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Doing More With Less:

From Sample to Population

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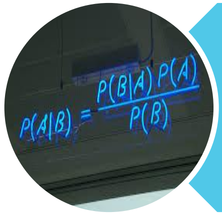
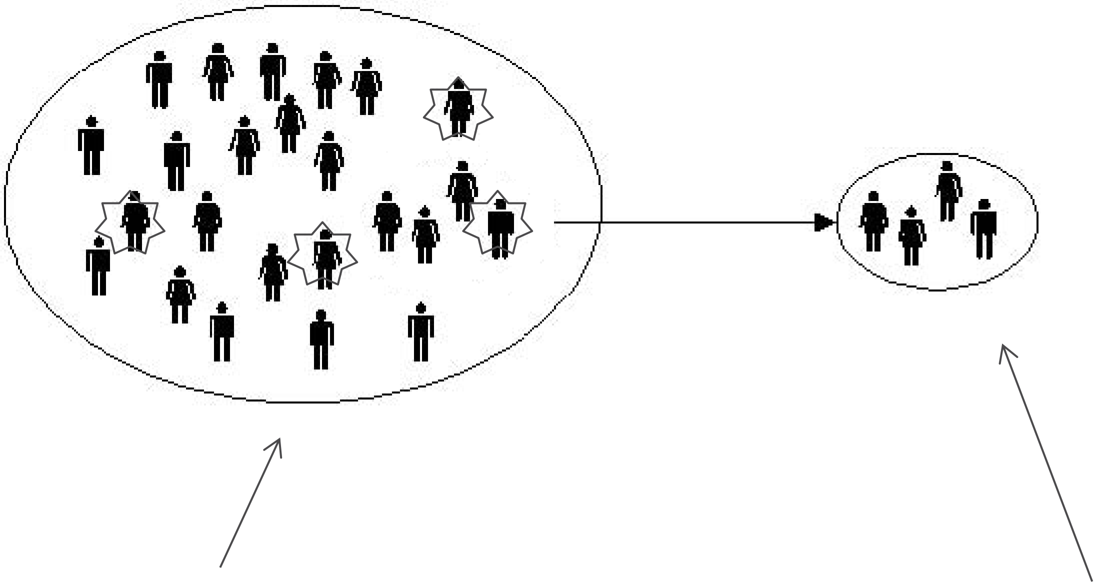


Bayesian Analysis

Population and Sample

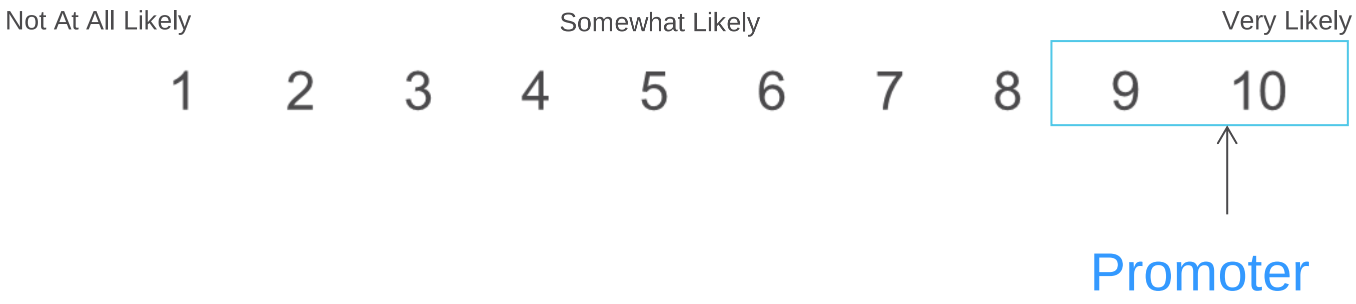


Current Customers

Surveys Completed

Measure Proportion of Promoter

What is the likelihood that you would recommend insurance from XXX company to a friend or colleague?



Not At All Likely

Somewhat Likely

Very Likely

1 2 3 4 5 6 7 8 9 10

Promoter

We want to know the proportion of Promoters p.

Hypothesis Testing

Hypothesis testing is a formal process to determine whether to reject a null hypothesis or not, based on the sample data.

- H_0 is null hypothesis
- H_a is alternative hypothesis

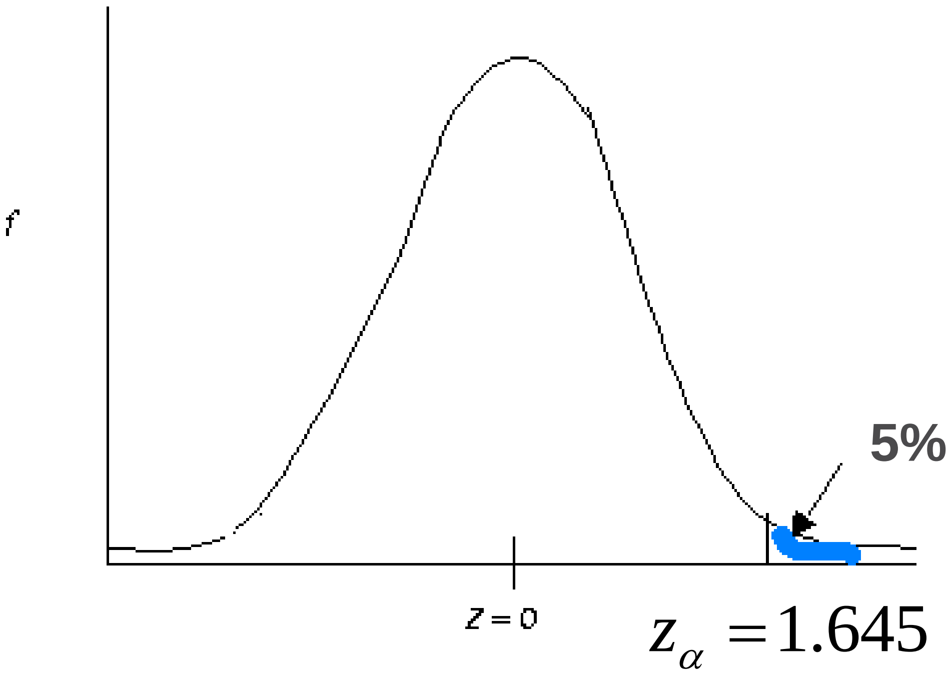
In the statistical framework, we either reject a null hypothesis or fail to reject a null hypothesis.

Type I and Type II Error

		Decision	
		Reject H_0	Fail to reject H_0
Truth	H_0	**Type I Error**	Correct Decision
	H_a	Correct Decision	**Type II Error**

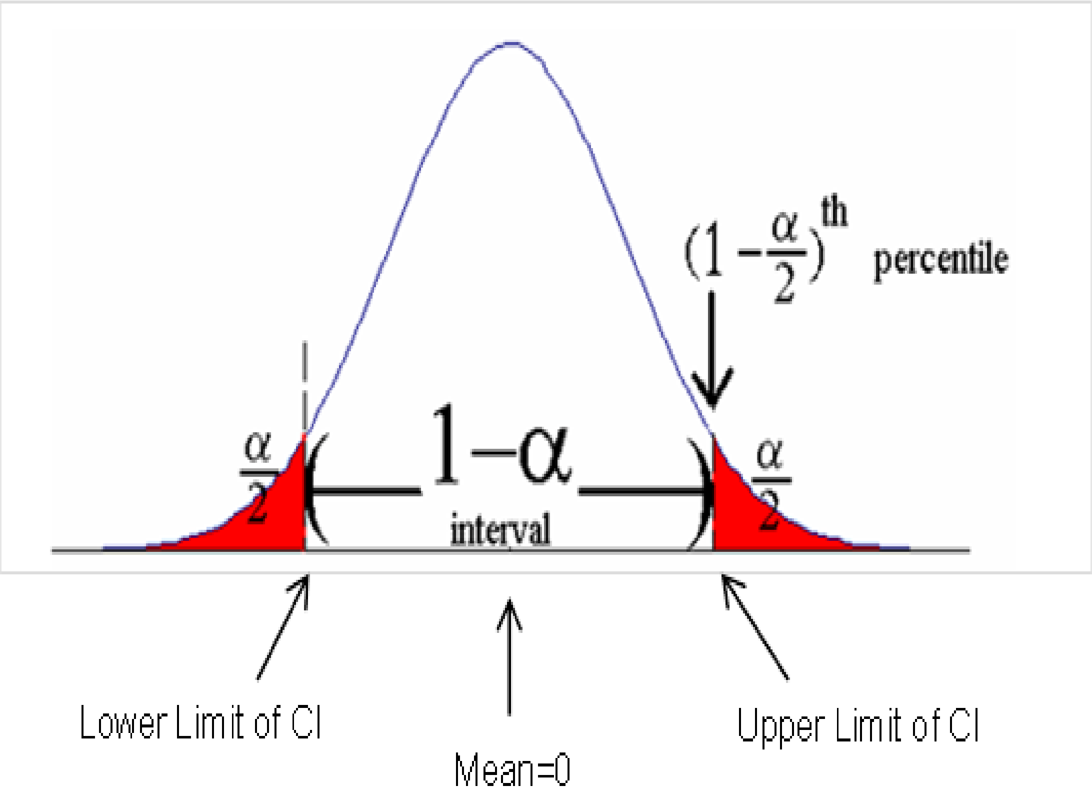
What is z_α

A z-score indicates how many standard deviations an element is from the mean.



$$z_\alpha = 1.645$$

Confidence Interval



Traditional Sample Size

The point estimate of p is $\hat{p} = \frac{y}{n}$ and an approximate $1 - \alpha$ confidence interval for p is

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Here $y{\sim}binomial(n, p)$.

α is the Type I error.

$\varepsilon = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ is the maximum error of the point estimate $\hat{p} = \frac{y}{n}$.

Traditional Sample Size

$$\varepsilon = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Assume p is about to equal to p^*, then

$$Sample\ Size(SS) = n = \frac{z^2{}_{\alpha/2}\, p^*(1-p^*)}{\varepsilon^2}$$

$$SS = \frac{1.645^2 * 0.82(1-0.82)}{0.10^2} \approx \mathbf{39.94}$$

From 40 sample surveys, we are 90% confident that the true p belongs to the interval $\hat{p} \pm 0.10$.

Confidence Level	Z score
90%	1.645

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Test for Equality

How many surveys per agent are credible in order to reflect a statistically significant change on the Promoter Score?

Steps

1. Set up hypotheses: null and alternative

2. Identify the distributions and key variables

3. Select desirable confidence level and statistical power

4. Calculate sample size needed

Hypothesis on Promoter Score

$$H_0 : p_1 - p_2 = 0 \, vs \; H_a : p_1 - p_2 \neq 0$$

p1 is the current proportion of promoters among all customers for an agent.

p2 is the proportion of promoters for an agent in the next month.

$$\hat{p} = \frac{y}{n} \; and \; y \sim binomal(n, p)$$

Derivation

We reject the null hypothesis if

$$\left|\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{(\hat{p}_1(1-\hat{p}_1) + \hat{p}_2(1-\hat{p}_2))/_{n}}}\right| > z_{\alpha/2}$$

$$\frac{|p_1 - p_2|}{\sqrt{(\hat{p}_1(1-\hat{p}_1) + \hat{p}_2(1-\hat{p}_2))/_{n}}} - z_{\alpha/2} = z_\beta$$

$$n_1 = n_2 = \frac{(z_{\alpha/2} + z_\beta)^2[p_1(1-p_1) + p_2(1-p_2)]}{(p_1 - p_2)^2}$$

Sample Size Needed

$$n_1 = n_2 = \frac{(z_{\alpha/2} + z_{\beta})^2[p_1(1-p_1) + p_2(1-p_2)]}{(p_1 - p_2)^2}$$

$z_{\alpha/2}$ is z statistic given Type I error α

z_{β} is z statistic given Type II error β

Illustrative Example

$$H_0 : p_1 - p_2 = 0 \, vs \; H_a : p_1 - p_2 \neq 0$$

$$n_1 = n_2 = \frac{(z_{\alpha/2} + z_\beta)^2 [p_1(1-p_1) + p_2(1-p_2)]}{(p_1 - p_2)^2}$$

$$n_1 = n_2 = \frac{(1.645 + 0.84)^2 [0.80(1-0.80) + 0.86(1-0.86)]}{(0.80 - 0.86)^2} \approx 480.98$$

To conclude a significant change, more samples are needed.

Aggregate the Results

We want to aggregate the results to a regional level.

We can use weighted average by number of customers to scale it back in order to minimize the sample size needed.

	Promoter Proportion	Agency Satisfaction
Big Sample	59.3%	70.2%
Weighted Avg w/ 40 surveys per agent	59.0%	70.5%

Sample Size Calculation

Survey Analysis

Hypothesis Testing

Collect right amount of sample

Bootstrapping

Random sampling with replacement.

Illustrative Example Cont.

- Customer Experience Survey

Bootstrap

When Sample is not Sufficient

Establish Error Bounds

Statistical Inference with a Small Sample

Downsampling

Down Sampling zero-claims can reduce the time of model convergence during the model development phase.

The standard deviation for parameter estimator will likely increase with the small sample. It won’t necessarily reverse modeling decisions.

Modeler can periodically run the model with the entire training data set to verify the model structure.

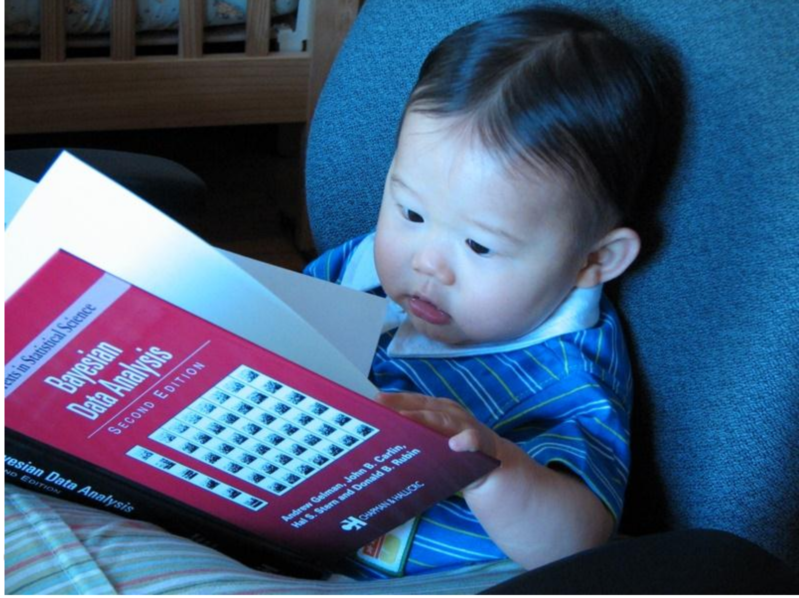
Once the mode structure is finalized, the entire data set should be used for the final model.

Downsampling

During Model Development

Save Time

Bayesian Analysis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SECOND EDITION
Andrew Gelman, John B. Carlin,
Hal S. Stern and Donald B. Rubin
CHAPMAN & HALL/CRC
esian Data Analysis

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Posterior is proportional to “likelihood*prior”

We use known knowledge as input for the prior distribution, and use data collected to calculate the likelihood. The posterior will be updated as more and more data is collected.

As a result, the sample size needed will be reduced.

Bayesian Analysis

Need more flexibility

Less Sample Needed

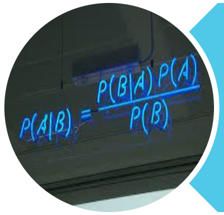
Allow distributions for parameters

Summary

Calculate Sample Size

Bootstrapping

Downsampling



Bayesian Analysis

Questions