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## Actuarial Portfolio Management of Infrastructure Service Contracts

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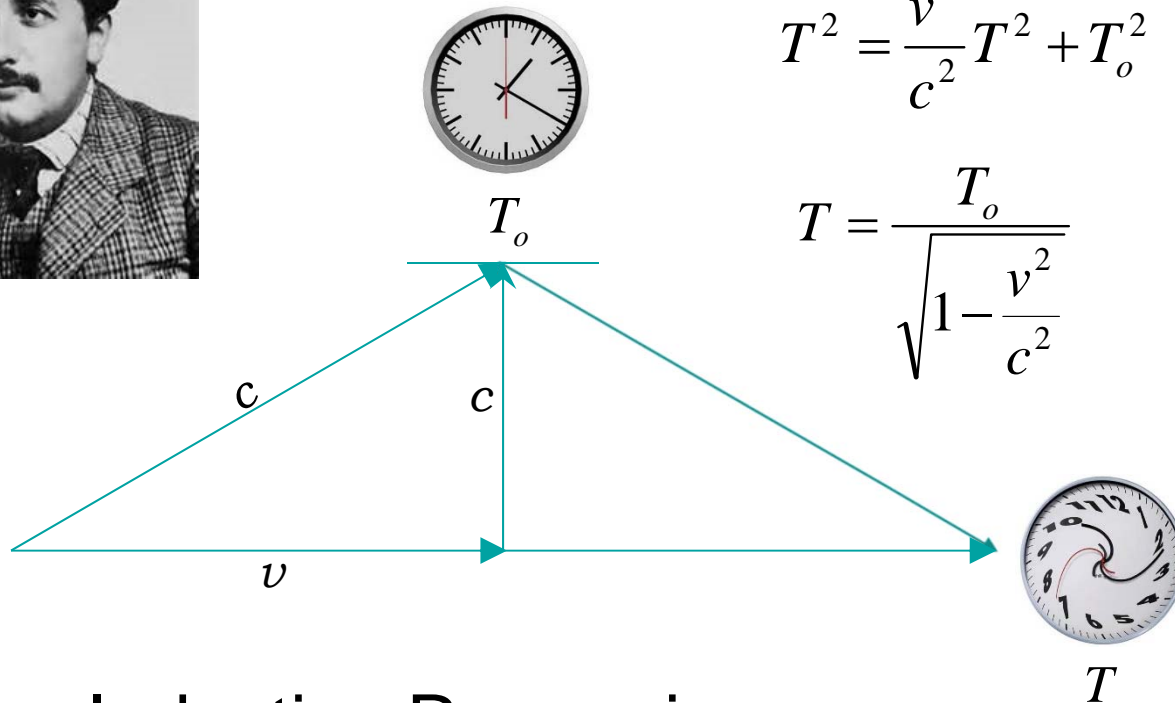


# A theoretical application of actuarial science....

- This session will feature a theoretical application of actuarial science to the world of sustainability in industry and infrastructure. All the world's scarce time, energy, and materials are consumed by aging cohorts of physical assets, not people.
- Managing the mortality of physical assets in the face of changing technology and energy environments can recover vast efficiencies throughout society, and may become a stock-in-trade of property and casualty actuaries.



# Physics Framework



$$c^2 T^2 = v^2 T^2 + c^2 T_o^2$$

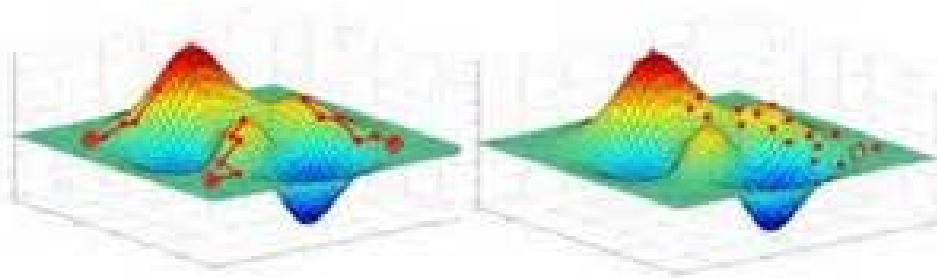
$$T^2 = \frac{v^2}{c^2} T^2 + T_o^2$$

$$T = \frac{T_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Inductive Reasoning
- Few auxiliary hypotheses

# Actuarial Framework

- The notion of ‘actuarial present value’ is unique to actuaries.
- Use it for optimization problems in the physical world currently not on the radar screen of operations research or industrial engineering.

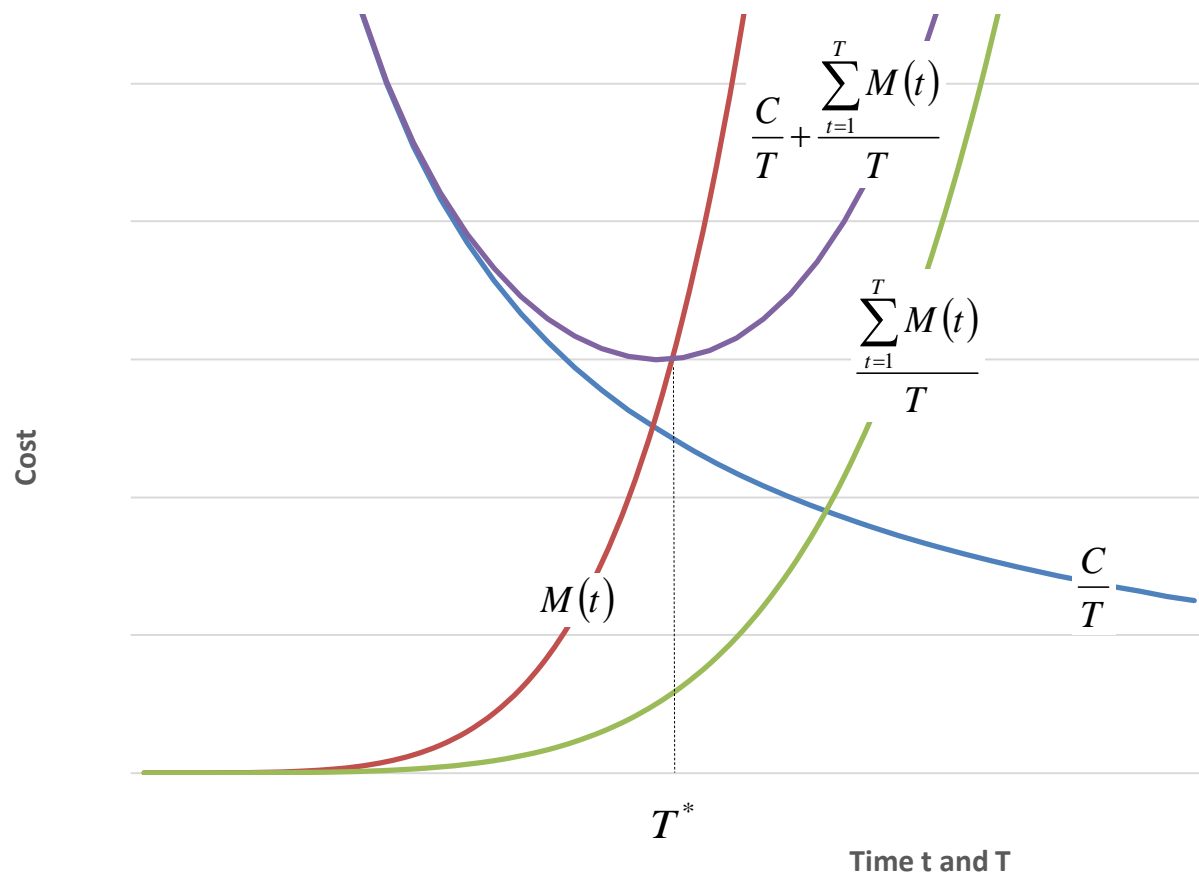


# Physical Assets

- We are surrounded by tens of trillions of dollars of fixed, physical assets.
- Machinery and equipment consume the world's scarce time, energy, and materials, not people.
- Nothing impacts an asset's efficiency more than its replacement with a new one!



# Physical Assets



# Physical Assets

$$M(t) \geq iC + \frac{C}{s_{\overline{T^*}|}} + \frac{(1+i)^{T^*} \sum_{t=1}^{T^*} M(t)v^t}{s_{\overline{T^*}|}}$$

$i$  = owner's real cost of capital

$$v^t = \frac{1}{(1+i)^t}$$

$$s_{\overline{T}|} = \frac{(1+i)^T - 1}{i}$$



# Approximation #1

$$M(t) \geq iC + E \left[ \frac{C}{s_{\overline{T^*}|}} + \frac{(1+i)^{T^*} \sum_{t=1}^{T^*} M(t)v^t}{s_{\overline{T^*}|}} \right]$$

$$M(t) \geq iC + E \left[ \frac{C}{s_{\overline{T^*}|}} + 0 \right]$$

$$\frac{M(t)}{C} \geq i + E \left[ \frac{1}{s_{\overline{T}|}} \right]$$





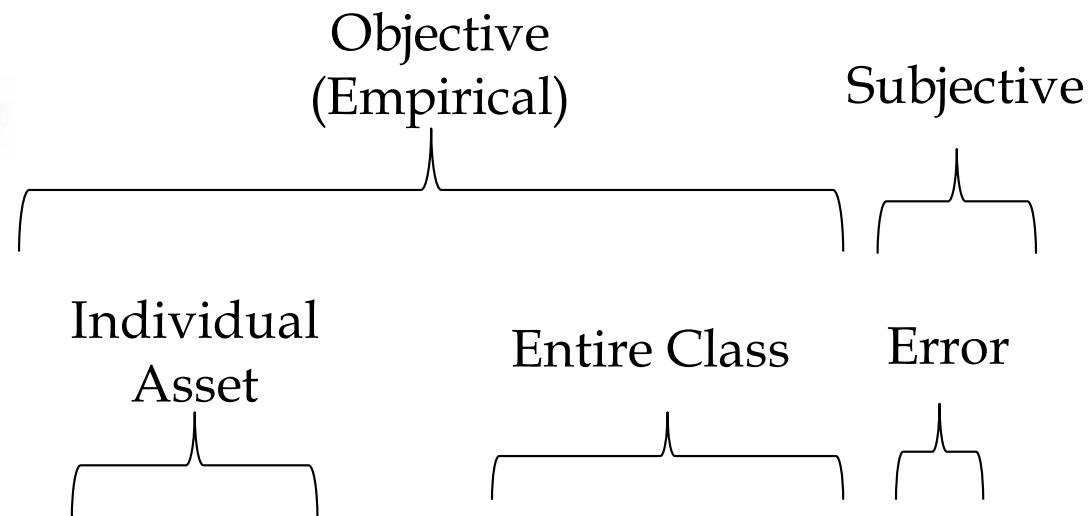
# Approximation #2

$$\frac{M(t)}{C} \geq i + E \left[ \frac{1}{s_{\overline{T}|}} \right]$$

$$E \left[ \frac{1}{s_{\overline{T}|}} \right] = \sum_{i=1}^N \left[ \frac{1}{s_{\overline{T}_i|}} \right] / N$$

Where  $T_1, T_2, \dots, T_N$  are empirical ages at the time of replacement (asset mortality data) from a homogeneous class of assets.





$$\frac{M(t)}{C} \geq i + \sum_{i=1}^N \left[ \frac{1}{s_{T_i|}} \right] / N + \beta$$

Where  $T_1, T_2, \dots, T_N$ , are empirical ages at the time of replacement (asset mortality data) from a homogeneous class of assets.

# 3 Principles of Efficient Physical Asset Mortality

1. There is only one instant in time that an asset must be replaced in order to minimize the present value cost impact.
2. This efficient instant is observable, and a function of both the owner's cost of capital and readily obtainable current calendar year information.
3. The time to this efficient instant is random. It may be infinite.



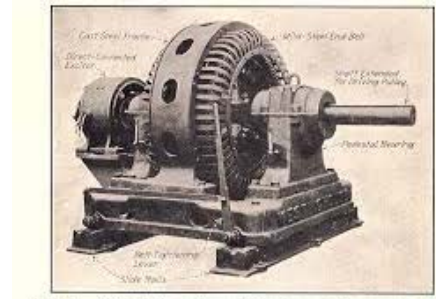
# Efficient Physical Asset Mortality

- ...is fortuitous and aleatory, not planned or scheduled.
- ...is a portfolio optimization problem.
- ...requires capital adequacy or risk transfer of volatile, uncertain, future costs - perhaps through insurance?
- Extended service contracts can enforce efficient physical asset replacement policy and provide the capital, legal, and tax benefits of insurance.



# Infrastructure Service Contracts

- Long-term extended service contracts of 10 to 30 year durations.
- Provide comprehensive long-term maintenance of public and industrial infrastructure, including replacement of physical assets as they age and become obsolete.
- Require the calculation of actuarially sound premiums and reserves.
- They already exist, but in limited contexts.



# Infrastructure Service Contracts

- Replacement of physical assets can be treated like life insurance....

....for efficiency.

....from a tax perspective.

