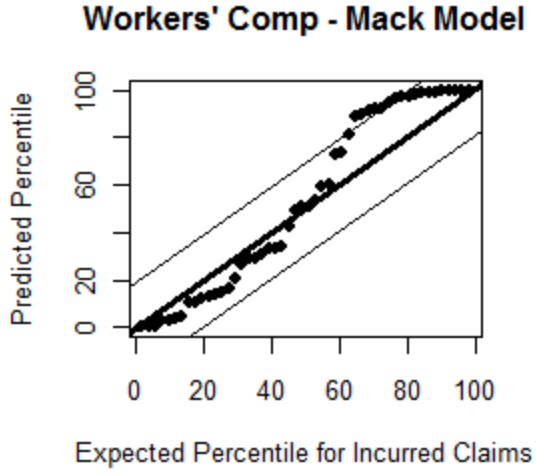
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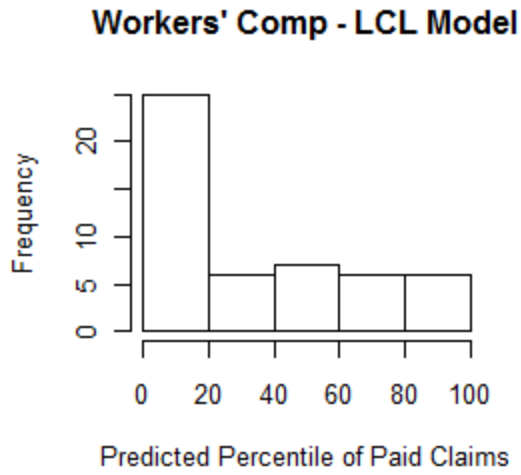
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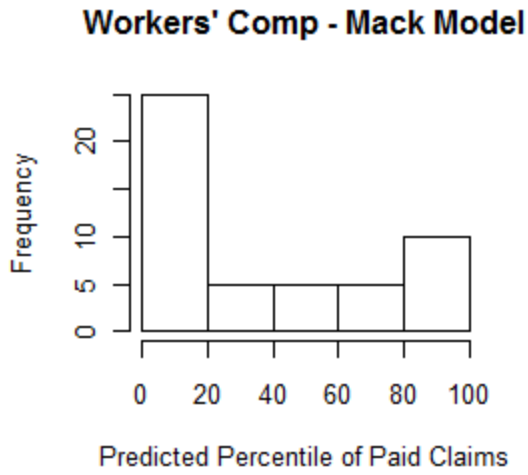
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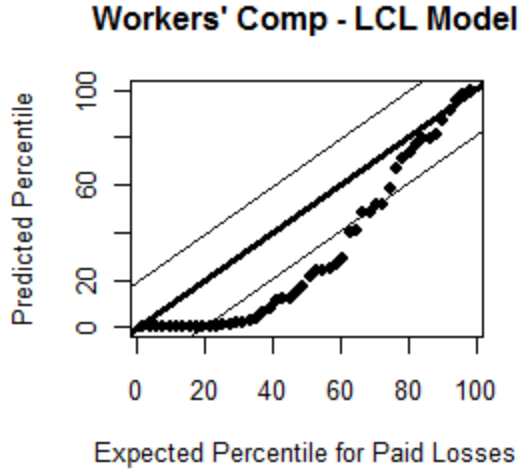
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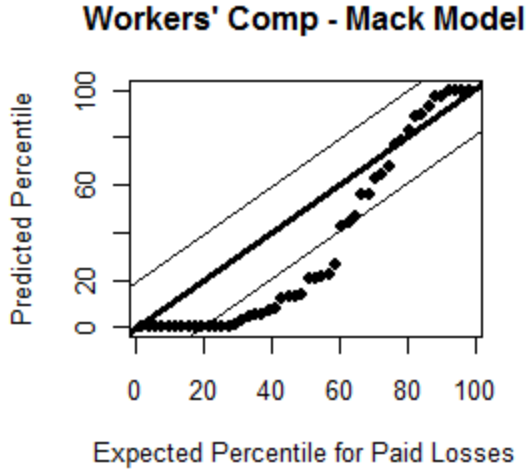
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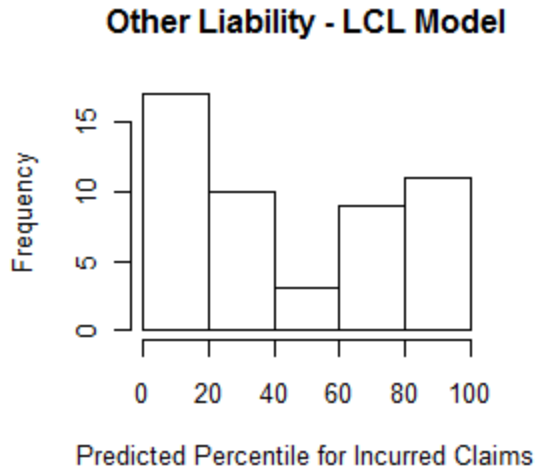
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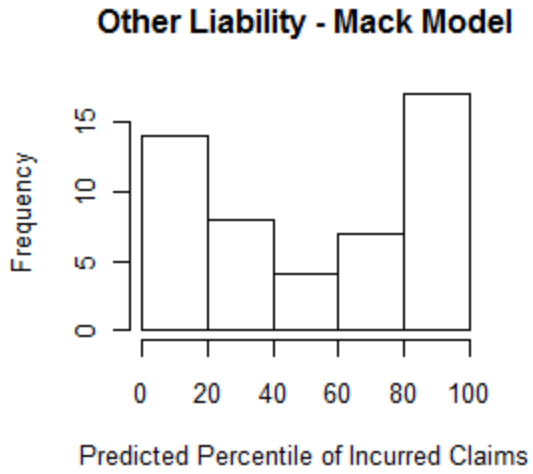
Predicted Percentile for Incurred Claims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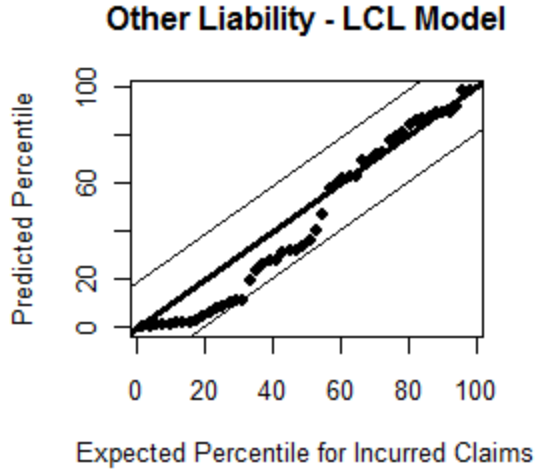
Predicted Percentile of Incurred Claims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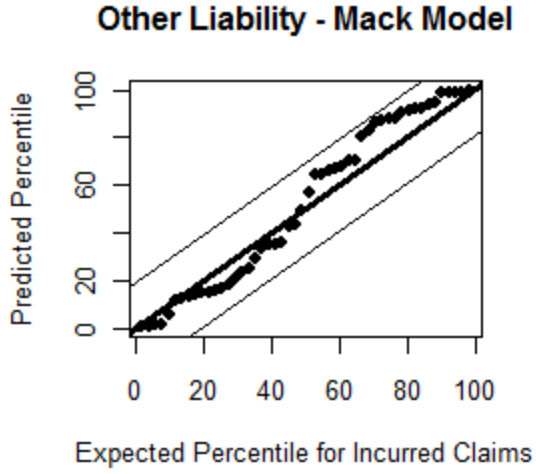
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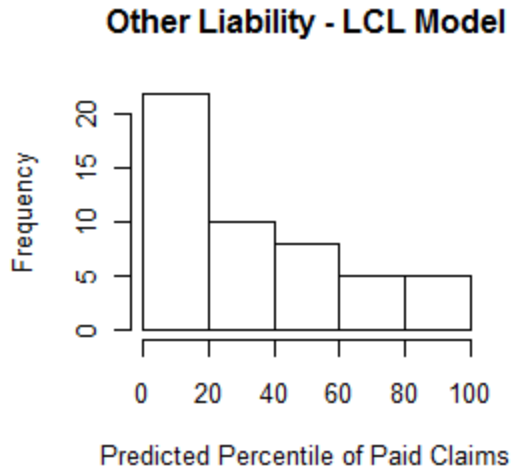
Other Liability - LCL Model
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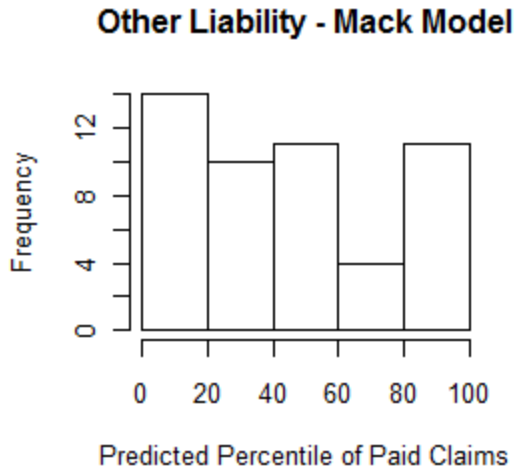
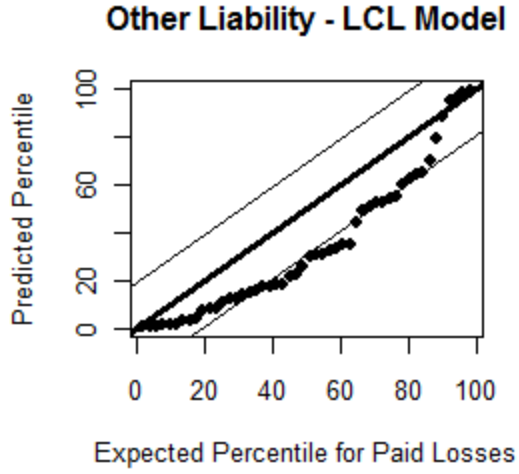
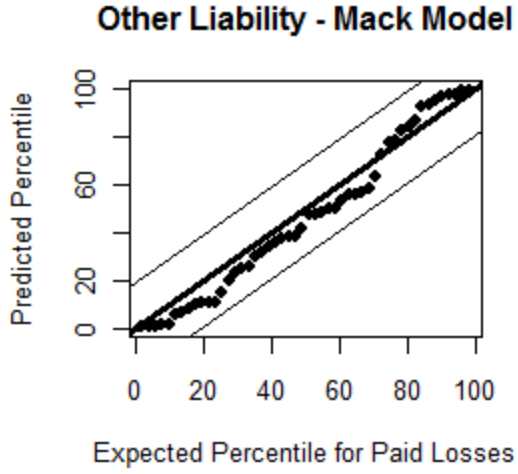
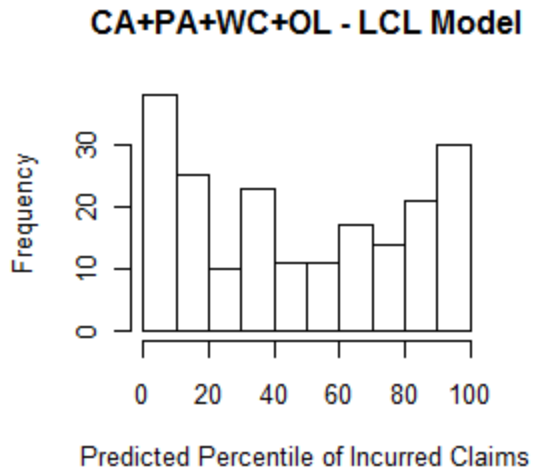
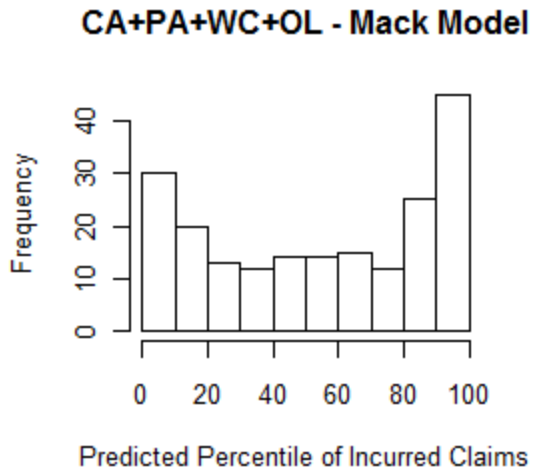
20
0
20
40
60
80
100
Predicted Percentile of Paid Claims

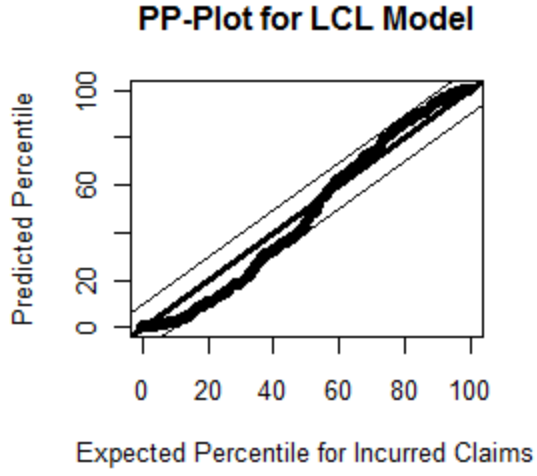

Other Liability - Mack Model
Frequency
0
4
8
12
0
20
40
60
80
100
Predicted Percentile of Paid Claims

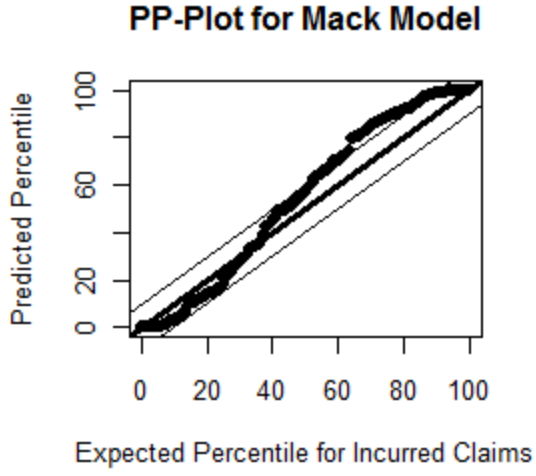

Other Liability - LCL Model
Predicted Percentile
0
20
60
100
0
20
40
60
80
100
Expected Percentile for Paid Losses

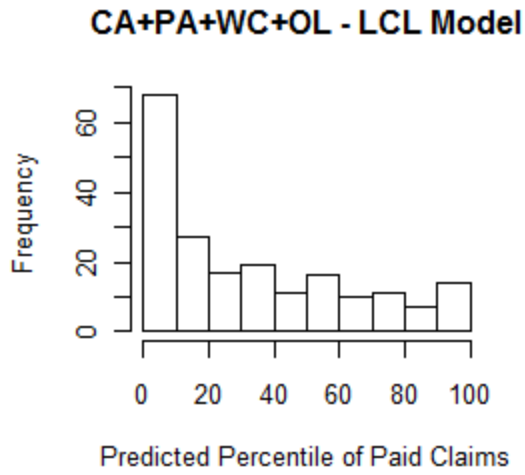

Other Liability - Mack Model
Predicted Percentile
0
20
60
100
0
20
40
60
80
100
Expected Percentile for Paid Losses

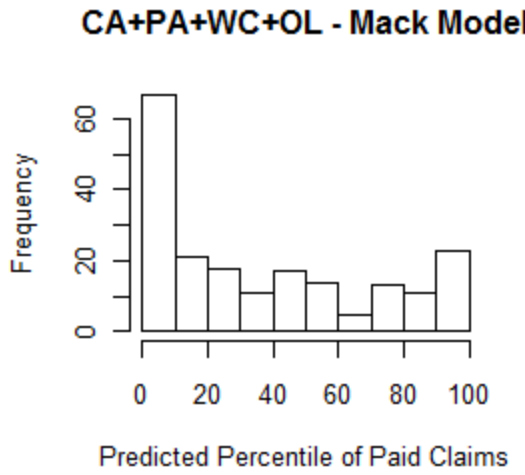

CA+PA+WC+OL - LCL Model
Frequency
0
10
20
30
0
20
40
60
80
100
Predicted Percentile of Incurred Claims

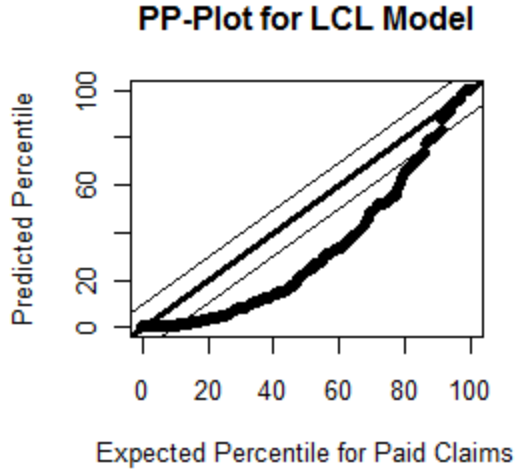

CA+PA+WC+OL - Mack Model
Frequency
0
10
20
30
40
0
20
40
60
80
100
Predicted Percentile of Incurred Claims

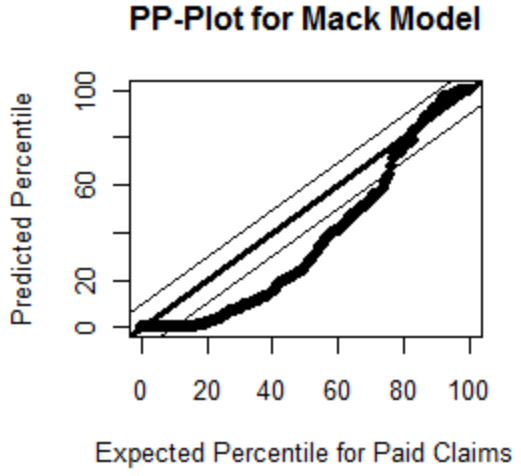

PP-Plot for LCL Model
Predicted Percentile
0
20
60
100
0
20
40
60
80
100
Expected Percentile for Incurred Claims


PP-Plot for Mack Model
Predicted Percentile
0
20
60
100
0
20
40
60
80
100
Expected Percentile for Incurred Claims


CA+PA+WC+OL - LCL Model
Frequency
0
20
40
60
0
20
40
60
80
100
Predicted Percentile of Paid Claims


CA+PA+WC+OL - Mack Model
Frequency
0
20
40
60
0
20
40
60
80
100
Predicted Percentile of Paid Claims


PP-Plot for LCL Model
Predicted Percentile
0
20
60
100
0
20
40
60
80
100
Expected Percentile for Paid Claims


PP-Plot for Mack Model
Predicted Percentile
0
20
60
100
0
20
40
60
80
100
Expected Percentile for Paid Claims

Conclusion

- Level Chain Ladder is an improvement over Mack Chain Ladder on cumulative incurred data.
- The conclusion that the predicted range is too narrow still holds.
- Both models perform poorly on cumulative paid data.