Risk-Adjusted Underwriting Performance Measurement

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Outline

- Focus on profits generated by premium (and its investment)
  - Exclude investment gains on capital
- Focus on ultimate economic profit
  - Trace a policy (or policy year) to its full closure
- Establish a break-even value for the ultimate profit
  - Break-even value is a function of policy risk
- Link the break-even profit to the cost of capital (COC)
  - The COC is a function of policy risk
- Link the two pricing approaches
  - Direct approach: discount loss
  - Indirect approach: solve premium from a target IRR

Motivation – a one-year example

Assume:

- Premium collected at time 0, loss fully paid at time 1
- Loss is random L, mean loss E[L]=100
- Risk-free rate = 4%, premium invested in risk-free

Question: what is the fair profit at time 1?

- Fair premium = 100/(1+0.04) + risk load = 96.15 + risk load
- Assume fair premium = 97.09 = 100/(1+0.03)
- Fair profit = 97.09*(1+0.04) – 100 = 0.97

Risk-adjusted discount rate

- Actual profit = actual premium * (1+r) – actual paid loss
- Profit > 0.97 => profitable; Profit < 0.97 => unprofitable

3%: risk-adjusted discount rate

- Risk-adjusted discount rate ≤ risk-free rate
- Spread = risk-free rate – risk-adjusted discount rate
- Spread ≈ 0 for less risky loss, large for very risky loss

Spread quantifies risk

The break-even profit

- Notation:
  - risk-free rate = r
  - risk-adjusted loss discount rate = r_L
  - usually $r_L < r$, $r - r_L$ = spread
- Profit (random) = $p(1+r) - L$
- Fair premium = $E[L]/(1+r_L) = MV(L)$
- Break-even profit = $MV(L)*(r-r_L)$

A policy is unprofitable, if actual profit < break-even profit

- Reasons for unprofitability:
  - Actual premium < MV(L)
  - Actual paid > E[L]

The cost of capital

Assume:

- Initial capital = c
- Capital invested in risk-free

- Profit generated by premium = $p(1+r) - L$
- Profit generated by capital = cr
- Break-even profit = $E[L]/(1+r_L)*(r-r_L) + cr$
- Break-even return on capital = $r + 1/c*MV(L)*(r-r_L) = COC$

COC is an increasing function of spread $r-r_L$ (representing risk) and decreasing function of c

Multiyear model

- Premium p collected at time 0
- Losses paid (random): L_1, L_2, …, L_n
- Capitals carried: c_0, c_1, …, c_{n-1}
  - Redundant capital paid out as dividend
- Assets (premium, capital) invested risk free
  - Constant risk free rate r
- Constant loss discount rate r_L
  - $MV(L) = \sum E[L_i]/(1+r_L)^i$
- Constant tax rate t

Treatment of expenses and taxes

Assume expenses are paid at time 0; premiums are net of expenses

Income tax complicates calculation

- Premium increases by present value of taxes
- Taxable income depends on how reserves are set and capital gains are realized
- Premium needs to cover taxes on income generated by capital investment
  - Less capital => less tax => lower premium => more competitive

Simplified tax calculation

Assume

- Tax loss reserve = expected unpaid losses discounted at r_L
  - This is MV of unpaid losses
- Unrealized capital gain = 0
  - If investable assets = A, then taxable income = Ar and tax = tAr

Capital account activities

Will separately track balances in policy account and capital account

- Capital account balances c_0, c_1, …, c_{n-1}, 0 (given exogenously)
- Investment income: rc_0, rc_1, … , rc_{n-1}
- Dividends: $c_0(1+r)-c_1$, $c_1(1+r)-c_2$, … , $c_{n-2}(1+r)-c_{n-1}$
- Tax (to be covered by premium and its investment): trc_0, trc_1, … , trc_{n-1}

Policy account balances

$A_0 = p$

$A_1 = p(1+r) - L_1$ prem + inv inc - loss

$\quad - t(p(1+r) - L_1 - V_1)$ tax on policy account

$\quad - trc_0$ tax on capital account

$A_2 = \ldots$

$\ldots$

A_n = the terminal asset

Fair premium

Fair premium

$$=MV(L) + tr/(1-t)*(1+r)$$

$$*[\ c_0 + c_1/(1+(1-t)r) + \ldots + c_{n-1}/(1+(1-t)r)^{n-1}\ ]$$

To solve for the fair premium, set the market value of A_n to zero

Break-even terminal assets

a_n = expected terminal assets when premium equals the fair premium

$$a_n = [\ MV(L) - PV^{tax}(L)\ ] * (1-t)(r-r_L)(1+(1-t)r)^n / ((1-t)r-r_L)$$

where $PV^{tax}(L) = \sum E[L_i]/(1+(1-t)r)^i$

Note: capital c_i's don't show up in the formula because of the assumption of constant investment return r and tax rate t

Example - assumptions

Time	Premium	Expense	Loss	Capital
0.0	1000.00	275.00	0.00	428.75
0.5	0.00	150.00	0.00	362.62
1.0	0.00	0.00	0.00	149.53
1.5	0.00	0.00	0.00	122.54
2.0	0.00	0.00	0.00	94.77
2.5	0.00	0.00	0.00	79.84
3.0	0.00	0.00	650.00	0.00

Risk-free rate = 4%

Risk-adjusted loss discount rate = 3%

Tax rate = 35%

Example - results

- Fair premium net of expense = 569.08
- Fair premium including expense = 988.31

  => The 1000 premium is excessive

- Break-even terminal asset = 24.37

Other profit measures

- CY metrics (combined ratio, ROE, etc.) mix up policies issued across years, not useful for pricing or measuring a pricing or underwriting decision
- PY combined ratio: ignores time value of money
- PY economic CR PV(loss+exp)/PV(prem): ignores risk
- ROE or RAROC:
  - Needs a risk-adjusted hurdle rate
  - Does not measure policy account profit
- EVA (=Income – COC*Capital):
  - Needs a risk-adjusted COC
  - Does not measure policy account profit

What determine the fair premium and a_n?

Besides the expected loss and payment pattern, key factors are

1. Risk adjusted loss discount rate r_L
   - $r_L < r$, smaller r_L means higher risk
   - Smaller r_L => greater MV(L) => greater premium
   - Smaller r_L => greater a_n
2. Capital sequence (required or desired): $c_0, c_1, \ldots, c_{n-1}$
   - More capital => higher tax => greater premium
   - More capital means less competitive on the market

Cost of capital (COC)

- At time 0, contributed capital = c_0
- Expected dividend (released capital) stream: $c_0(1+r)-c_1$, $c_1(1+r)-c_2$, … , $c_{n-2}(1+r)-c_{n-1}$
- Final expected dividend: $c_{n-1}(1+r)+a_n$
- The IRR of the expected capital flow

$$c_0 = (c_0(1+r)-c_1)/(1+IRR) + (c_1(1+r)-c_2)/(1+IRR)^2 + \ldots + (c_{n-1}(1+r)+a_n)/(1+IRR)^n$$

COC = the IRR

- Since $a_n > 0$, COC > r
- Greater policy risk => greater a_n and greater COC

Example (cont'd) – COC

Time	Premium	Expense	Loss	Capital
0.0	1000.00	275.00	0.00	428.75
0.5	0.00	150.00	0.00	362.62
1.0	0.00	0.00	0.00	149.53
1.5	0.00	0.00	0.00	122.54
2.0	0.00	0.00	0.00	94.77
2.5	0.00	0.00	0.00	79.84
3.0	0.00	0.00	650.00	0.00

- $a_n = 24.37$
- COC = 5.62%

- Compute the IRR of the actual capital flow. Operation is profitable ⇔ the IRR > 5.62%

The COC is determined by risk of operation

- Policy risk quantified by the spread $r - r_L$: greater spread ⇔ greater risk
- Total risk of operation also affected by amount of capital: less capital $c_0, c_1, \ldots, c_{n-1}$ ⇔ greater risk

- The COC is determined internally
- If the shareholder wants higher return, management can
  - Write more risky business
  - Reduce capital held
  - Look for mispriced policies and investments (cannot rely on it)

Pricing – calculation of MV(L)

Suppose (1) expected loss and (2) payout pattern are accurately obtained (not trivial, of course). Our concern: how to calculate MV(L)

In general,

$$\text{Premium} = MV(L) + PV(\text{expense}) + PV(\text{tax})$$

Under our assumptions,

$$\text{Premium} = MV(L) + tr/(1-t)*(1+r)$$
$$*[\ c_0 + c_1/(1+(1-t)r) + \ldots + c_{n-1}/(1+(1-t)r)^{n-1}\ ]$$

Two approaches

The direct approach

$MV(L) = \sum E[L_i]/(1+r_L)^i$ – r_L = ?

$= \sum E[L_i]/(1+r)^i$ + Risk load – Risk load = ?

The indirect approach: MV(L) back-solved from a target IRR for the dividend flow

- Key input is the target IRR, should be equal to the COC

The two approaches should give the same MV(L)

Example (cont'd)

Scenario	r_L	IRR	MV(L)
More risky	3%	5.62%	544.36
Less risky	3.39%	5%	532.26

To get the correct MV(L) or premium, have to use the correct IRR

Cannot "set" an arbitrary target IRR, need to start from analyzing risk

Further research

Formulas clean and simple for performance measurement and premium calculation.

Key inputs – how to get them?

- r_L or risk load
- Target IRR = the COC
- Capital allocated to policy c_0, c_1, …, c_{n-1}

Appendix – formulas for the indirect approach

Step 1. For a given target IRR, solve a_n from the following

$$c_0 = (c_0(1+r)-c_1)/(1+IRR) + (c_1(1+r)-c_2)/(1+IRR)^2 + \ldots + (c_{n-1}(1+r)+a_n)/(1+IRR)^n$$

Step 2. Solve MV(L) from

$$a_n = [\ MV(L) - PV^{tax}(L)\ ] \times (1-t)(r-r_L)(1+(1-t)r)^n / ((1-t)r-r_L)$$

Step 3. Calculate the fair premium (net of expense)

$$Premium = MV(L) + tr/(1-t)*(1+r) *[\ c_0 + c_1/(1+(1-t)r) + \ldots + c_{n-1}/(1+(1-t)r)^{n-1}\ ]$$