Validating Stochastic Scenario Sets by Comparing to History

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Validation Context

- Looking at stochastic generators that generate thousands of scenarios
- Right now focusing on interest rates
- Risk management context – risk to portfolio value (including possibly liability proxies) of holding a particular mix of assets – need probability distribution, often tail focus
- This part of validation looks only at the scenarios, not the generating process
- Looks for realism of distribution of yield curves compared to historical statistical properties – statistical properties related to getting a realistic distribution of changes in portfolio values – such as term structure of volatility, distribution of shapes, etc.
- Did historical statistical study for US, UK, Germany, Japan but will show US results
- For purpose of illustrating testing methodology only, look here at three scenario sets that were available:
  - BK model – Calibrated to market prices of swaptions thus has high volatility
    - Issue with BK is it is popular for risk-neutral scenarios for pricing but real-world??
  - 3 factor CIR model with Hull-White-like adjustment – good for real world but risk-neutral??
  - Free public model from American Academy of Actuaries (AAA) – aiming at long-term simulation for reserving but will test to see if it can also be useful for portfolio risk analysis
- Statistical properties were compiled over 50 years as well as for last 6 years, where rates were lower – stochastic scenarios can be compared to both

Arbitrage-Free Scenario Sets – Risk-Free Yield Case

- Arbitrage means making a net investment of $0 by borrowing and investing in various instruments that have an expected profit and no possibility of loss.
- Arbitrage-free scenario sets are important: scenarios sets that allow arbitrage are not realistic and can distort analysis – e.g., ALM would say to take the arbitrage
- Consider strategy of borrowing at 1Y rate, buy a 5Y bond and sell in 1 year – often would make a profit but could lose money if bond goes down enough in value
- If 4Y rate one year later is lower than current implied 4y forward rate in *every* simulated scenario, model says there is no possibility this strategy can have a loss – it is arbitrage
- Real arbitrage problems in models likely to be more complicated than this
- But having such strategies available would pull investment strategy to them
- So for real-world risk analysis, need set of yield curves to be arbitrage-free
- Modeling issue is to make the dynamic evolution of yields over time consistent with the cross-sectional shape of the yield curve at any point in time
- Dangerous to do ALM analysis using model that like AAA is not arbitrage-free – but that model now has an arbitrage-free extension
- Arbitrage-free generation is embedded in CIR and BK models
- Even if don't allow borrowing as an investment strategy, including liabilities in ALM is like borrowing

Model 1. *Extended 2 Factor Black-Karasinski*

- Instantaneous short rate stochastic model

$$d \log r(t) = a[m(t) - \log r(t)]dt + c dW_r(t)$$

$$d\, m(t) = b[\mu \quad - m(t)]dt + s dW_m(t)$$

- Rate reverts to temporary mean that itself reverts to long term mean, which can control short rates and long rates behavior
- Simulated nominal rates are strictly positive (log-normally distributed)
- Fits the initial yield curve by adding a deterministic function of time
- Flexible price of risk structure to match interest rates properties
- Does not have closed form solutions for yield curve so some work to simulate
- Calibration targets (B&H)
  - Initial yield curve
  - Long term rates level
  - Dispersion or volatility around long-term mean at given horizons
  - Volatility of changes in rates while approaching horizon
  - Correlation between short and long rates
- No target for tail behavior

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Model 2. *3-Factor Cox-Ingersoll-Ross (CIR) model*

- Instantaneous short rate stochastic model – sum of 3 unobserved CIR processes

$$d\, r_i(t) = [b_i + a_i r_i(t)]dt + \sigma_i r_i(t)^{0.5} dW_i(t)$$

- $r(t) = r_1(t) + r_2(t) + r_3(t) + \phi(t)$, $\phi(t)$ – deterministic function to match initial yields
  - Mean reverting, controls short rate dynamics
  - Simulated nominal rates are strictly positive before deterministic shift
  - Fits the initial yield curve by $\phi(t)$ (BK does similar)
  - Flexible price of risk structure to match interest rates properties
  - Provides closed form solutions to whole yield curve
  - Same equation for real-world and risk-neutral rates, but dt drift higher for risk neutral
  - Same thing holds for BK
- Calibration targets (AIG implementation)
  - Expected curve shape at the horizon
  - Term structure of volatility at the horizon
  - Correlations between selected rates
  - Regression of long spreads over short rates

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Model 3. *AAA Model*

- r_t is 20 year rate at time t, $i_t = \log[r_t]$, short 1y rate = $r_t - \alpha_t$. The 20 year rate has stochastic volatility σ_t with $\nu_t = \log[\sigma_t]$. These all evolve as:

$$\nu_t = \sigma_\nu r_{t-1}^{\gamma} Z_{\nu t} + 0.96\nu_{t-1} + .04\bar{\nu}$$

$$i_t = \sigma_t Z_{it} + \max(min\{\langle 0.995 i_{t-1} + 0.005\bar{\imath} + [\bar{\alpha} - \alpha_{t-1}]\beta_\alpha\rangle, i_{max}\}, i_{min})$$

$$\alpha_t = \sigma_\alpha r_t Z_{\alpha t} + 0.975\alpha_{t-1} + 0.025\bar{\alpha} + [i_{t-1} - \bar{\imath}]\beta_i$$

- Subject to min and max, i slowly reverts to its mean, with a drift up or down depending if slope α is below or above its mean. The drift of α responds similarly to i.
- Then all the other rates are generated by Nelson-Siegel functions that go through these two rates. For any tenor $r(t)$ at time T:

$$r(t) = b0 + b1\left(\frac{1-e^{-kt}}{kt}\right) + \left(\frac{b2}{k}\right)\left(\left(\frac{1-e^{-kt}}{kt}\right) - e^{-kt}\right)$$

- Here *b0* and *b1* are selected to match 1y and 20y rates, while *b2* =0, *k* is calibrated from the historical data, currently $k = 0.4$
- Parameters are calibrated to long historical series and economical projections
- Matches today's 1y and 20y rates (i_0 and α_0)
- Used for multi-decade projections

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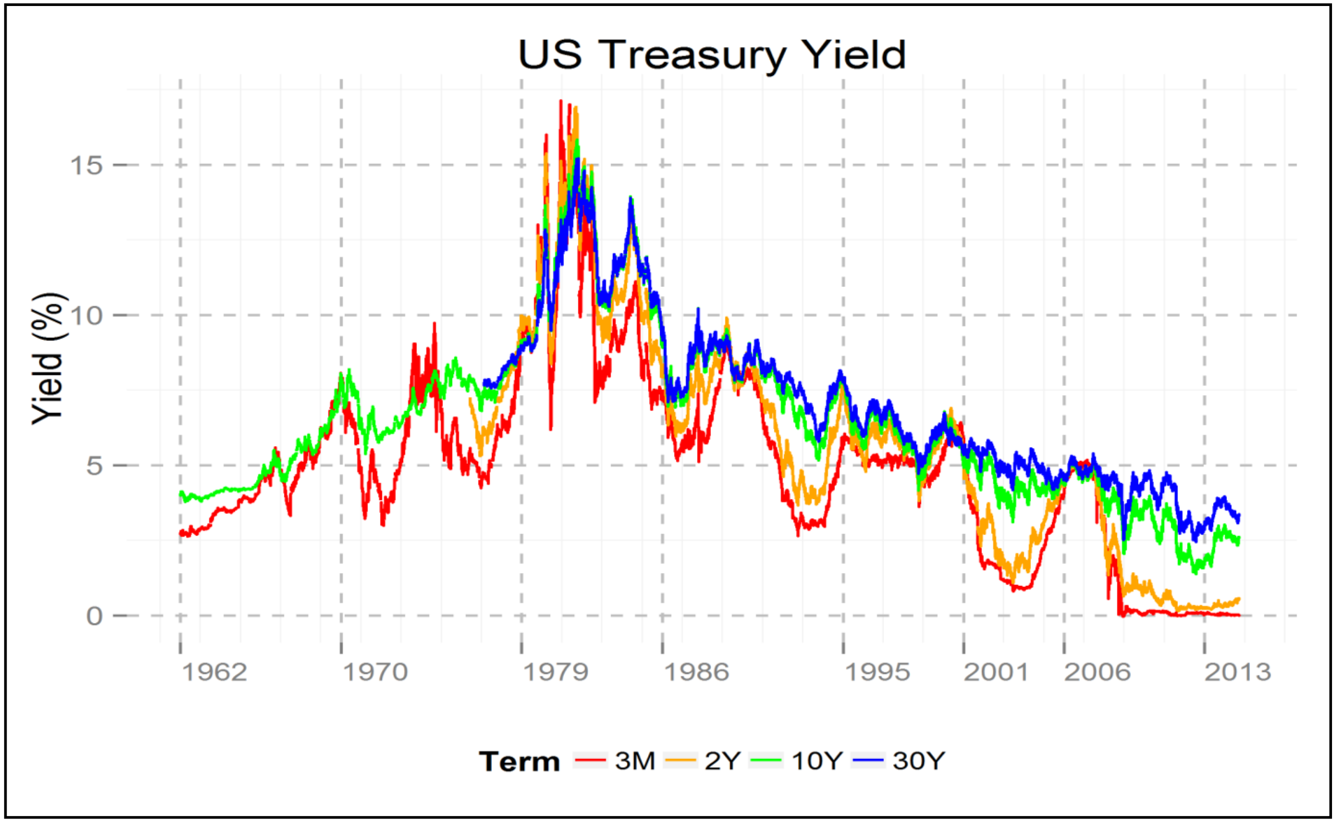
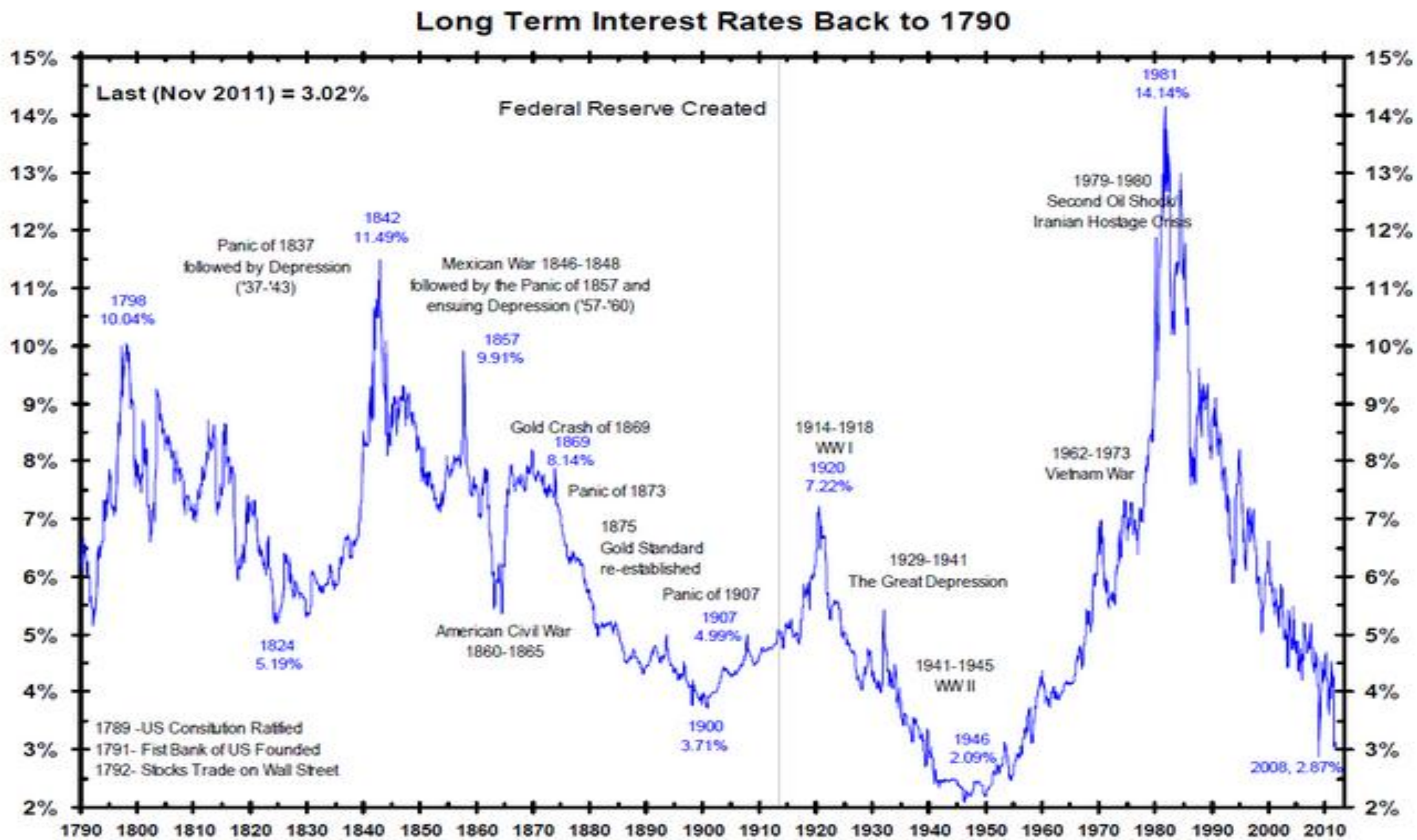
History of US Treasury Rates 1962-present

- **Interest rates today are at historically low levels**
- **Negative short rates have been observed**
- **Some interest rate models cannot handle new rates environment accurately**



History of US Interest Rates 1790 - Present

- **Long cycles – 50 – 75 years**
- **1940's look a lot like 2010's – both slow economic recovery eras**
- **Rates were pretty low for 30 years last time – so now low simulated scenarios reasonable**



Source: What Drives The Bond Market?
Chicago CFA Handout by Bianco Research LLC
January 18, 2011

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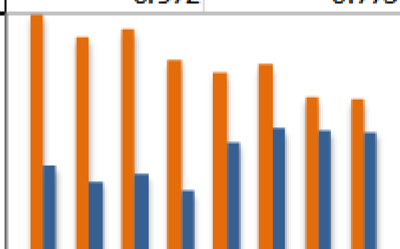
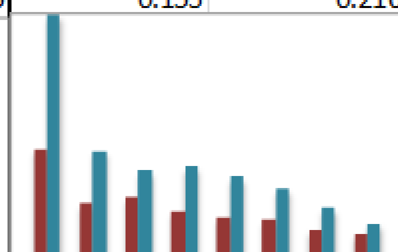
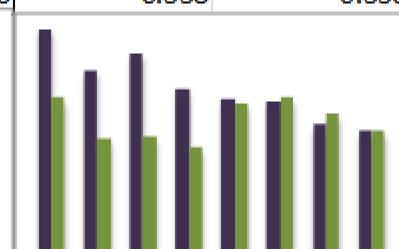
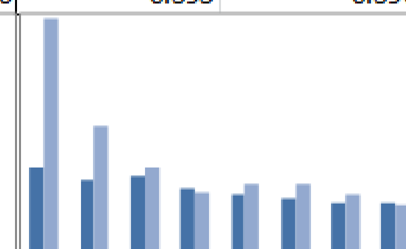
Facts 1. Term Structure of Volatility

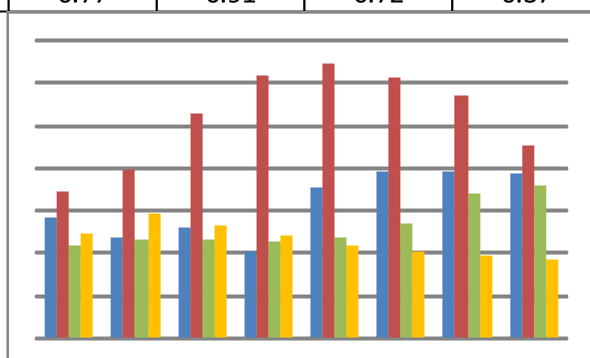
- Historical annual volatilities downward sloping across maturities
- Low rates now: shape more complicated – but log volatilities opposite change
- Looking for more stable volatility pattern over time led to look at 2/3 power of rates – especially mid and long term bond volatility term structures look the same historically and recently by this measure
- Similar story for standard deviation divided by the square root of the rate, as in CIR model

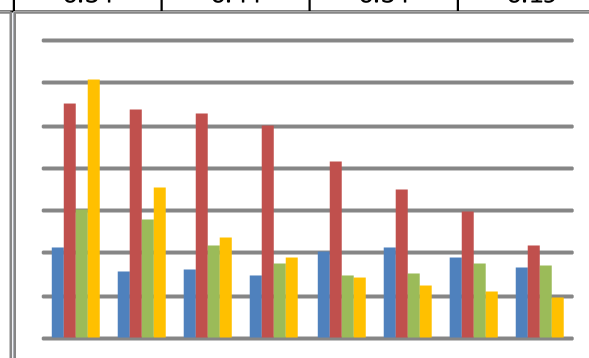
	Abs Vol (%)	Since Jan 2009	Log Vol	Since Jan 2009	Abs Vol of 2/3 Power of Rates (%)	Since Jan 2009	Abs Vol / Sqrt Rate	Since Jan 2009
3m	1.487	0.569	0.669	1.482	0.607	0.424	0.678	1.833
1y	1.345	0.472	0.335	0.657	0.498	0.316	0.579	0.987
2y	1.395	0.516	0.379	0.534	0.543	0.322	0.612	0.675
3y	1.209	0.411	0.288	0.560	0.449	0.294	0.512	0.475
5y	1.130	0.708	0.258	0.498	0.422	0.408	0.474	0.544
7y	1.179	0.788	0.242	0.430	0.416	0.424	0.438	0.545
10y	0.984	0.782	0.179	0.311	0.354	0.380	0.405	0.458
30y	0.972	0.775	0.153	0.216	0.338	0.336	0.395	0.390
Charts								

Test 1. Volatility Term Structure

- Since some tenors have higher volatilities, parallel shifts are rare
- Getting realistic volatilities by tenor is one key step to getting a reasonable distribution of portfolio values
- For absolute volatilities BK is high, and at the long end CIR a bit low and AAA quite low
- For the more historically stable 2/3 power volatilities, CIR is fairly similar to both the long-term and recent volatilities while BK is much more volatile than either
- AAA is too volatile at the short end and not volatile enough at the long end

	Absolute Volatility (%)				Volatility of R^2/3 (%)			
	2009+	E2BK	CIR3	AAA	2009+	E2BK	CIR3	AAA
3m	0.57	0.69	0.44	0.50	0.42	1.10	0.60	1.21
1y	0.47	0.80	0.46	0.59	0.32	1.08	0.56	0.71
2y	0.52	1.06	0.47	0.53	0.32	1.06	0.43	0.47
3y	0.41	1.24	0.46	0.49	0.29	1.00	0.35	0.38
5y	0.71	1.29	0.47	0.43	0.41	0.83	0.29	0.29
7y	0.79	1.22	0.54	0.41	0.42	0.70	0.30	0.25
10y	0.78	1.14	0.68	0.39	0.38	0.60	0.35	0.22
30y	0.77	0.91	0.72	0.37	0.34	0.44	0.34	0.19

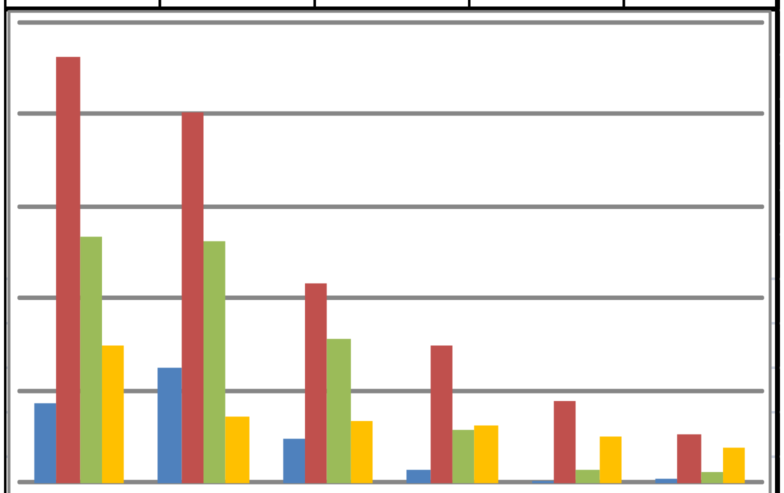




Test 1. Related test - Skewness

- For tail of distribution, skewness gives another test of reasonableness of simulations
- BK very highly skewed – would make tail too heavy even if volatilities were ok
- CIR also more skewed than history
- AAA best fit to history for shorter rates; CIR best for longer

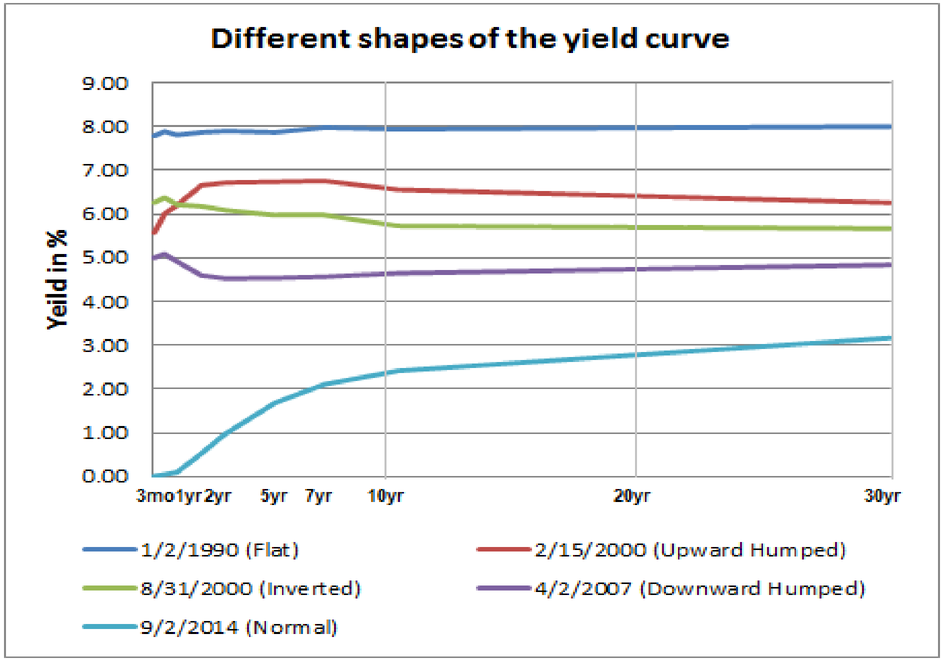
Skewness				
Tenor	2009+	E2BK	CIR3	AAA
3 MO	0.85	4.63	2.66	1.50
1 YR	1.24	4.01	2.61	0.71
3 YR	0.47	2.17	1.56	0.68
5 YR	0.14	1.48	0.56	0.62
10 YR	0.02	0.88	0.13	0.49
30 YR	0.05	0.51	0.12	0.37



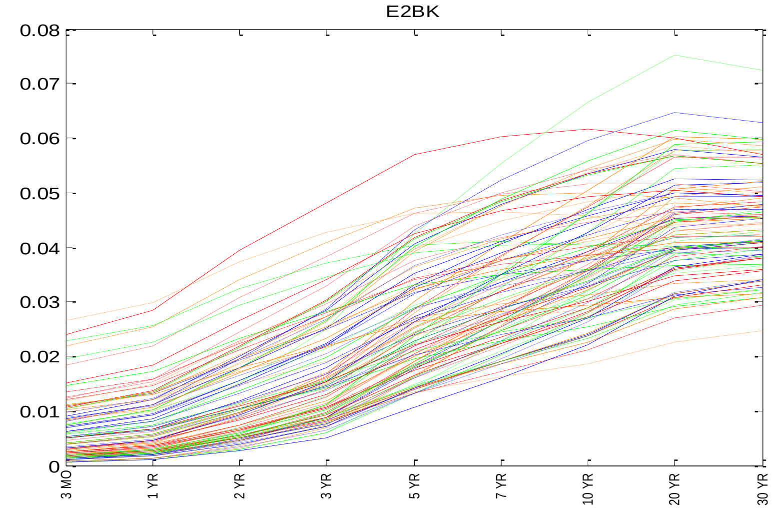
Historical Curve Shapes

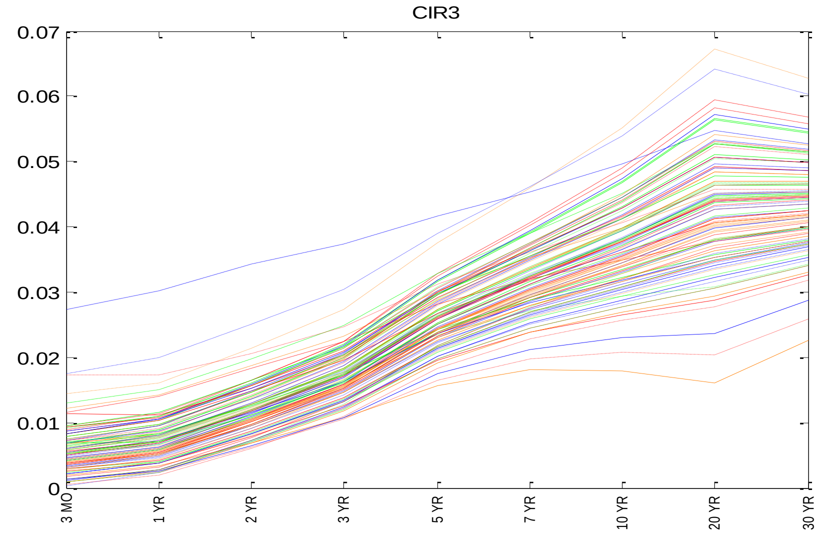
Yields curves are shaped by investors' expectations and risk premiums driven by economic conditions – short end driven largely by central bank, long end by inflationary expectations and risk attitudes

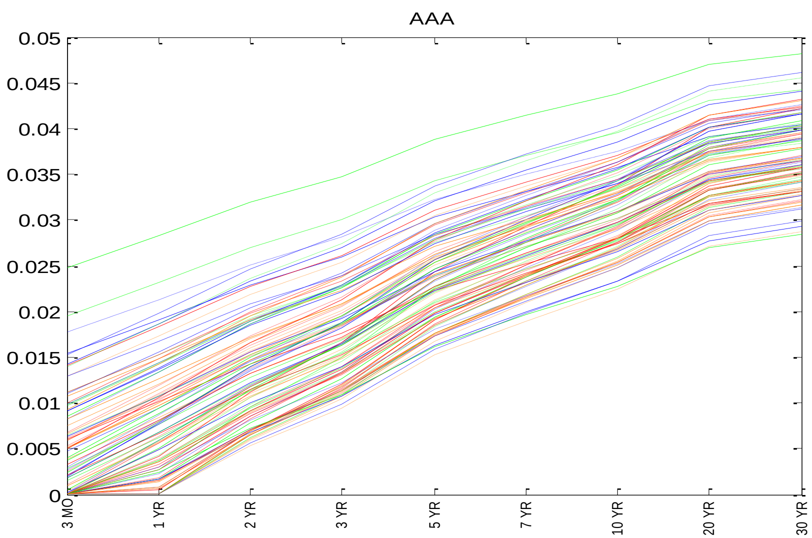
- Normal, upward sloping curve – accommodative Fed policy with fairly low short rate
- Inverted, flat or downward humped curve – quite tight Fed policy pushing short rate as high or higher than long rate, with different market views of mid rates
- Upward humped curve – medium tightness with current and mid-term risky outlook, so move to long rates becomes low risk strategy; often transitional between normal and flat

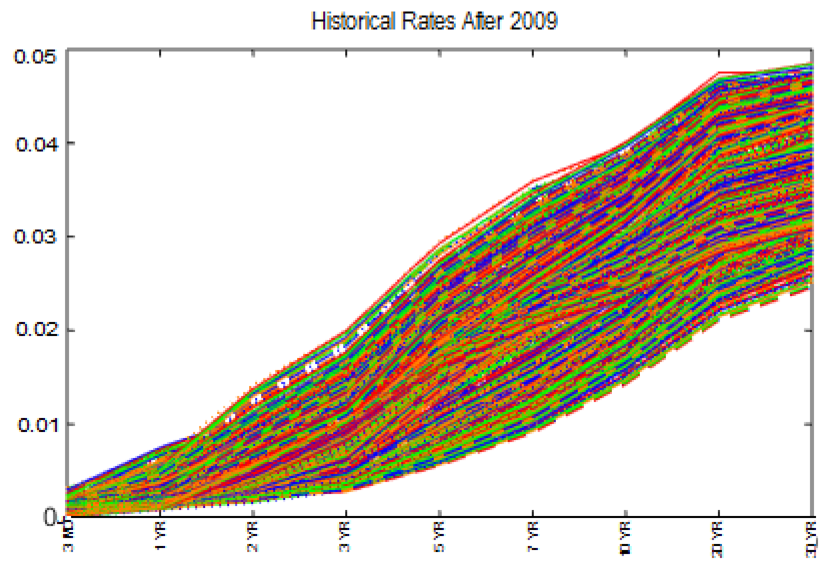


Simulated Curve shapes









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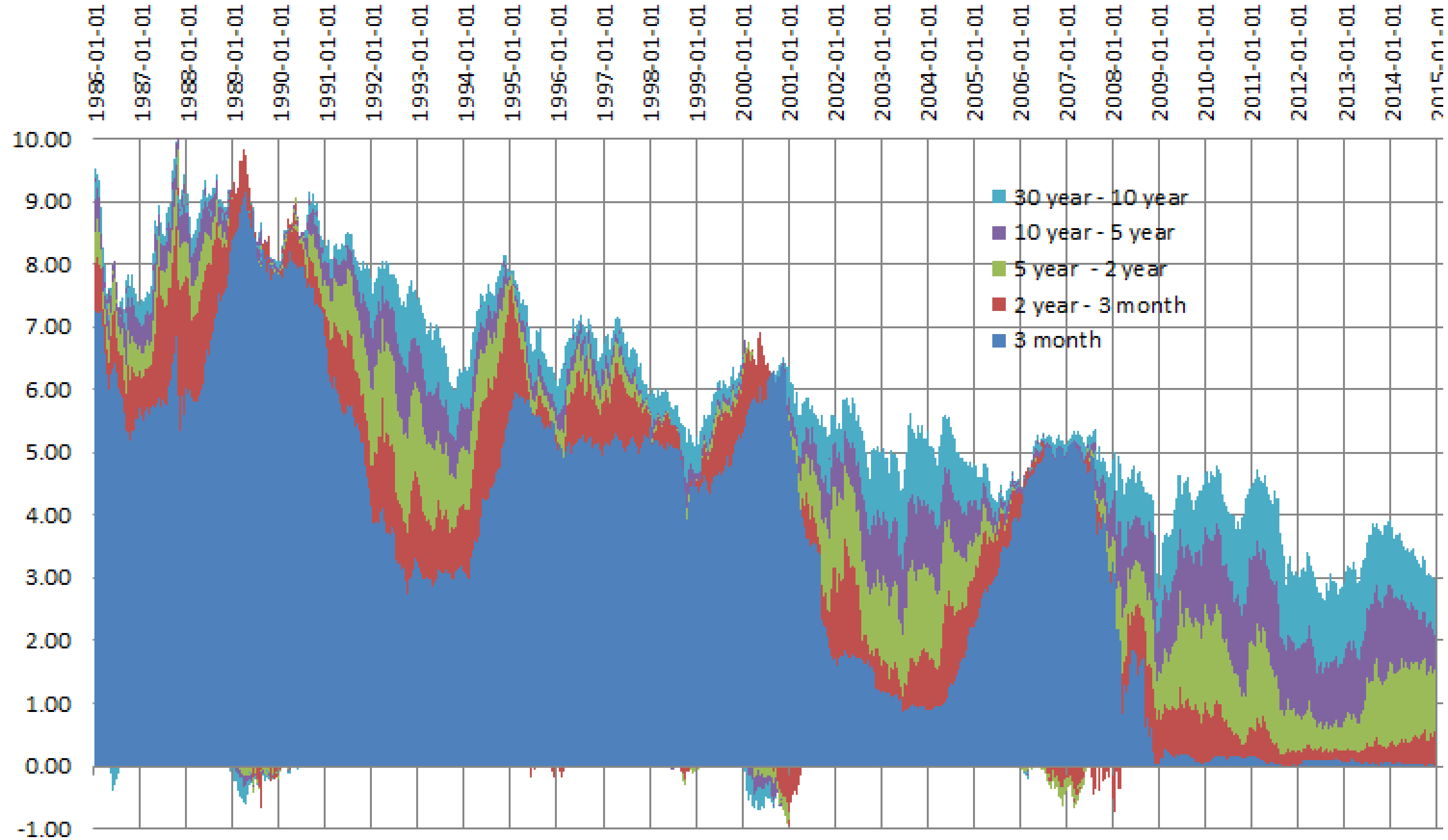
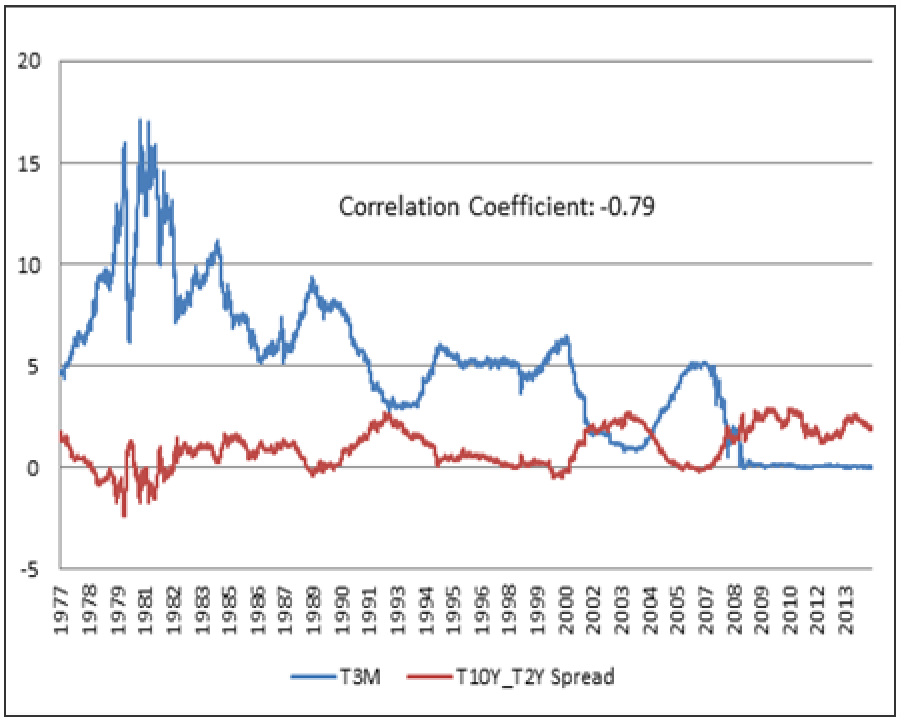
History of Spreads

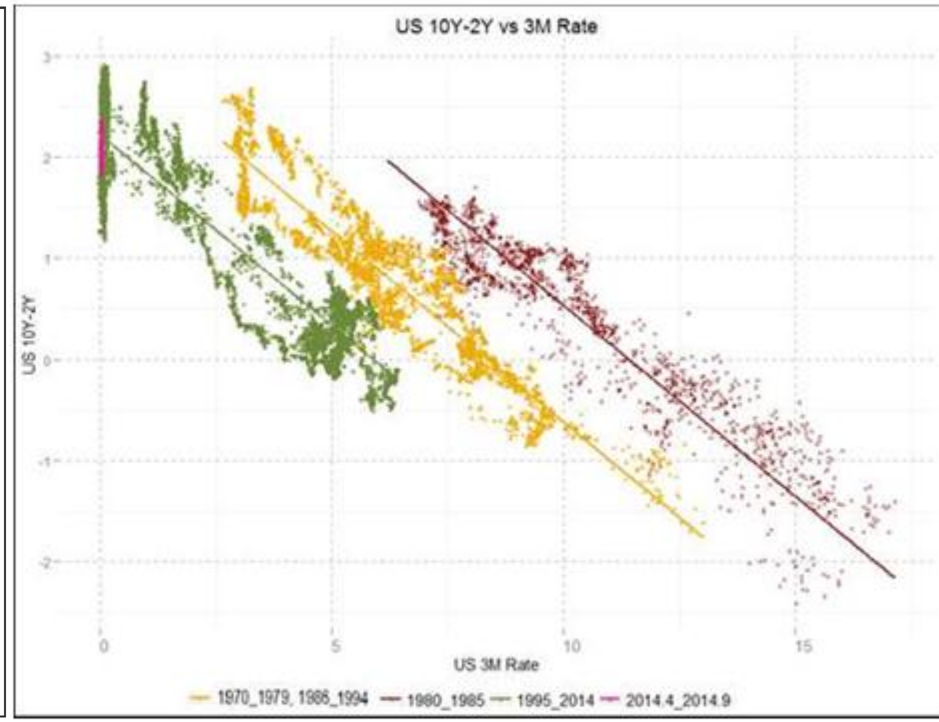
- Rates have been moving down over this period.
- The short rate moves up and down in waves, with spreads moving in the opposite direction, so spreads widen when the short rate decreases and narrows when it increases. Thus the yield curve flattens when the short rate goes up and steepens when it declines. Thus the longer rates have not moved as much as the shorter rates have over the period.



Curve Shapes – Distribution of Spreads around Typical

- Typical 10 – 2 spread defined using regression against 3 month rate
- Combined measure with 3 points on curve defines basic shape – Higher short rates, lower spreads
- Regressions for several historical periods but same slope used for all
- Different intercepts probably relate to inflationary expectations
- Residual distributions measured for total period





Test 2. Shape of Yield Curves

- Intercepts all fairly reasonable; slopes all used identical historical slopes
- Standard deviations of spreads very high for BK, so it generates many unrealistic yield curves, which can distort distribution of asset values
- About right for CIR for 10 – 2 and curvature but variability too low for 30 – 10
- Upper and lower percentiles for BK too extreme so tails have too many overly steep curves and too many overly flat; just the opposite for AAA – not enough different curve shapes; CIR 30 – 10 spreads too constant but other two spreads have reasonable percentiles.

	Model	Intercept	Beta	Res_SD	Skew
Spread 10Y-2Y (%)	Historical	2.11	-0.39	0.39	-0.45
	E2BK	2.74	-0.39	0.70	0.61
	CIR3	2.74	-0.39	0.46	0.06
	AAA	1.92	-0.39	0.16	0.31
Spread 30Y-10Y (%)	Historical	0.96	-0.14	0.16	-0.11
	E2BK	0.75	-0.14	0.35	-0.54
	CIR3	0.75	-0.14	0.08	-0.08
	AAA	0.70	-0.14	0.06	0.31
Curvature (%)	Historical	-1.16	0.25	0.38	0.54
	E2BK	-1.99	0.25	0.77	-0.85
	CIR3	-1.99	0.25	0.38	-0.07
	AAA	-1.22	0.25	0.10	-0.31

	Model	p0.5%	p2.5%	p5%	p25%	p50%	p75%	p95%	p97.5%	p99.5%
Spread 10Y-2Y (%)	Historical	-1.23	-0.83	-0.70	-0.26	0.02	0.29	0.59	0.68	0.80
	E2BK	-1.46	-1.17	-1.02	-0.49	-0.07	0.42	1.26	1.58	2.27
	CIR3	-1.17	-0.89	-0.75	-0.31	-0.01	0.31	0.77	0.93	1.21
	AAA	-0.38	-0.30	-0.25	-0.11	-0.01	0.10	0.28	0.34	0.46
Spread 30Y-10Y (%)	Historical	-0.50	-0.35	-0.26	-0.10	0.00	0.10	0.26	0.31	0.47
	E2BK	-1.14	-0.78	-0.61	-0.21	0.03	0.24	0.53	0.62	0.81
	CIR3	-0.21	-0.15	-0.13	-0.05	0.00	0.05	0.13	0.15	0.20
	AAA	-0.14	-0.11	-0.09	-0.04	0.00	0.04	0.10	0.12	0.17
Curvature (%)	Historical	-0.80	-0.63	-0.54	-0.28	-0.03	0.21	0.71	0.91	1.12
	E2BK	-2.64	-1.77	-1.42	-0.43	0.10	0.54	1.07	1.21	1.44
	CIR3	-1.01	-0.77	-0.65	-0.26	0.01	0.26	0.62	0.74	0.97
	AAA	-0.29	-0.21	-0.18	-0.06	0.01	0.07	0.16	0.19	0.24

Relevance of historical facts and tests

Most of the models can not pass all tests. What test is the most significant? It depends on purpose. Consider fixed income asset portfolio and ALM risk or economic capital analysis. Does the duration-matched asset - liability portfolio have interest rate risk?

Duration – measures portfolio sensitivity to parallel shift of the yield curve

Even the duration-matched assets – liabilities portfolio may have sensitivities to non-parallel yield curves shifts

Non parallel movements of yield curves could be modeled through the individual shifts for different tenors (key rates). Key rates are chosen to fit for purpose, say 6M, 1Y, 3Y, 7Y, 10Y, 20Y, 30Y

Key rate duration is sensitivity of portfolio value to a small change in a single key rate. $KRD_i = -\frac{1}{P}\frac{\Delta P_i}{\Delta r_i}$

Change in the total portfolio value for non-parallel shifts of the entire yield curve (different Δr_i by i gives non-parallel)

$$\Delta P = -P * \{\sum KRD_i * \Delta r_i + 0.5 * Convexity * (\Delta r_D)^2\}$$

Volatilities of the key rates is the main factor for risk and economic capital analysis
Key rates movements are not independent. Correlations explaining co-movements between key rates is another factor

$$vol(\Delta P) = P\sqrt{\sum_{i=1}^{M}\sum_{j=1}^{M} KRD_i * KRD_j * vol(\Delta r_i) * vol(\Delta r_j) * corr(\Delta r_i, \Delta r_j)}$$

For P&C insurance significant key rates for liabilities and matching assets lay within 3Y-15Y.

For trading strategies curve shapes could be the most significant test.

A similar approach is called economic duration – for an important key rate like 5 year, assume a 25 basis point shift, and for all the other tenors estimate the expected shift conditional on that either from the ESG or history, and compute the resulting change in portfolio value. This can be scaled up or down for a larger or smaller shift, and can be extended to include expected shifts in credit spreads, equity values and ABS drivers.

Having realistic curve shapes, rate volatilities and correlations are important for doing this from an ESG

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Facts and Test 3. Correlations

- Correlations across rates are another area that could affect realism of the distribution of asset values
- For medium and longer tenors, BK correlations tend to be too low, while overall those from CIR are slightly high but about the same as 2009-2014.
- Traditional portfolio sensitivity to interest rates (duration, IR01) assume 100% correlation of tenors.
- Partial correlation methods, such as empirical duration and correlated key rate durations, seek more realistic sensitivities by using plausible correlations – could do from ESG.
- Slightly high correlations like CIR's would give sensitivities between traditional and realistic; AAA a bit higher than CIR
- Low correlations like BK's would not give meaningful sensitivities of asset values to changes in yields

Historical 2009-2014

Tenor	3y	5y	10y	30y
3y	1			
5y	0.95	1		
10y	0.85	0.96	1	
30y	0.7	0.86	0.96	1

E2BK

Tenor	3y	5y	10y	30y
3y	1			
5y	0.97	1		
10y	0.78	0.90	1	
30y	0.46	0.64	0.91	1

CIR3

Tenor	3y	5y	10y	30y
3y	1			
5y	0.97	1		
10y	0.84	0.95	1	
30y	0.77	0.91	0.99	1

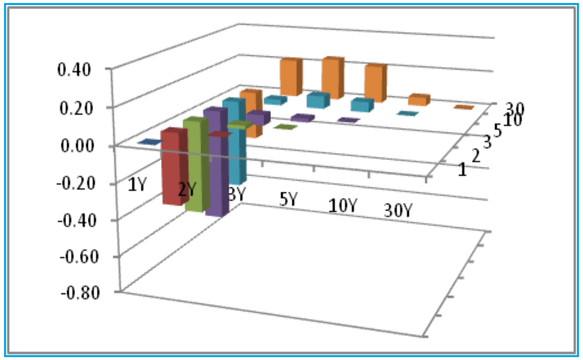
AAA

Tenor	3y	5y	10y	30y
3y	1			
5y	0.99	1		
10y	0.92	0.97	1	
30y	0.80	0.88	0.97	1

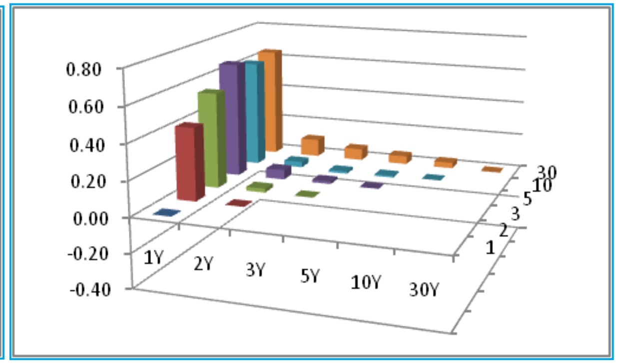
Correlation Tests – Model Compared to History

- CIR and AAA 1Y correlations much higher than recent history; long-rate correlations similar to whole history
- BK correlations match less well

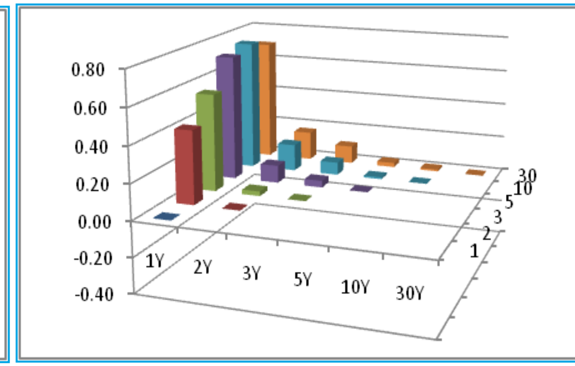
E2BK / 2009-2014



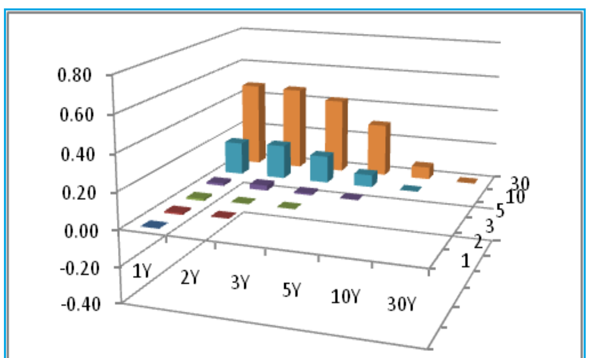
CIR3 / 2009-2014



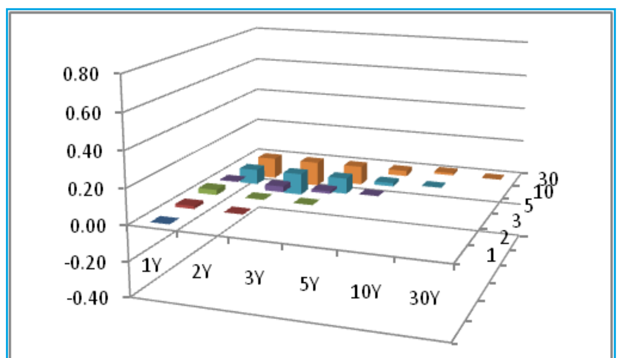
AAA / 2009-2014



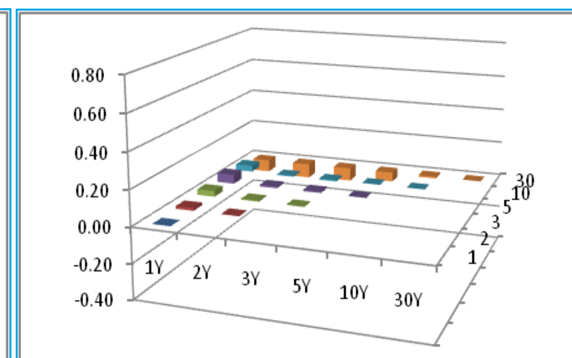
E2BK / 1982-2014



CIR3 / 1982-2014



AAA / 1982-2014



Facts and Test 4. Moments of Spreads

- Yield spreads are usually positive and have higher volatility at middle tenors; AAA mean spreads deviated more from history than other models but none match closely.
- CIR spreads have lower volatility than observed, especially for the 3M/1Y and 10Y/30Y spreads, while BK is a little more volatile than observed at the long end. AAA short spreads have too much volatility and long spreads too little.
- This is the biggest problem CIR has as it generally gives a good distribution of overall yield-curve shapes and rate volatilities and correlations, but the low volatility of some adjacent spreads is somewhat problematic
- Certainly you could construct options that this model would price wrong, but the overall portfolio value distribution might not be overly distorted by this problem.
- Still it is an issue users should be aware of if they come up with innovative uses for an ESG
- Empirical skewnesses of the spreads are all slightly negative but the models do not match these very well – AAA is closest.

Moment	Model	3M/1Y	1Y/3Y	3Y/10Y	10Y/30Y
Mean (%)	2009+	0.14	0.61	1.83	1.03
	E2BK	0.17	1.00	1.98	0.67
	CIR3	0.17	1.00	1.98	0.67
	AAA	0.36	0.96	1.34	0.65
Std (%)	2009+	0.07	0.37	0.5	0.23
	E2BK	0.09	0.37	0.69	0.43
	CIR3	0.04	0.09	0.38	0.08
	AAA	0.19	0.14	0.20	0.09
Skewness	2009+	-1.1	-0.15	-0.12	-0.01
	E2BK	1.27	0.85	-0.32	-1.07
	CIR3	-0.70	-1.62	-0.60	-3.36
	AAA	-0.92	-0.34	-0.34	-0.34

Facts 5. Excess Returns and Persistence of Structure

- Excess returns: borrow for year at 1Y rate, buy n year bond, sell a year later
- Profitability potential identified by regressions since Fama (1984)

$$Y(t+1,n-1) - Y(t,n) = const + \phi_n \left[\frac{Y(t,n) - Y(t,1)}{n-1} \right] + res$$

- Coefficients negative and increasingly so by maturity, and significant
- Lower left side more profit, comes with steeper initial curve
- Also a forecast of next year’s yield from this year’s
- A little like forward predicting future – residuals higher for higher n
- Can test whether or not this structural feature is maintained by the simulation
- Look at distribution of residuals from simulation vs. from history
- Used last 6 years of history in the test
- Looked at standard deviation and 6 percentiles of residuals for tenors 2Y – 10Y
- Did graphs of percentage differences between models and history after recentering means
- Means might be different due to other reasons for mean forecast

Model	2y	3y	4y	5y	6y	7y	8y	9y	10y
E2BK/CIR3	0.8%	0.9%	1.0%	1.0%	1.0%	1.0%	1.0%	0.9%	0.9%
AAA	0.6%	0.8%	0.9%	0.9%	0.8%	0.7%	0.6%	0.5%	0.3%

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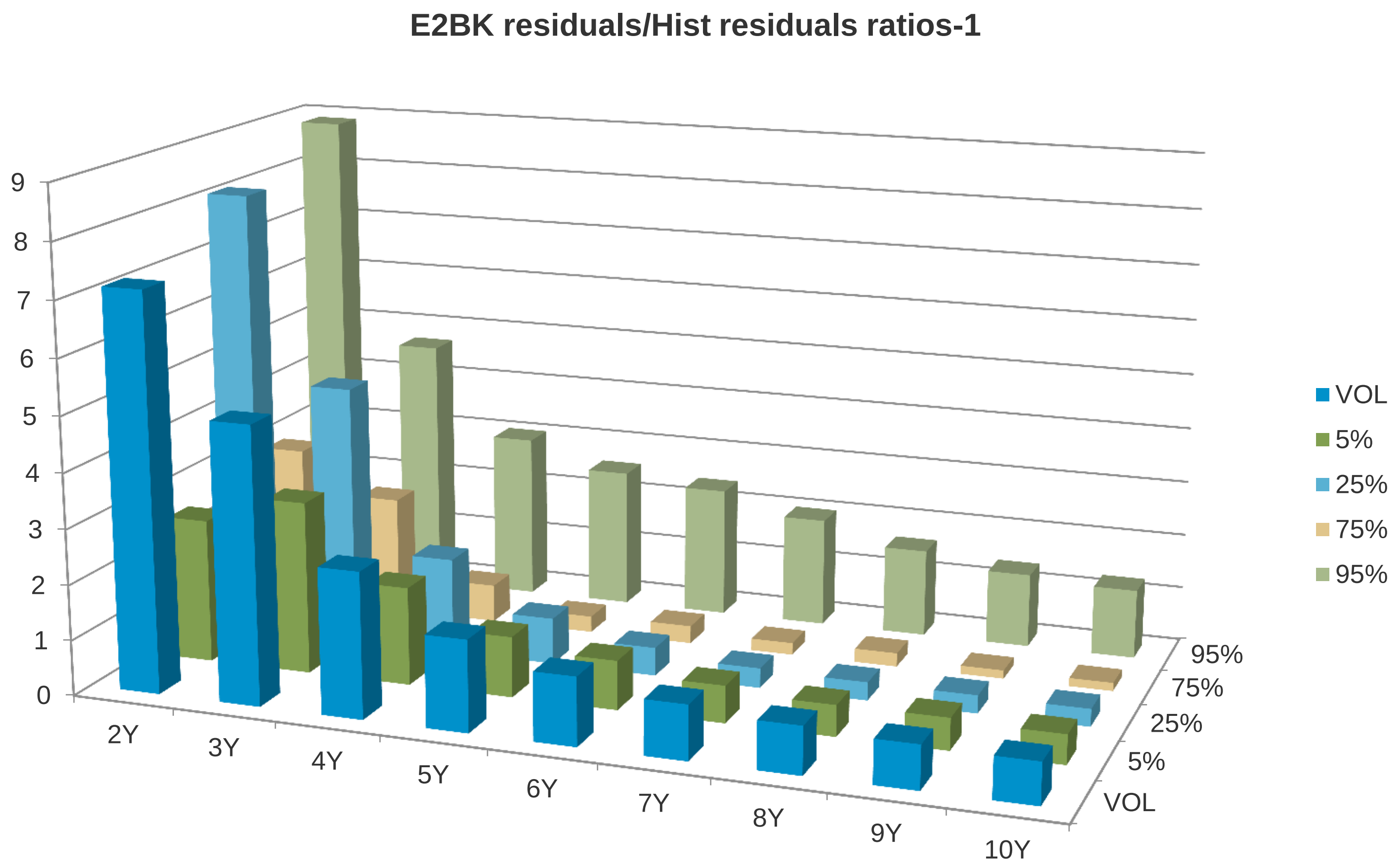
Test 5. Excess Returns and Persistence of Structure

BK had higher spreads around the forecast than history

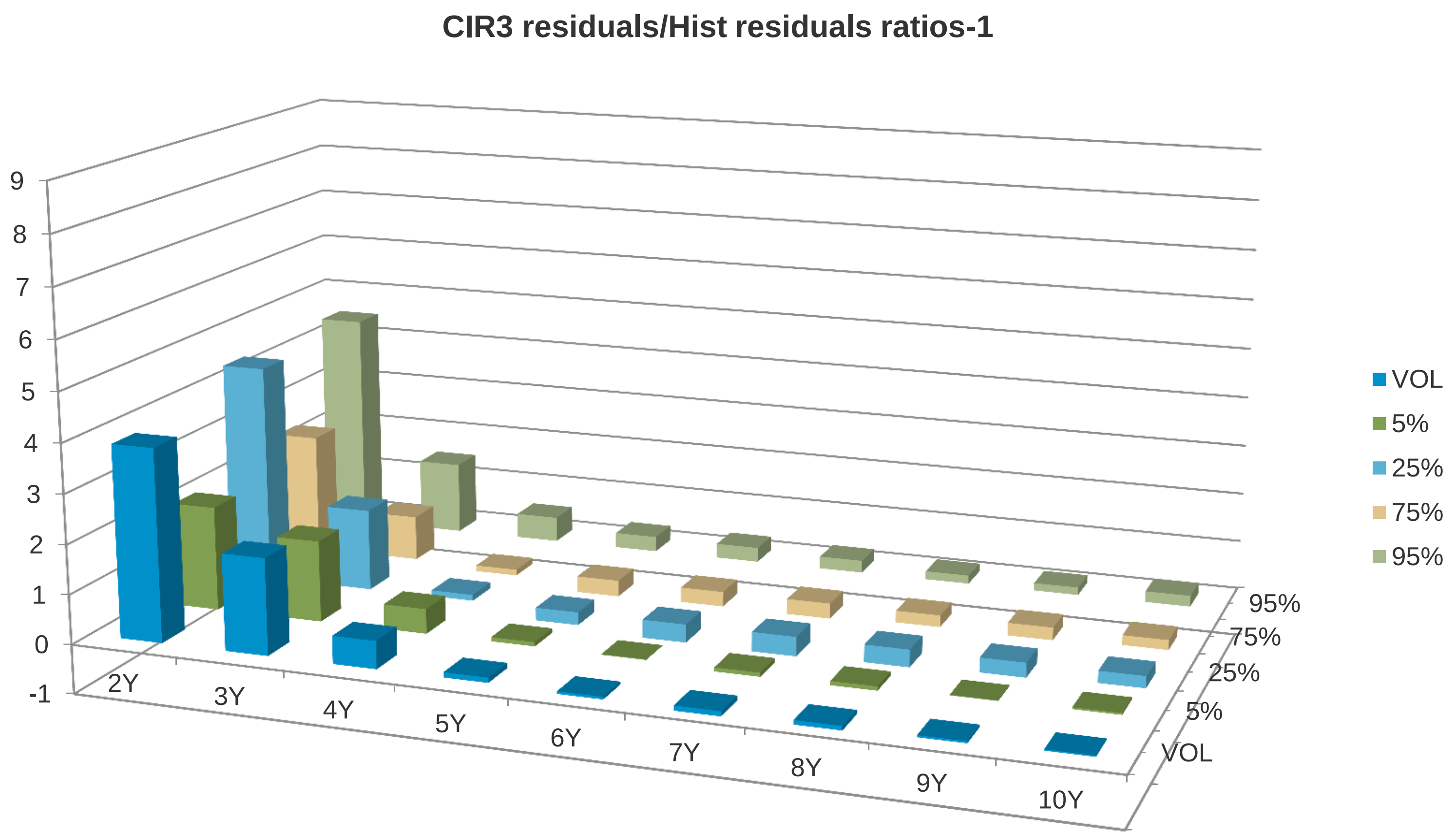
It doesn't really preserve this structure

Suggests some rates do not have realistic distributions, especially in the tails



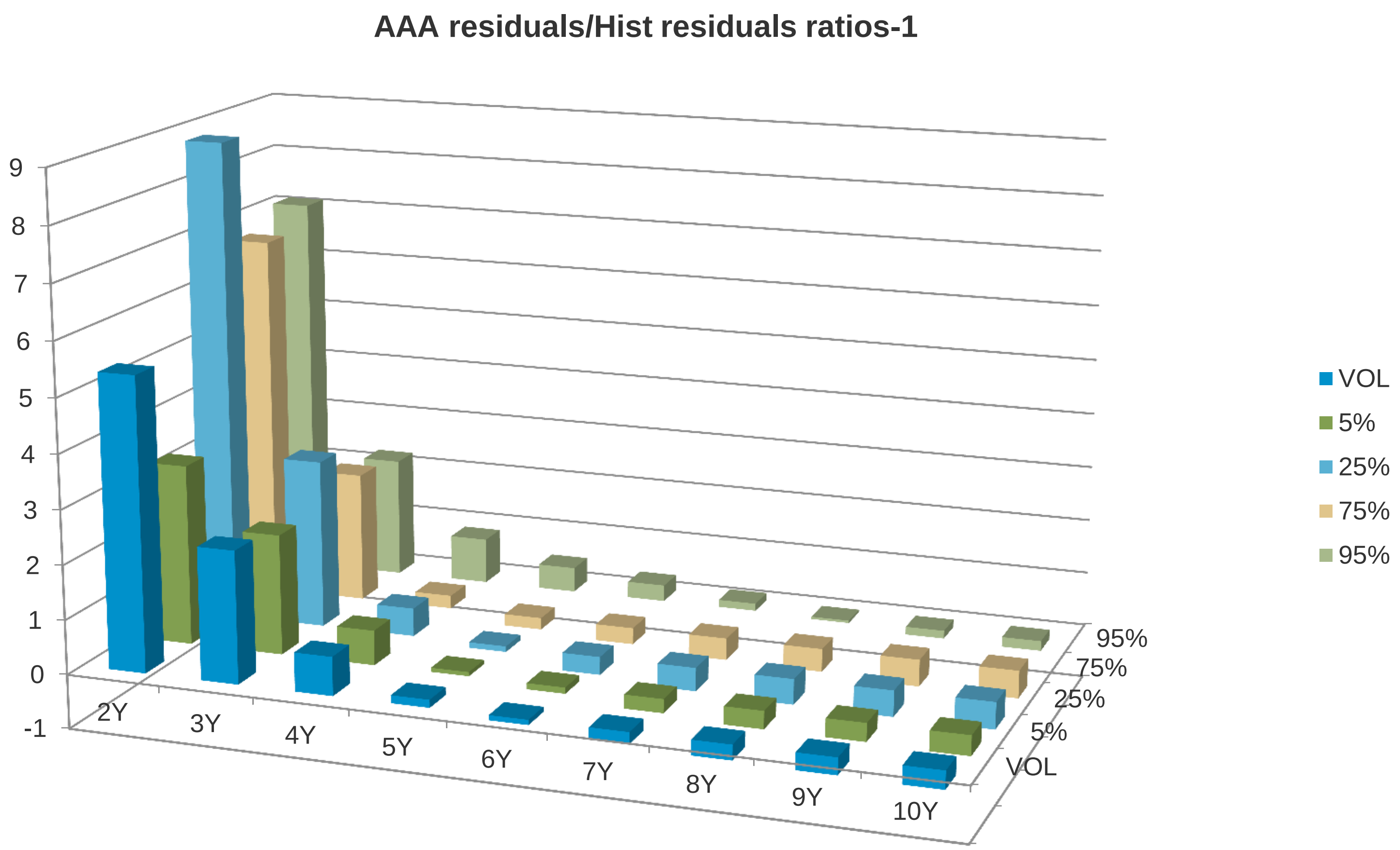
Test 5. Excess Returns and Persistence of Structure

CIR volatility and spread around this mean were less than historical for longer tenors



Test 5. Excess Returns and Persistence of Structure

AAA volatilities provide good fit for longer terms



Facts 6. Relationships among Yields – Naïve PCA

- Chapman and Pearson (2001) model 2Y and 10Y yields as functions of what they call Naïve PCA
- They use 5Y rates to represent level, 5Y / 1Y spread to represent slope, and 5Y/3Y – 3Y/1Y as curvature – these are the independent variables
- They get pretty good fits
- But equivalent to doing regressions on 1Y, 3Y, and 5Y rates
- So really looking at relationships among yields
- For instance for 2000-2004, 20Y = 23%*1Y+24%*3Y+63%*5Y+ε
- We did these regressions for the last 6 years (2009-2014) and tracked moments and percentiles of the residuals
- Using the fitted coefficients, the residuals were computed for the simulated scenario sets and their moments and percentiles tracked as well
- Tests if the scenarios maintain relationships across yields that occur in the data

Test 6. Relationships among Yields – Naïve PCA

- The models have considerably higher dispersion of yields than history except lower for 2Y – not unusual for out-of-sample vs. in-sample errors
- CIR always much closer to history than the other two models, which are different but have comparable errors
- Relationships among yields important for risk to various portfolio choices
- Maybe historical results are too tight due to some idiosyncratic effects

Residuals statistics for naïve PCA regressions (basis points)

Rate	Model	MEAN	STD	p1%	p5%	p25%	p75%	p95%	p99%
20Y RATE	HIST	0	14	-29	-21	-10	9	22	35
	E2BK	-75	136	-511	-227	-41	78	138	180
	CIR3	-75	64	-291	-109	2	25	44	69
	AAA	-83	107	-293	-194	-69	90	139	152
10Y RATE	HIST	0	8	-22	-13	-6	6	13	19
	E2BK	-23	56	-206	-91	-18	31	62	85
	CIR3	-23	27	-122	-46	0	11	19	30
	AAA	-32	46	-126	-84	-30	38	60	67
2Y RATE	HIST	0	3	-7	-5	-2	2	4	7
	E2BK	-4	1	-4	-2	-1	1	2	2
	CIR3	-4	2	-5	-3	-1	1	2	3
	AAA	3	4	-10	-7	-2	3	5	5

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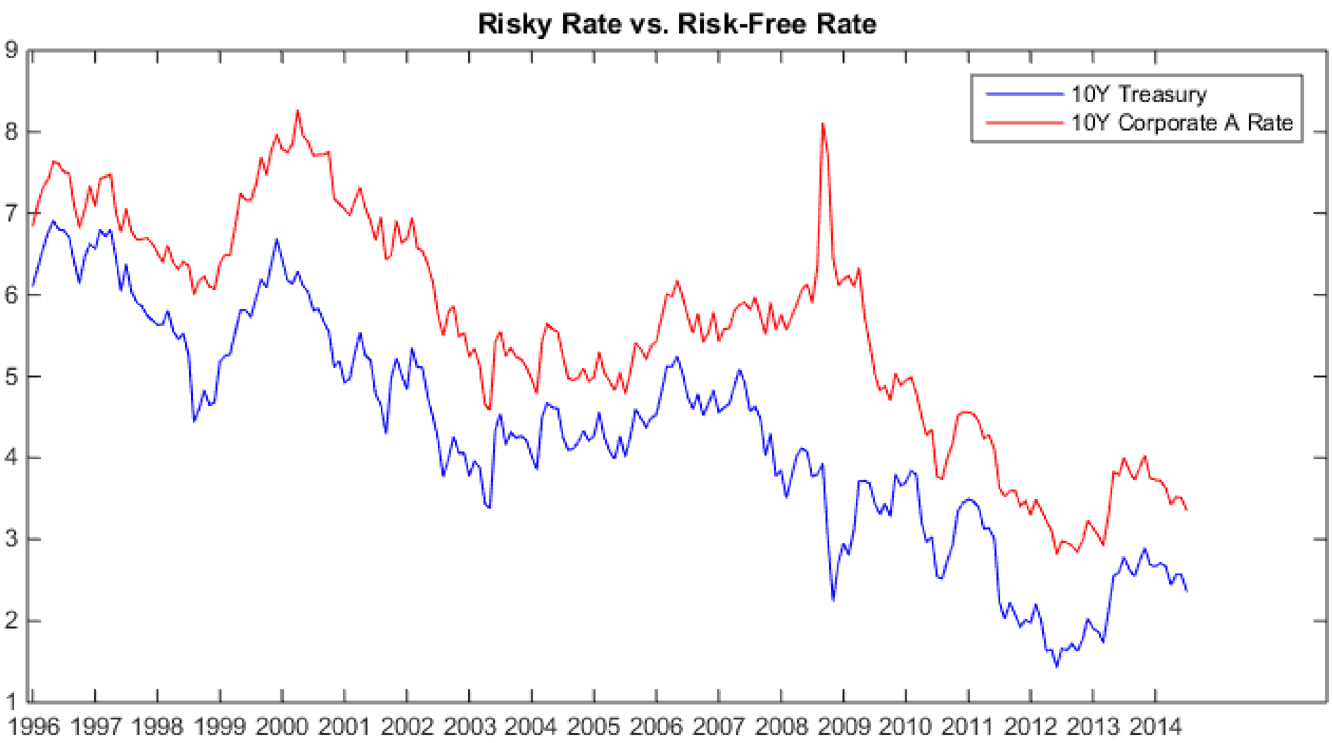
Correlation between risk-free and risky rates

Correlations between Risk-Free Rates and Risky Rates						
	1996-2014			2009-2014		
	"A" 5YR vs. 5YR Treasury	"A" 10YR vs. 10YR Treasury	"A" 20YR vs. 20YR Treasury	"A" 5YR vs. 5YR Treasury	"A" 10YR vs. 10YR Treasury	"A" 20YR vs. 20YR Treasury
YoY Changes	66.3%	59.4%	58.3%	64.7%	71.0%	73.2%
Detrended Rates	74.0%	60.9%	56.2%	68.9%	68.9%	68.8%



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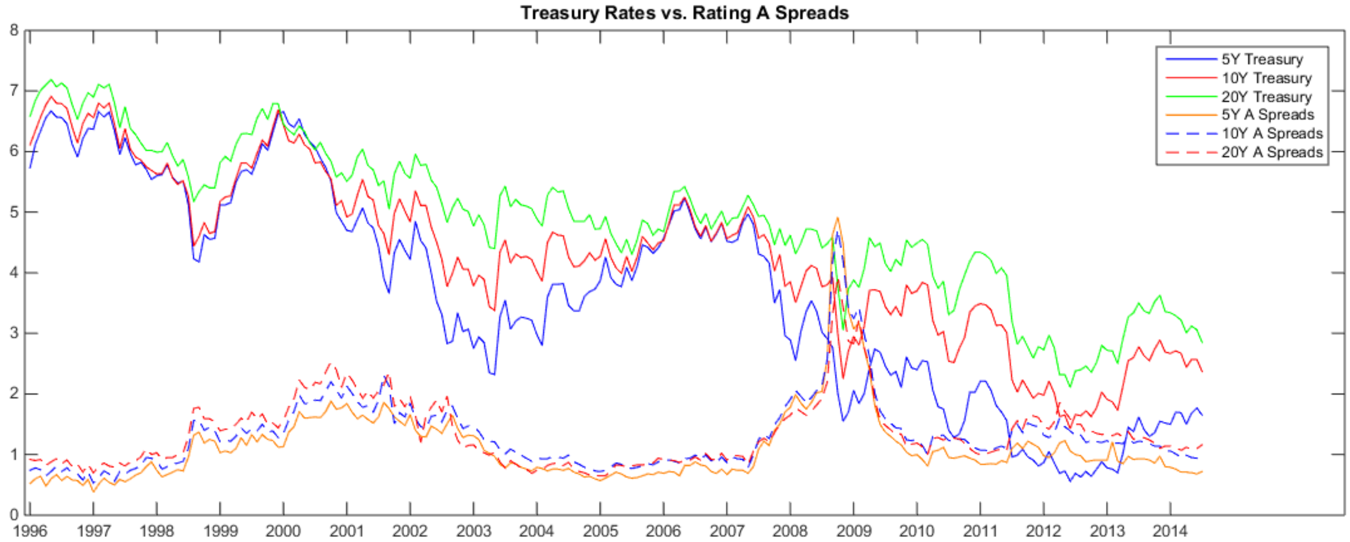
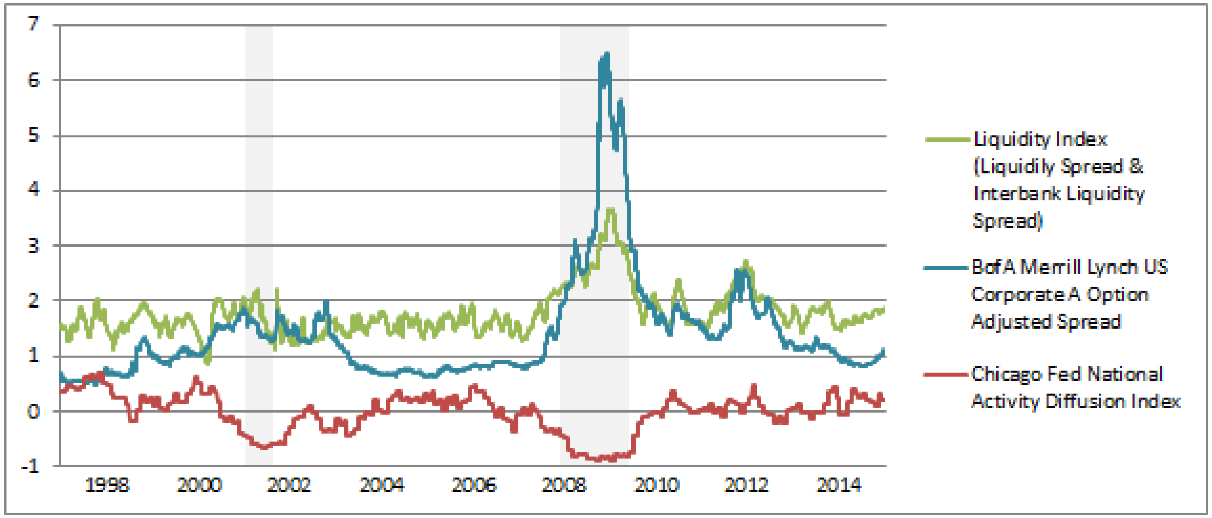
Credit spread properties



- Spreads mean-reverting, do not have long-term trend, but some inverse to rates
- Spread levels do not depend purely on risk-free rates level; also driven by economic conditions



Correlation	
Corporate A Spread vs. Activity Index	-0.682
Corporate A Spread vs. Liquidity Index	0.844

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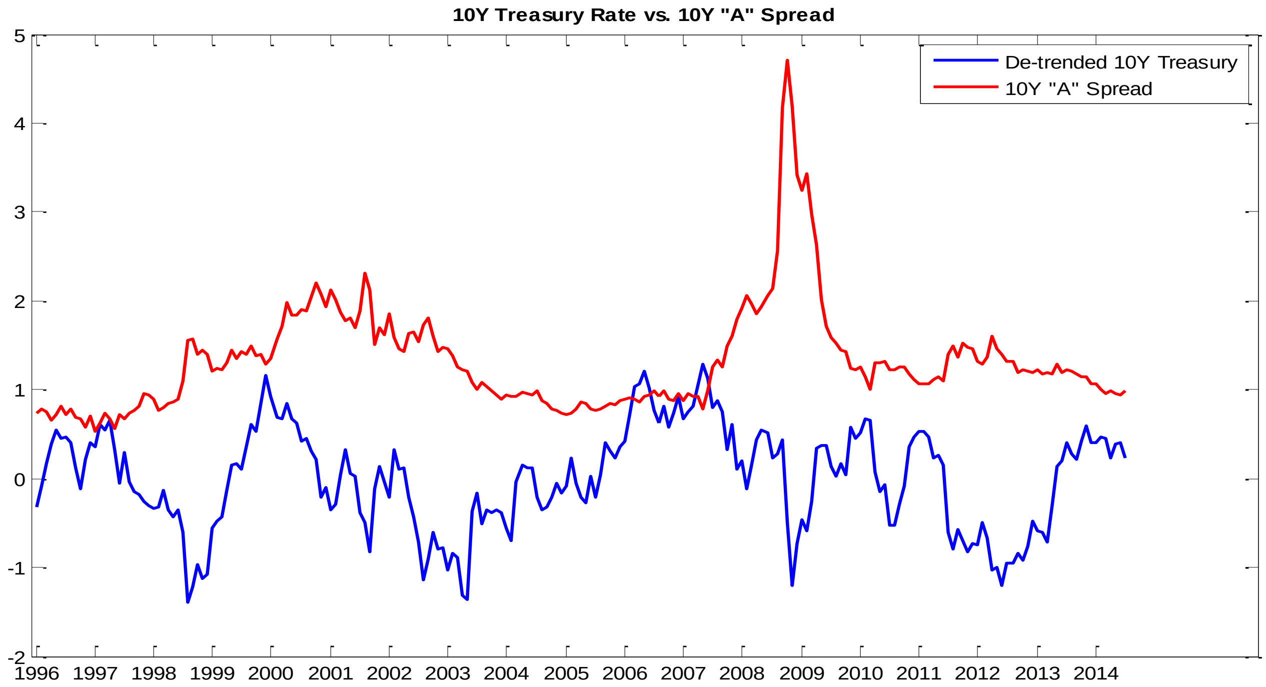
Correlation between risk-free rates and spreads

Correlations between YoY Changes in Treasury Rates and Spreads					
1996-2014			2009-2014		
"A" 5YR vs. 5YR Treasury	"A" 10YR vs. 10YR Treasury	"A" 20YR vs. 20YR Treasury	"A" 5YR vs. 5YR Treasury	"A" 10YR vs. 10YR Treasury	"A" 20YR vs. 20YR Treasury
-34%	-37%	-36%	-35%	-47%	-59%



YoY changes in risk-free rates and spreads are negatively correlated so risk-free rates have more market risk than risky rates do

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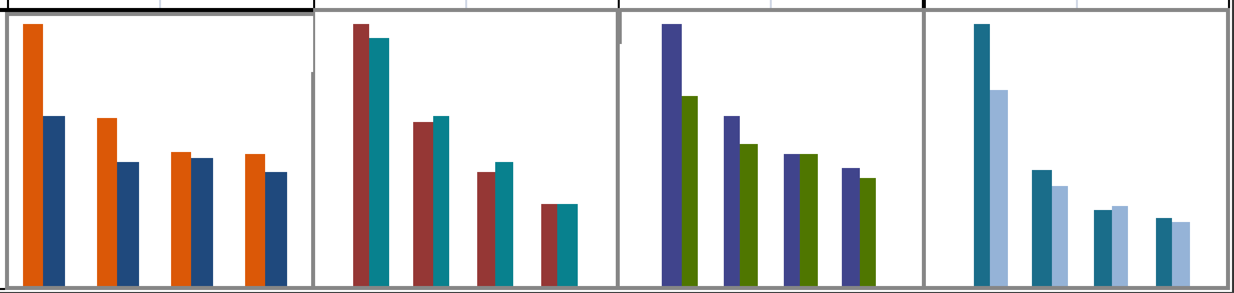
Risky Rates Term Structure of Volatility

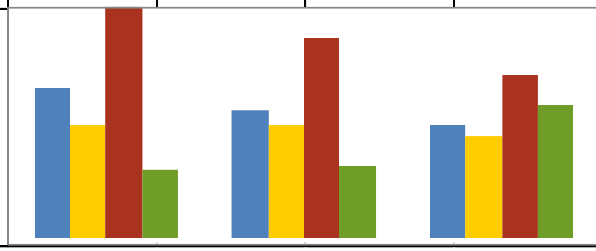
- Historical annual volatilities downward sloping across maturities
- Usually less volatile than risk-free rates (negative correlation between rates and credit spreads)
- Risky rates are more stable in the latest years, never go too close to zero, due to credit spread
- Volatilities in recent 6 years not too different from longer history
- Volatility of log of rates the most constant over time – gives the best target for testing simulated rate volatility

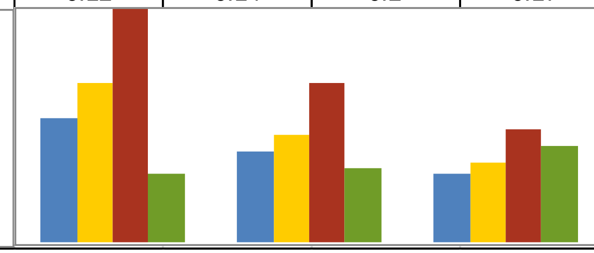
Corporate Bond (A)								
	Abs Vol (%)	Since Jan 2009	Log Vol	Since Jan 2009	Abs Vol of 2/3 Power of Rates(%)	Since Jan 2009	Abs Vol / Sqrt Rate	Since Jan 2009
2Y	1.52	0.98	0.43	0.41	0.63	0.46	1.23	0.92
5Y	0.97	0.72	0.27	0.28	0.41	0.34	0.55	0.47
10Y	0.78	0.74	0.19	0.21	0.32	0.32	0.37	0.39
20Y	0.76	0.67	0.14	0.14	0.28	0.26	0.32	0.30
Charts								

Volatility Term Structure of Risky Rates

Volatility of Risky Rates

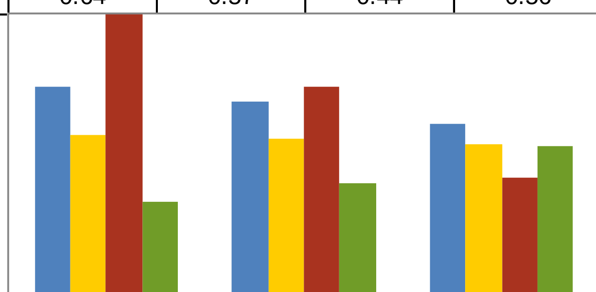
	Volatility (%)				Log Volatility			
	Historical 1996-2014	Historical 2009-2014	JLT	CIR3	Historical 1996-2014	Historical 2009-2014	JLT	CIR3
5 YR	0.99	0.74	1.52	0.45	0.22	0.28	0.41	0.12
10 YR	0.84	0.74	1.31	0.47	0.16	0.19	0.28	0.13
20 YR	0.74	0.67	1.07	0.88	0.12	0.14	0.2	0.17

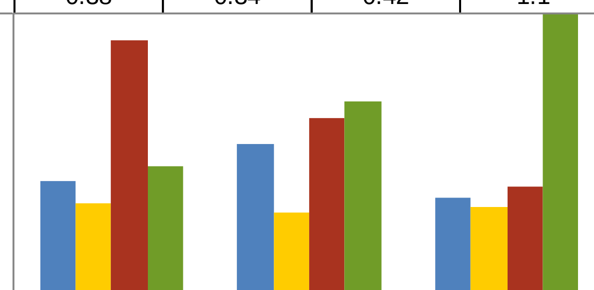




Volatility of Spreads

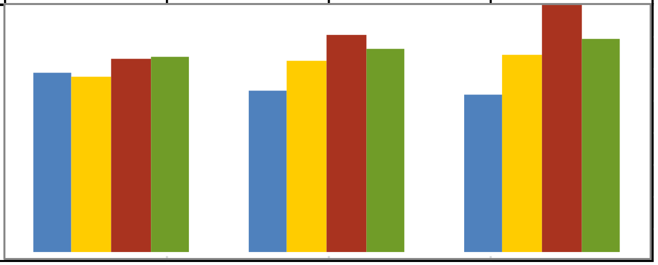
	Volatility (%)				Log Volatility			
	Historical 1996-2014	Historical 2009-2014	JLT	CIR3	Historical 1996-2014	Historical 2009-2014	JLT	CIR3
5 YR	0.78	0.6	1.06	0.35	0.44	0.36	0.99	0.5
10 YR	0.73	0.59	0.78	0.42	0.59	0.32	0.69	0.75
20 YR	0.64	0.57	0.44	0.56	0.38	0.34	0.42	1.1



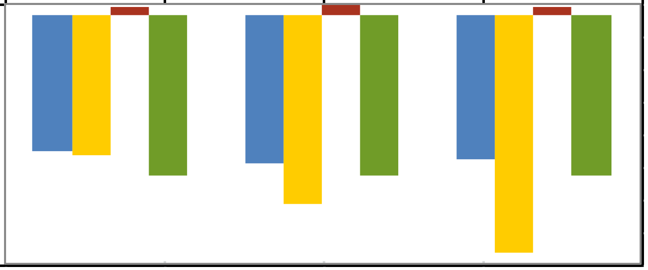


Correlations between Treasury Rates and Risky Rates

Correlations between Treasury Rates and Risky Rates				
	Historical 1996-2014	Historical 2009-2014	JLT	CIR3
"A" 5YR vs. 5YR Treasury	66%	65%	72%	72%
"A" 10YR vs. 10YR Treasury	59%	71%	81%	75%
"A" 20YR vs. 20YR Treasury	58%	73%	91%	79%



Correlations between Treasury Rates and Spreads				
	Historical 1996-2014	Historical 2009-2014	JLT	CIR3
"A" 5YR vs. 5YR Treasury	-34.0%	-35.0%	2.2%	-40.0%
"A" 10YR vs. 10YR Treasury	-37.0%	-47.0%	2.4%	-40.0%
"A" 20YR vs. 20YR Treasury	-36.0%	-59.0%	2.2%	-40.0%



Summary

- This is status of building a testing methodology for ESG output
  - For now looking at one-year horizon and emphasizing risk to portfolio value
  - Mostly interest-rate methodology with some look at correlations with risky rates
  - AIG assets more sensitive to risky rates so that will be emphasis going forward, using the tests shown for risk-free rates
- BK model seems to have been calibrated with too much volatility, which gives poor performance on most tests, but is not an inherent feature of BK
- CIR looks a lot better but doesn’t produce enough volatility of spreads
- AAA is not designed for portfolio risk over a short timeframe, and does not perform well for that