Parameters for a Loss Distribution

Estimating the Tail

CAS Spring Meeting

Seattle, Washington

May 17, 2016

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The Problem

- Estimate the Distribution of Losses for 2016
- We can use this to measure economic capital or regulatory capital (Solvency II)
- Test Different Reinsurance Plans
- Could be a book of Business – Auto, Marine, Property, or an entire company

Data

- Analysis Date – 13 Nov 2015
- Premium 2010 – 2014
- Loss Data 2010 – 2014
- Data as of Sep 30, 2015
- Why don't we use 2015 data

Summary of Loss Data

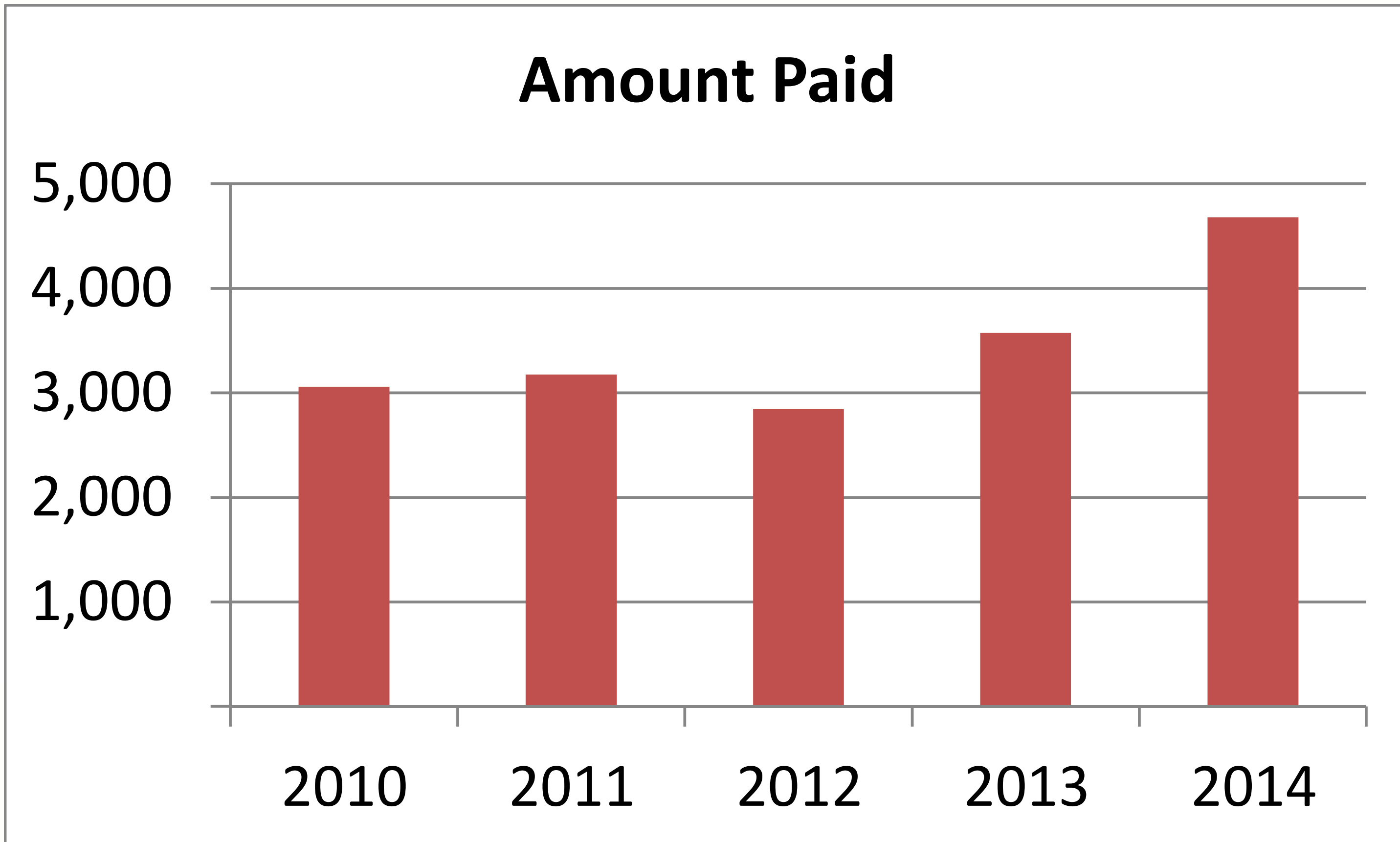
Year	Number of Claims	Amount Paid
2010	330	3,057,507
2011	312	3,177,057
2012	256	2,849,844
2013	272	3,571,991
2014	367	4,680,122

Summary of Data

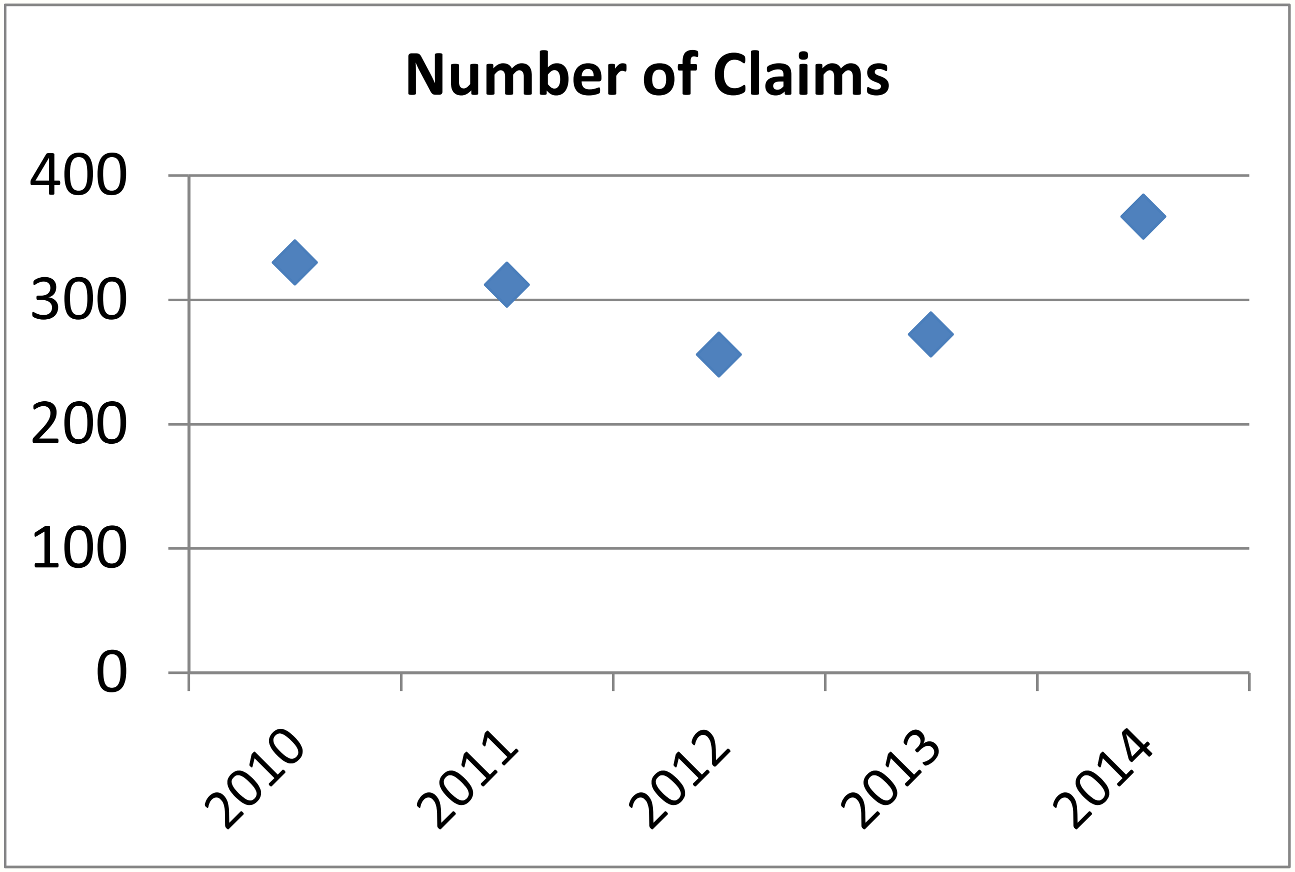
Assumptions

- Size of the book is unchanged
  - If it has, we would calculate **Frequency** of Claims, instead of number
- Loss Trend is zero – **0%**
  - If it isn't, we need to adjust the historical data
- If we have a sufficiently large book, or market data we can use a specific loss trend
- Otherwise, a general inflation estimate can be used

Summary of Data
Amount Paid



Summary of Data
Number of Claims

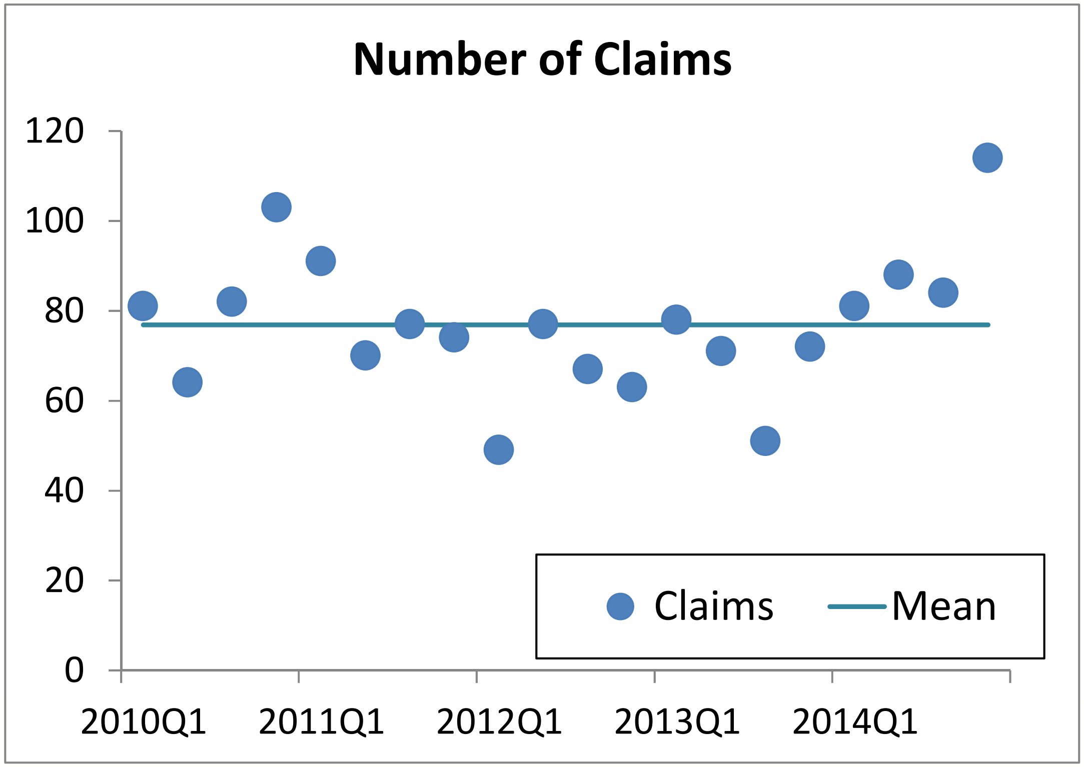


Summary of Data

Year	Number of Claims	Amount Paid	Severity
2010	330	3,057,507	**9,265**
2011	312	3,177,057	**10,183**
2012	256	2,849,844	**11,132**
2013	272	3,571,991	**13,132**
2014	367	4,680,122	**12,752**

- Severity appears to be increasing
- Speak to Underwriting, Product and Claims

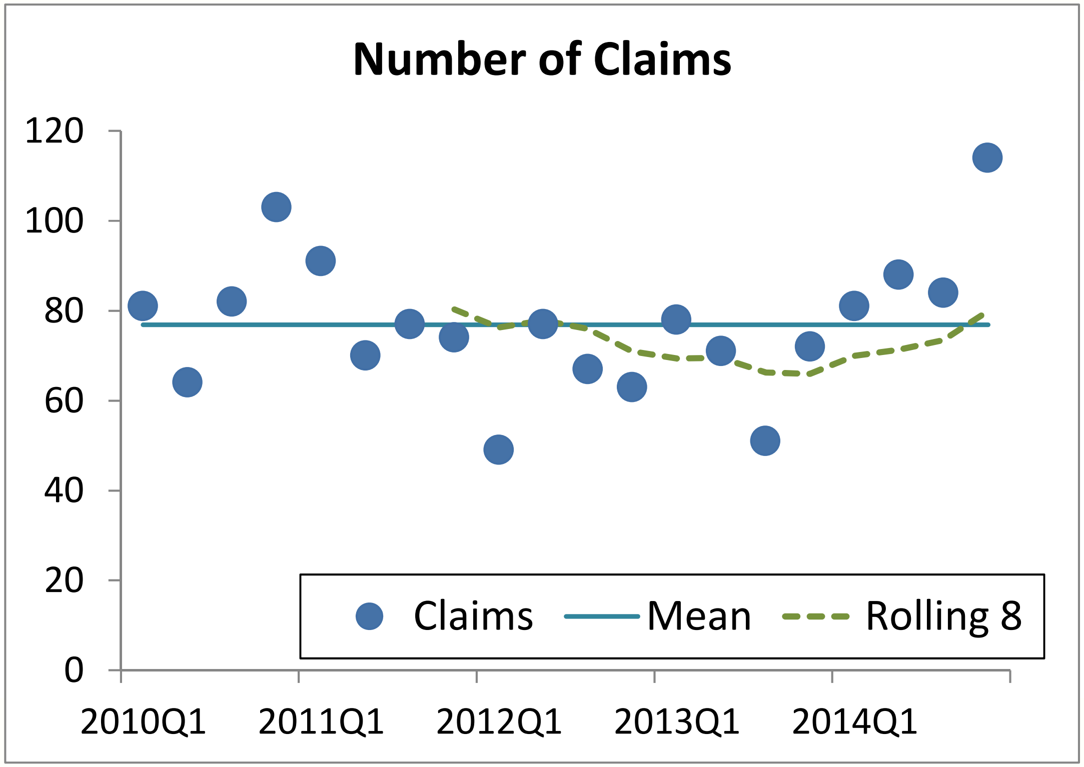
Summary of Data
Number of Claims (Quarterly)



Mean	77
Median	77
Min	49
Max	114
Std Dev	15

- There does not appear to be a trend
- We assumed exposures is constant over this period

Summary of Data
Number of Claims (Quarterly)



Mean	77
Median	77
Min	49
Max	114
Std Dev	15

- Rolling Averages are helpful for looking for trends
- Use annual periods to minimize seasonality effects

Steps

- Estimate Frequency Distribution
  - Generally, estimate frequency and apply to projected exposures
- Estimate Severity Distribution
  - First the Meat (belly) of the distribution
  - Then the Tail

Frequency Distributions

- **Negative Binomial**
- Poisson
- Overdispersed Poison
- Binomial

Frequency Parameters

Actual Data	
Mean	76.9
Standard Deviation	15.4

Negative Binomial

$$\mu = \frac{r \cdot p}{1 - p}$$

$$\sigma^2 = \frac{\mu}{1 - p}$$

Selected Parameters	
r	37
p	0.68

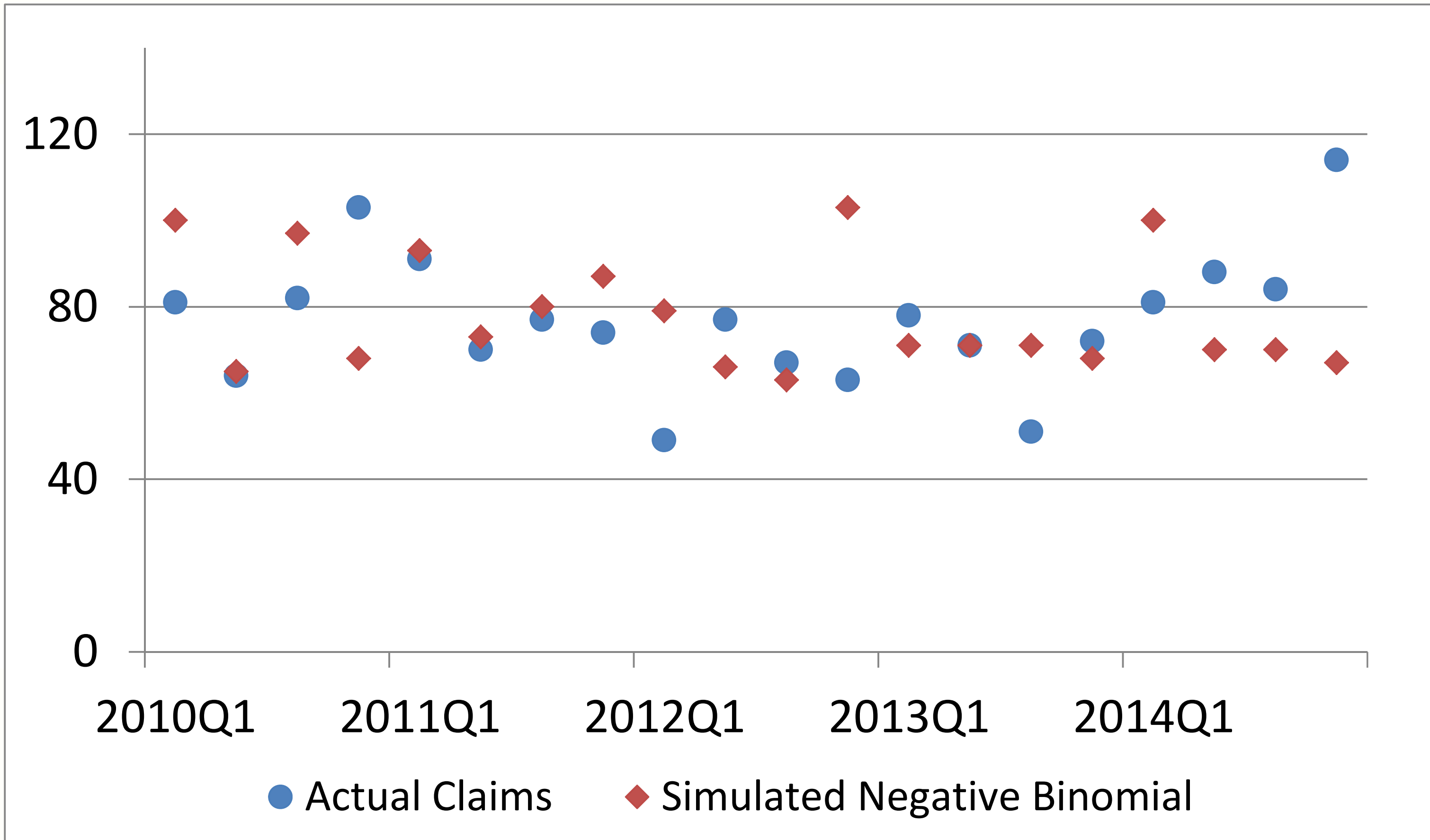
Frequency Parameters

Actual Data	
Mean	76.9
Standard Deviation	15.4

Estimated	
Mean	77.3
Standard Deviation	15.5

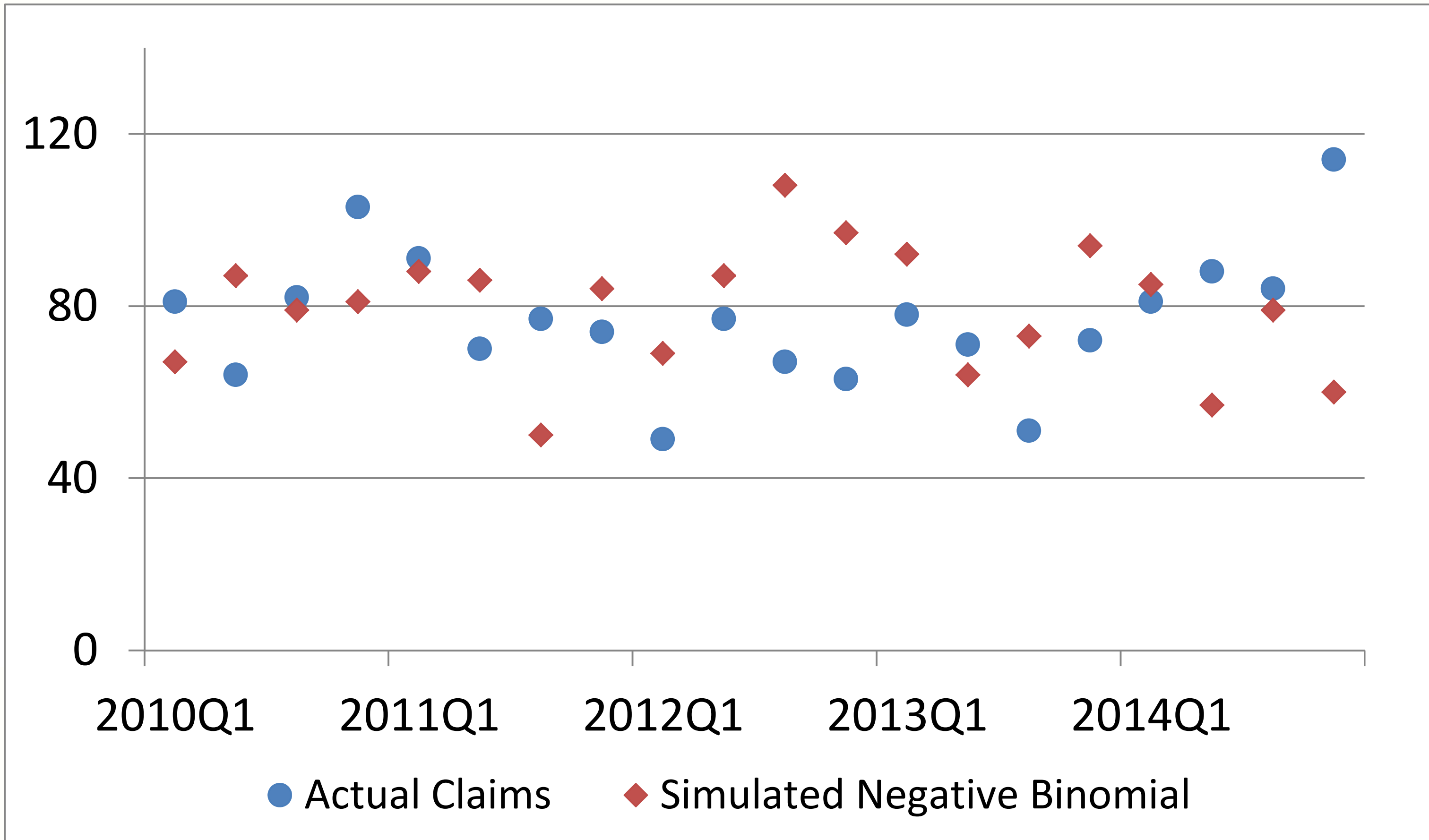
Selected Parameters	
r	37
p	0.68

Frequency Distribution Simulation 1



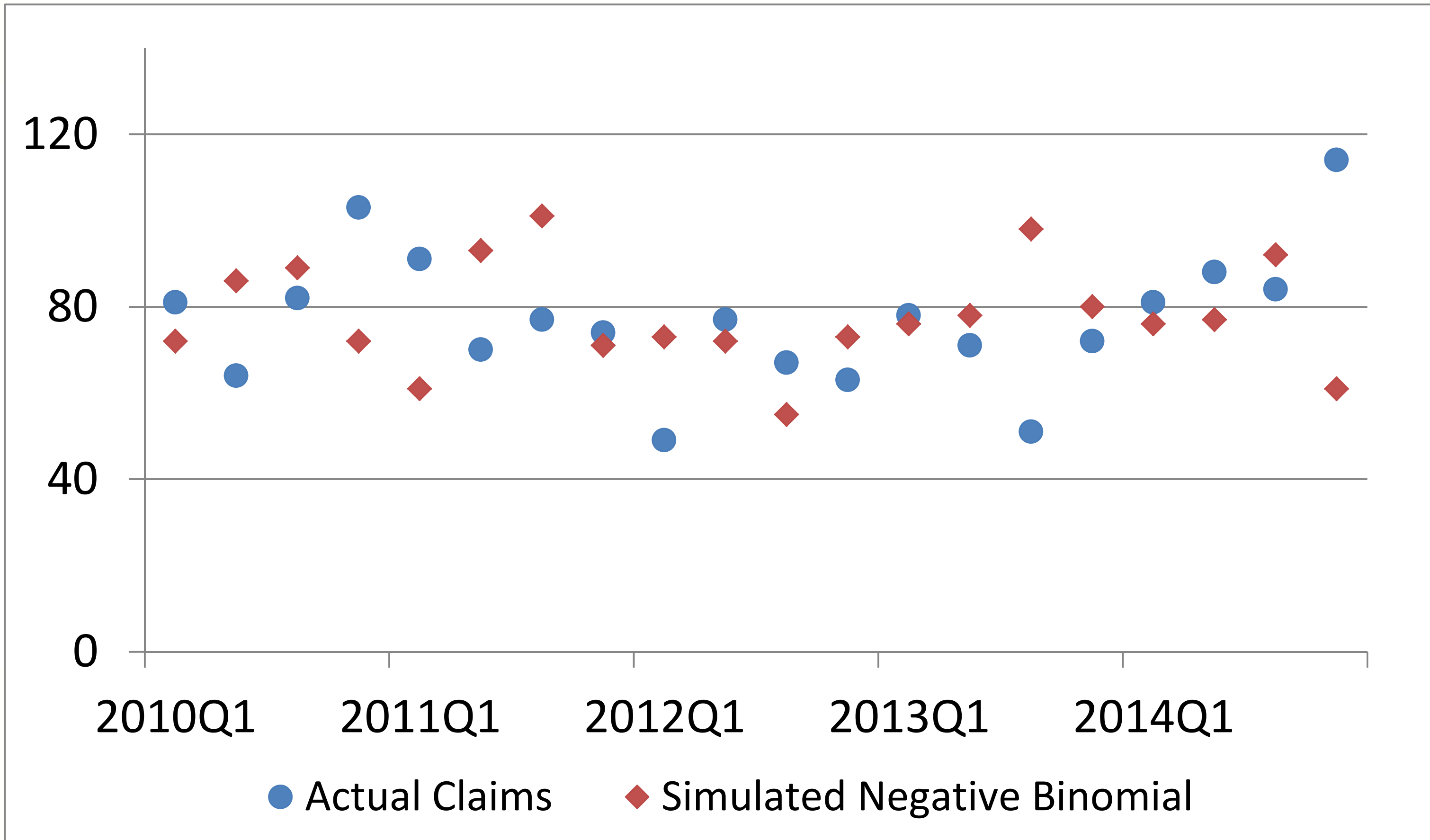
Frequency Distribution

Simulation 2



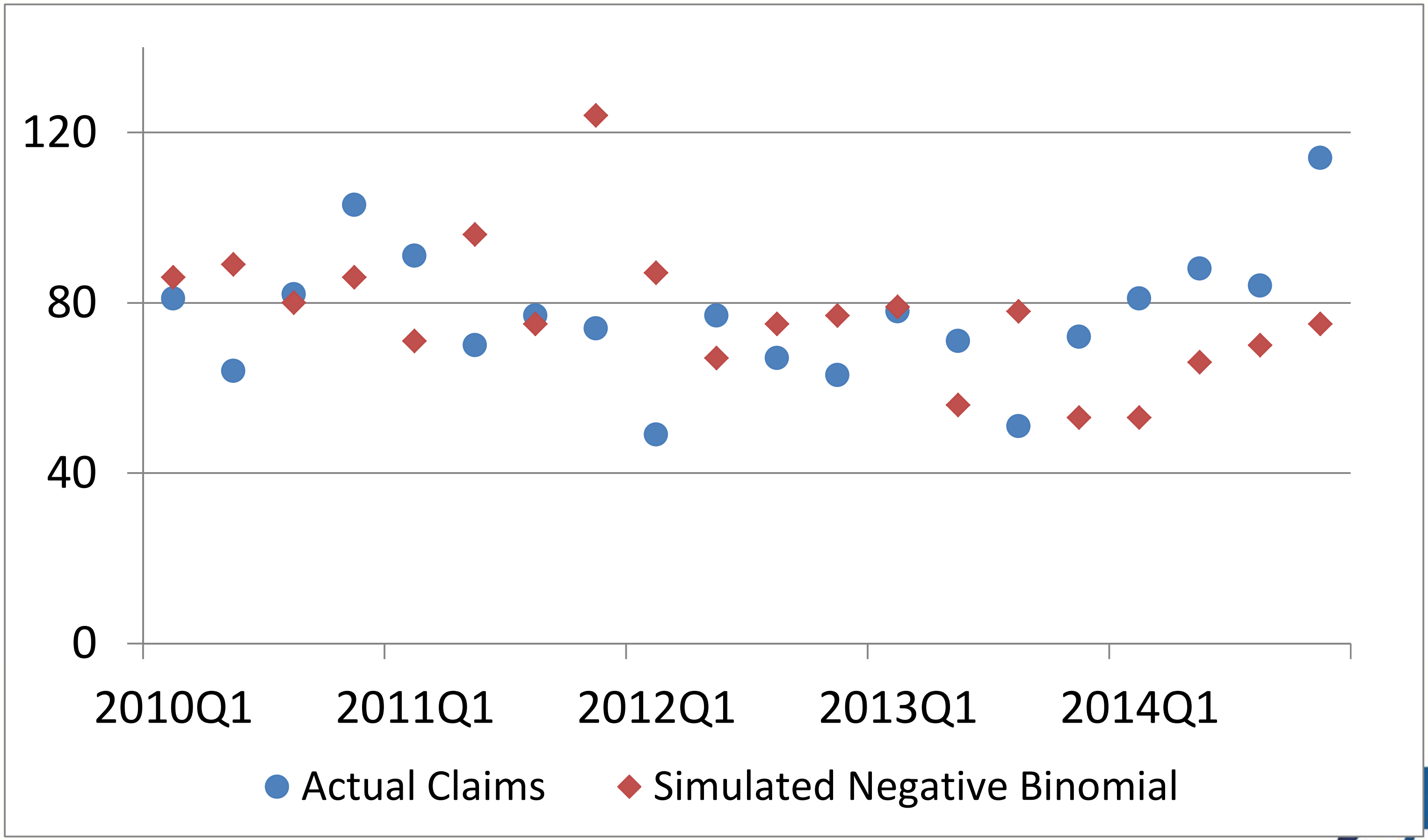
Frequency Distribution

Simulation 3



Frequency Distribution

Simulation 4



Steps

- Estimate Frequency Distribution
  - Generally, estimate frequency and apply to projected exposures
- Estimate Severity Distribution
  - **First the Meat (belly) of the distribution**
  - Then the Tail

Severity Distributions

- Weibull
- Gamma
- Normal
- LogNormal
- Exponential
- Pareto

Severity - Parameters

- Parameter Estimation
- Maximum Likelihood Estimators
- Graph Distribution of Losses with Estimated Distribution
- Graphical representation confirms if the modeled distribution is a good fit

Steps

- Estimate Frequency Distribution
  - Generally, estimate frequency and apply to projected exposures
- Estimate Severity Distribution
  - First the Meat (belly) of the distribution
  - **Then the Tail**

Excess Mean

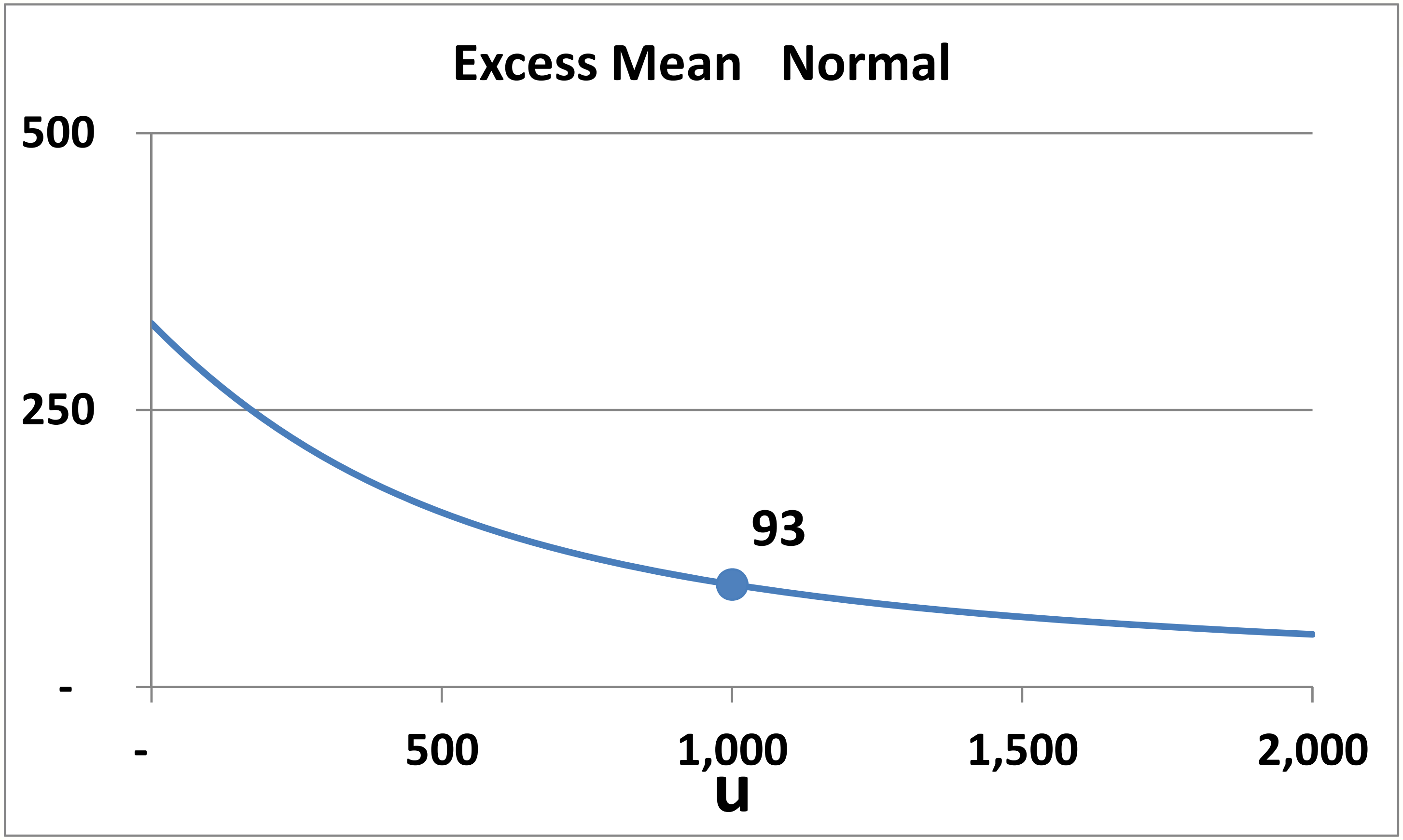
- For all Losses greater than u; the average amount by which they exceed u

$$E[X - u|X > u]$$

  - For any u

- When this increases with respect to u you have a fat tailed distribution

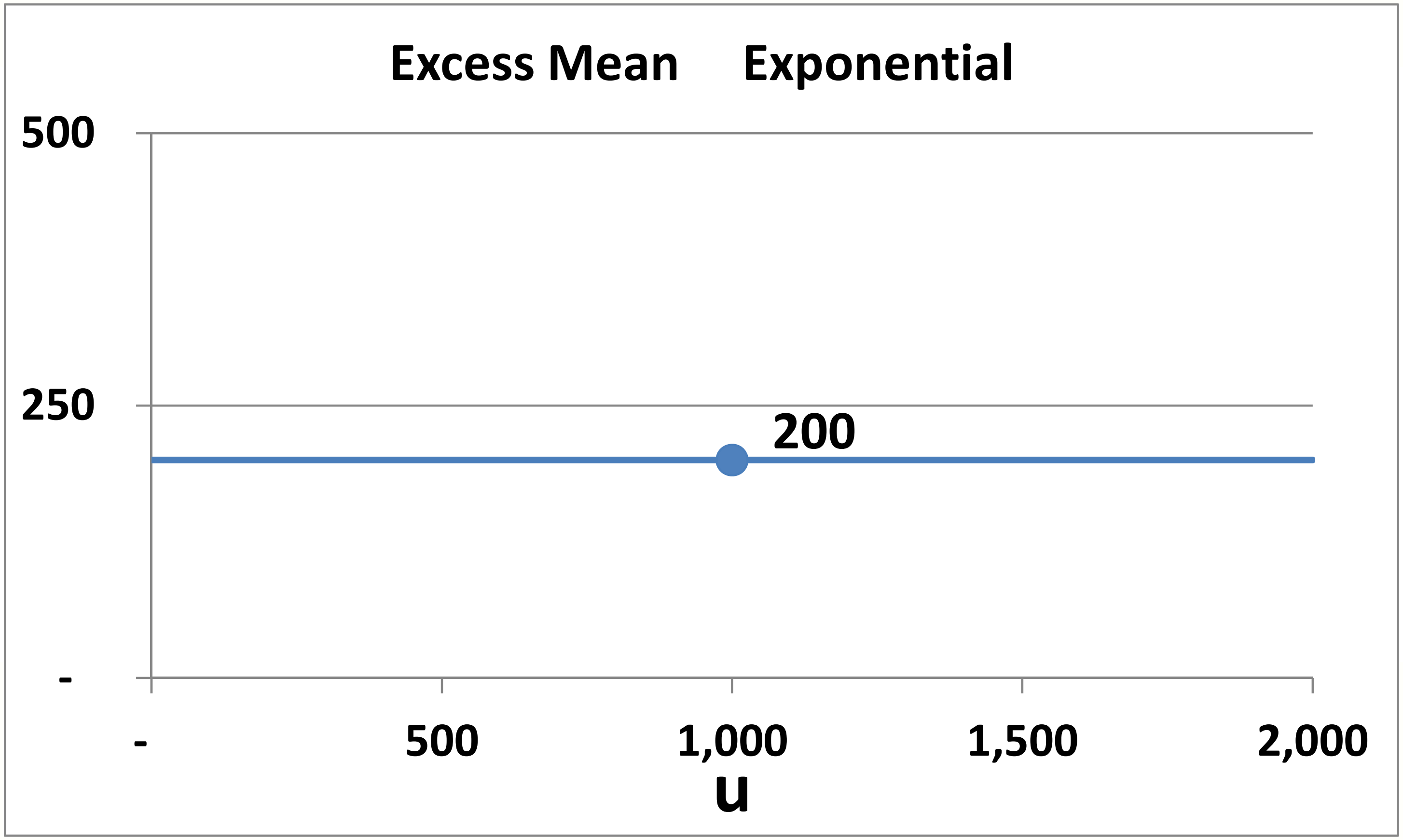
Excess Mean - Normal



$\mu = 200$

$\sigma = 300$

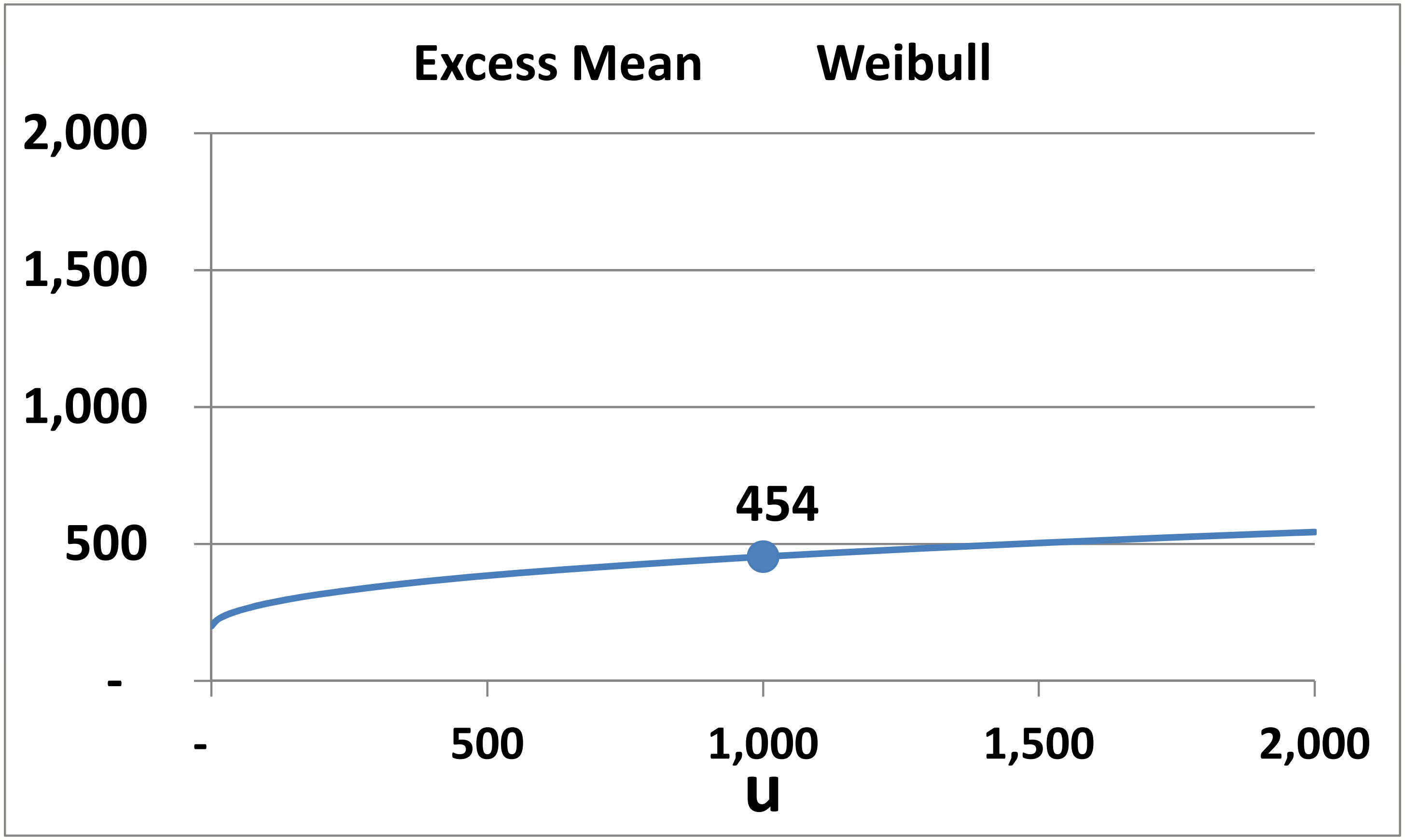
Excess Mean - Exponential



$\mu = 200$

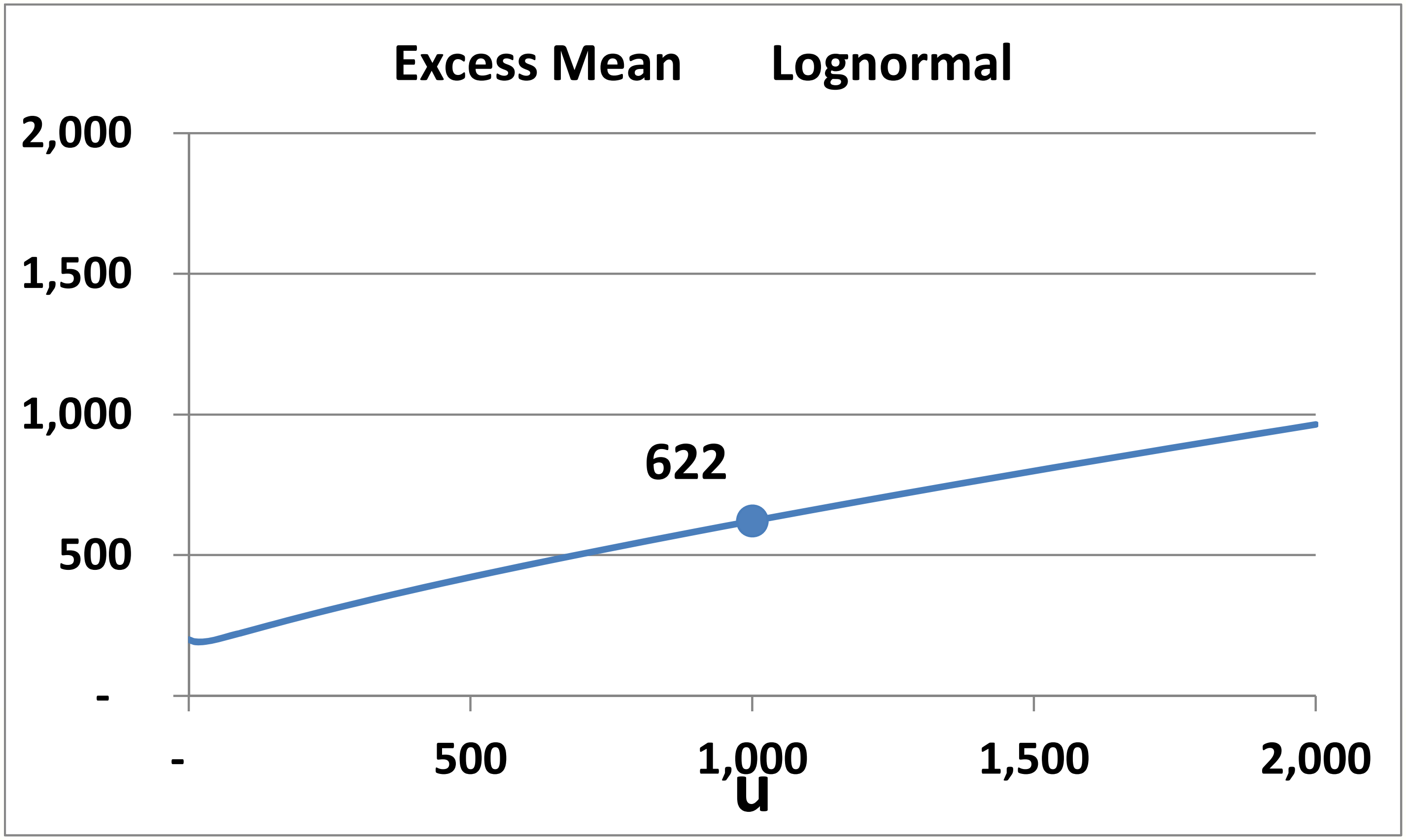
$\sigma = 200$

Excess Mean - Weibull



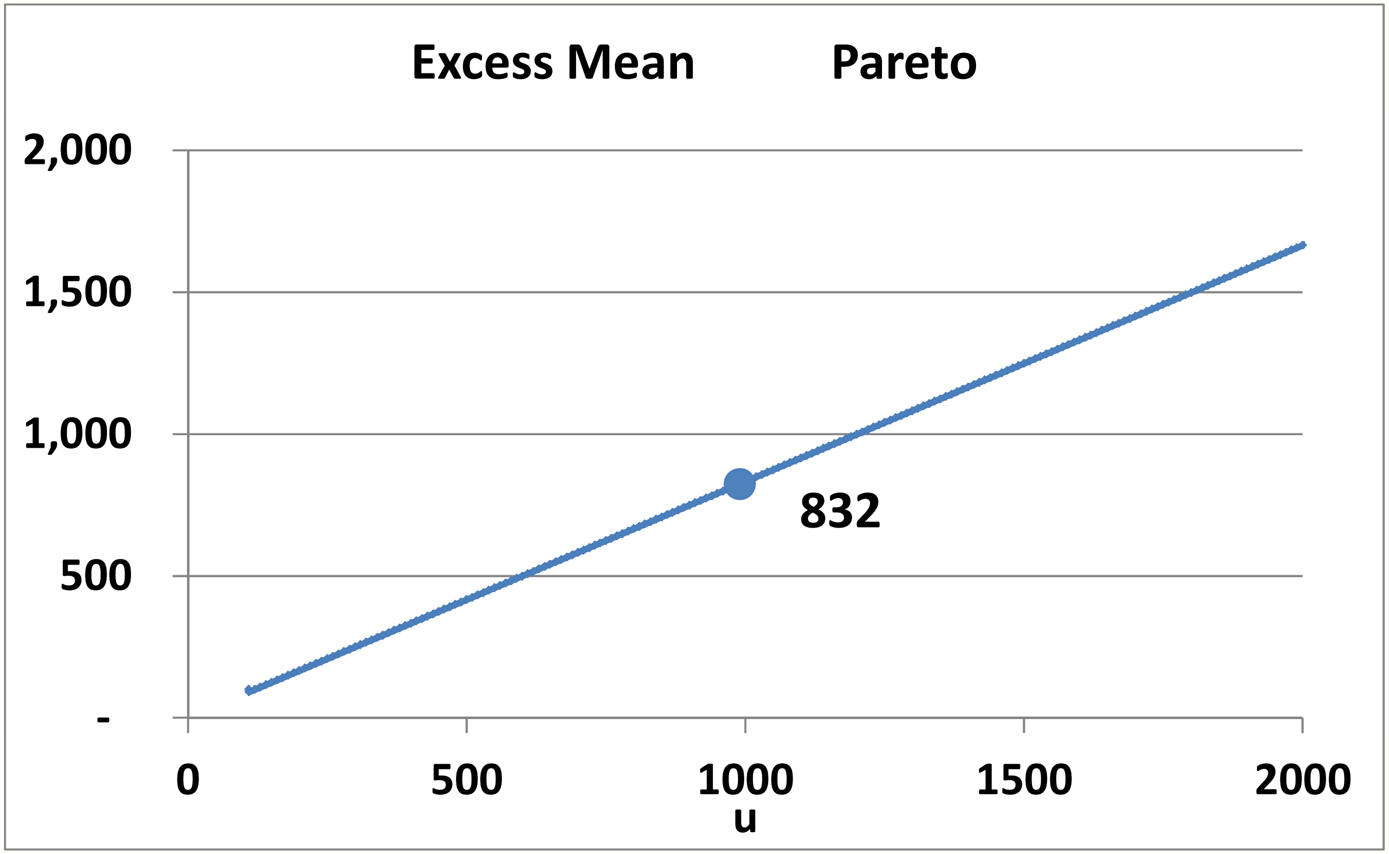
$\mu = 200$
$\sigma = 300$

Excess Mean - LogNormal



$$\mu = 200$$
$$\sigma = 300$$

Excess Mean - Pareto



$\mu = 200$
$\sigma = 300$

Linear

Fat Tailed

- Embrechts:
- Any Distribution with a **Fat Tail**
- In the Limit, has a **Pareto** distribution

- The girth (size, fatness) of the tail is controlled by the parameter ξ (Xi)

- $0.5 < \xi$ Variance does Not Exist
- $1 < \xi$ Mean Does not Exist

Fat Tailed

- Embrechts:
- Any Distribution with a **Fat Tail**
- In the Limit, has a **Pareto** distribution

- $1 < \xi < 2$ Insurance
- $2 < \xi < 5$ Finance (eg. stocks)

Pareto Distribution

$$F_u(x) = 1 - \left(1 + \frac{\xi x}{\beta}\right)^{-\frac{1}{\xi}}$$

$$S_u(x) = \left(1 + \frac{\xi x}{\beta}\right)^{-\frac{1}{\xi}}$$

Data - Severity

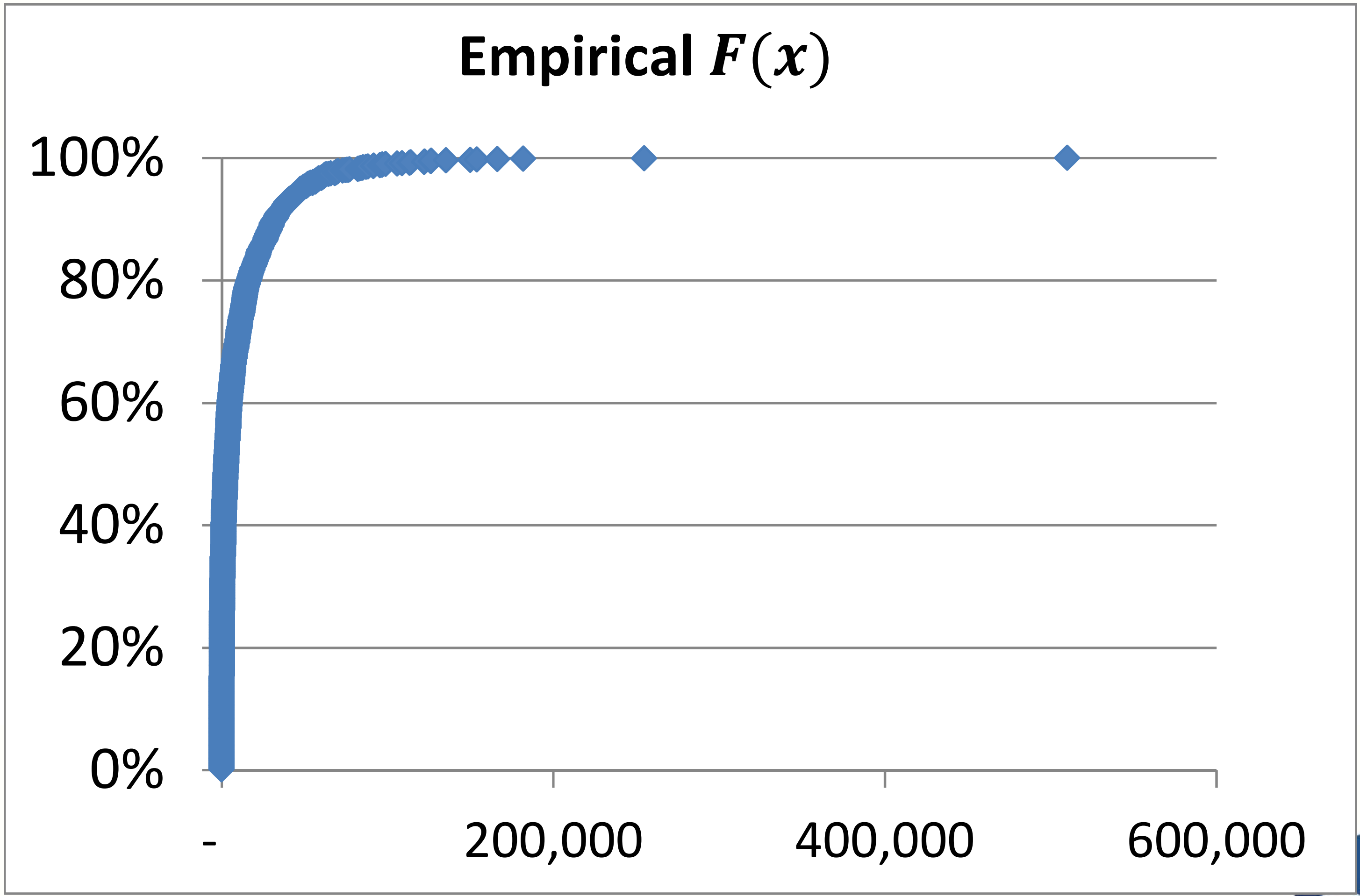
Claim	Quarter	Amount	Amount with Trend
1	2010Q1	14,101	14,101
2	2010Q1	1,824	1,824
3	2010Q1	688	688
…	…	…	…
1534	2014Q4	25	25
1535	2014Q4	32,574	32,574
1536	2014Q4	15,380	15,380
1537	2014Q4	1,016	1,016

Data - Severity

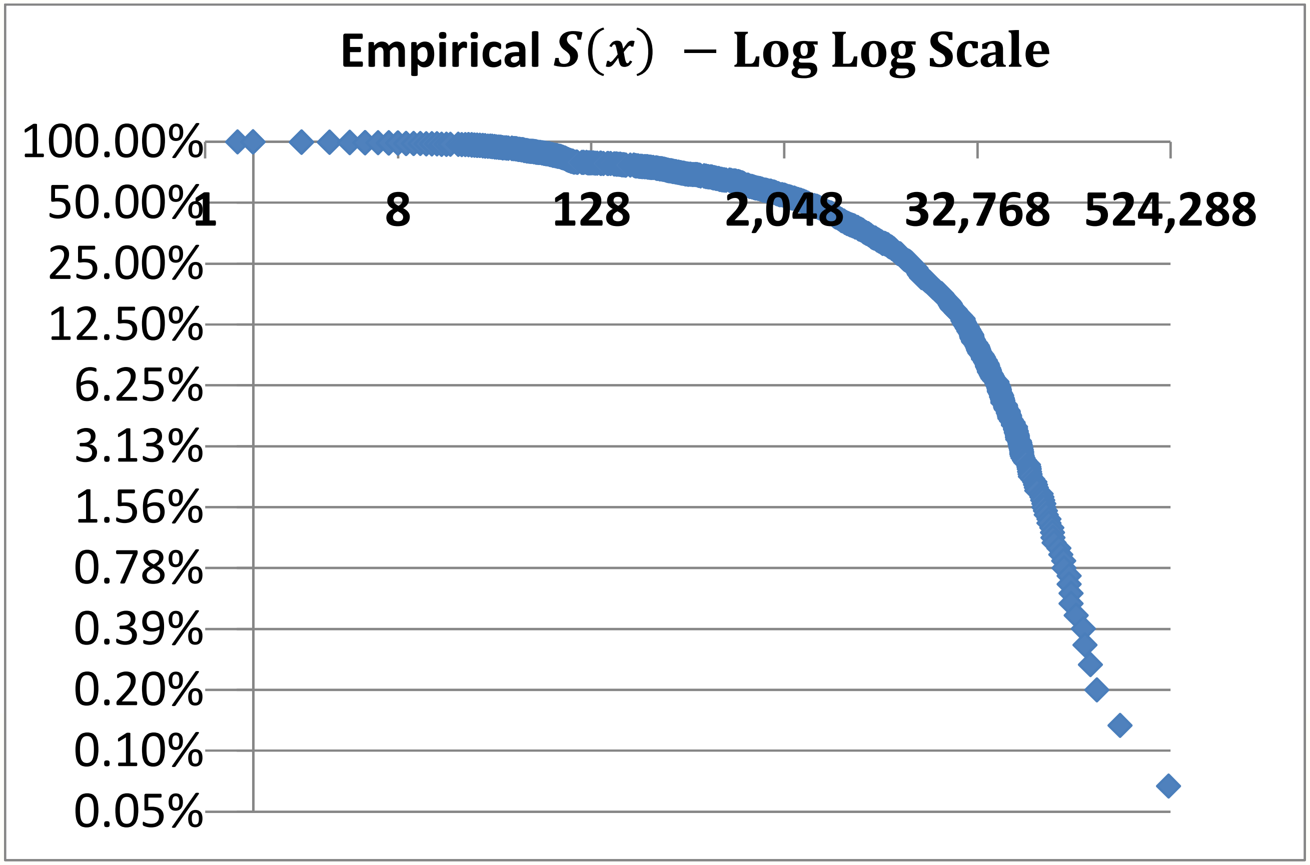
- Largest Claims

Claim	Quarter	Amount
1305	2014Q2	126,434
415	2011Q1	135,387
1392	2014Q3	149,925
1055	2013Q3	153,900
1423	2014Q3	166,335
225	2010Q3	181,881
1381	2014Q3	254,864
1310	2014Q2	**510,060**

Severity



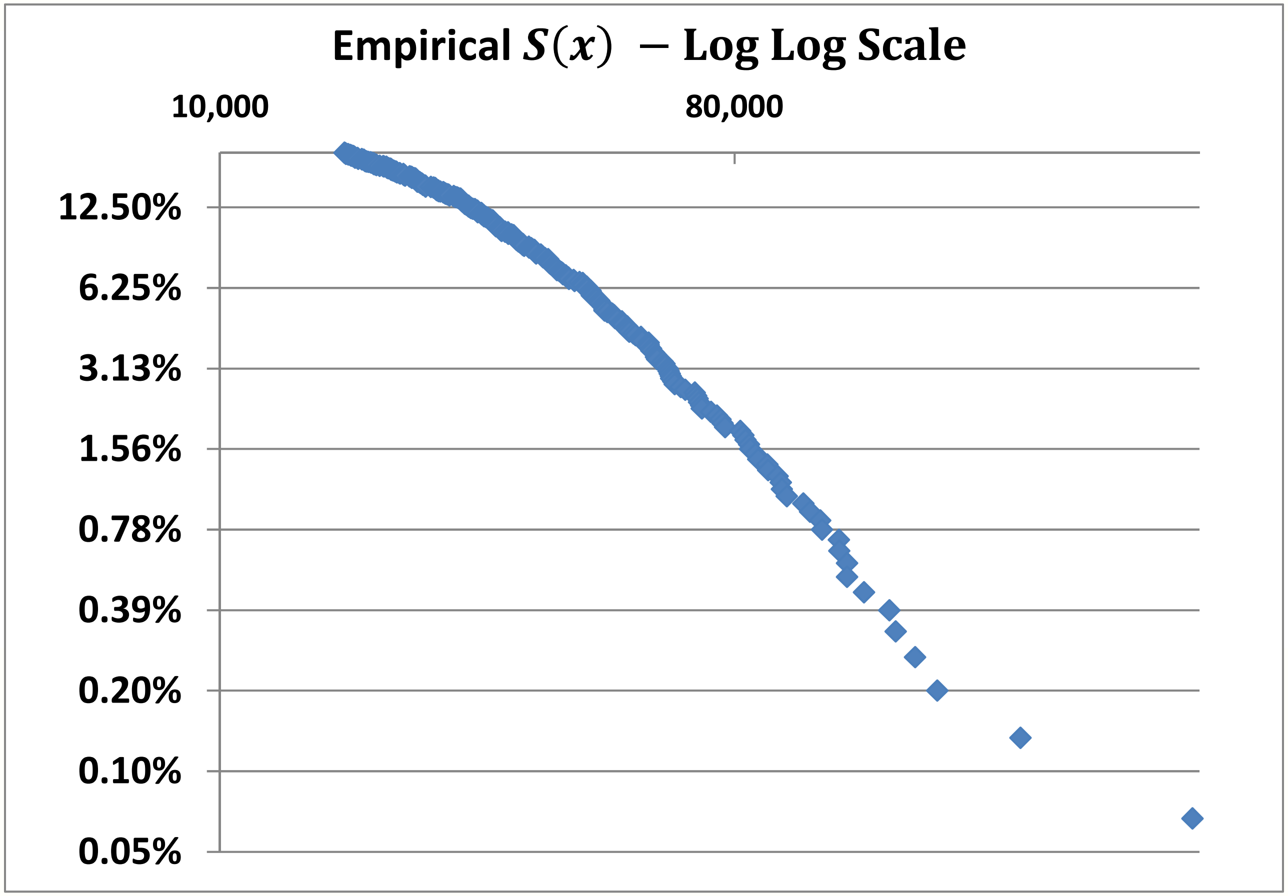
Survival Function – Log Log Scale



$S(x) = 1 - F(x)$

Survival Function – Log Log Scale

The Tail of the Pareto Distribution is a **Line** on a log log graph

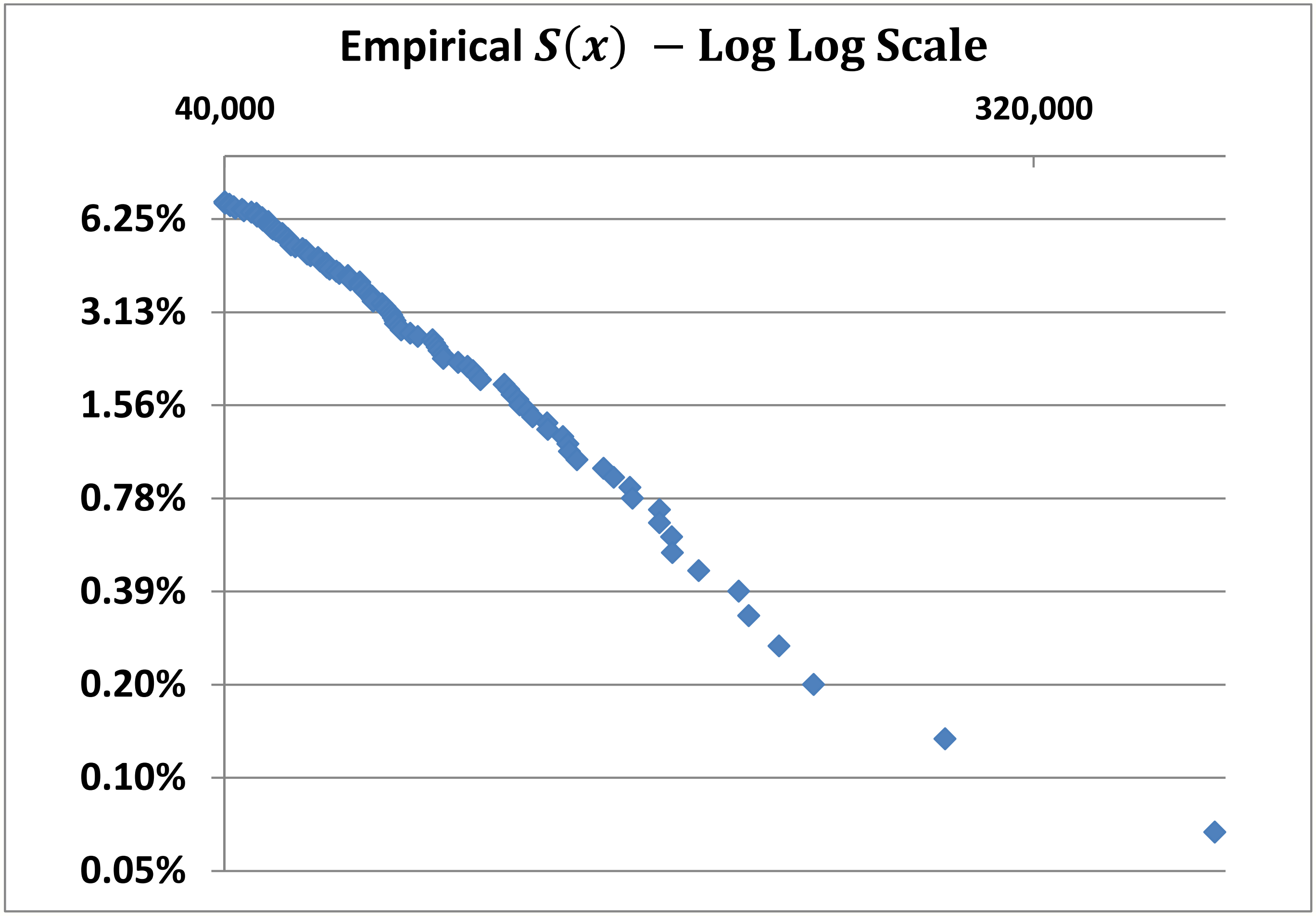


$S(x) = 1 - F(x)$

Look for the Turn

Survival Function – Log Log Scale

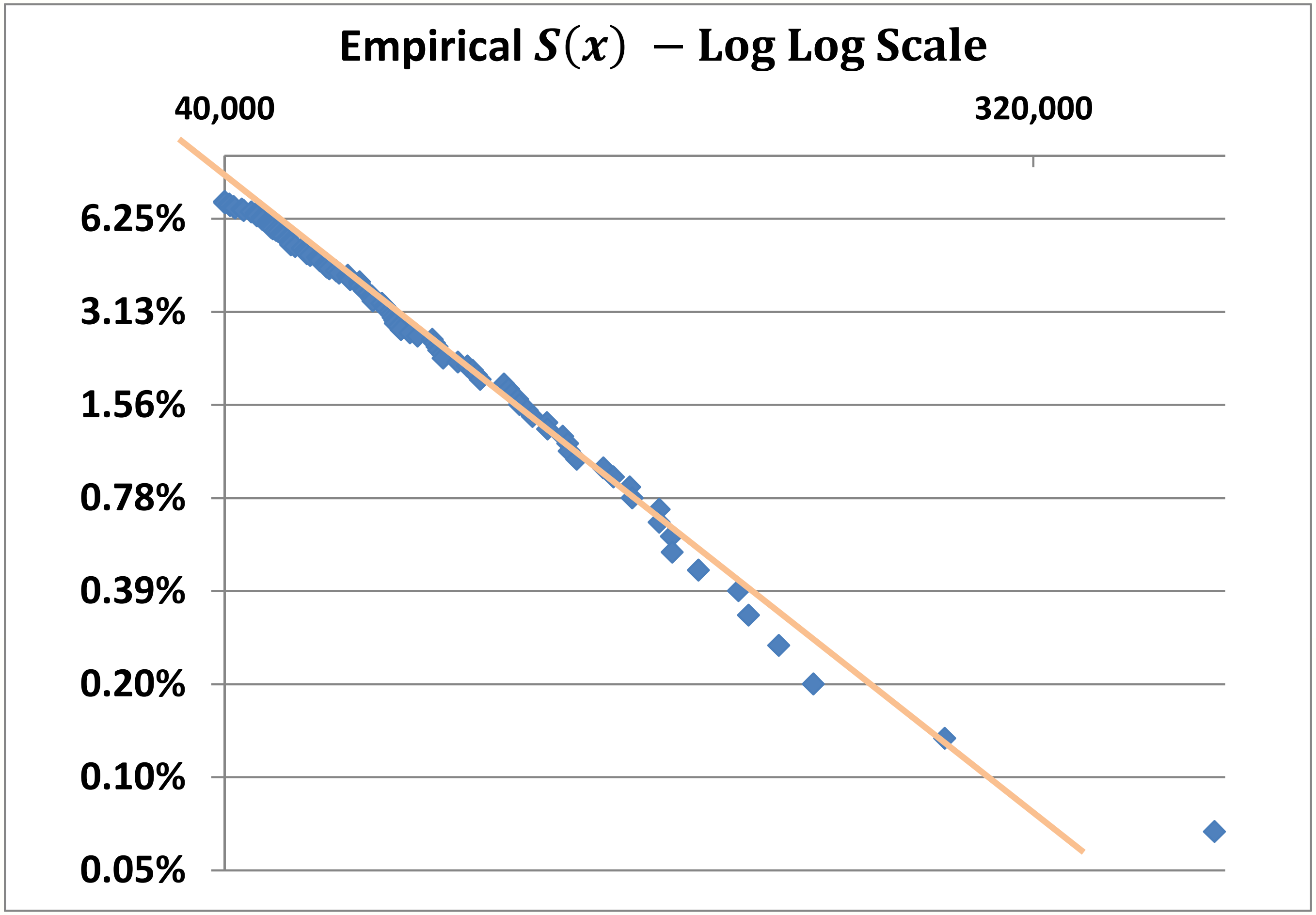
The Tail of the Pareto Distribution is a **Line** on a log log graph



$S(x) = 1 - F(x)$

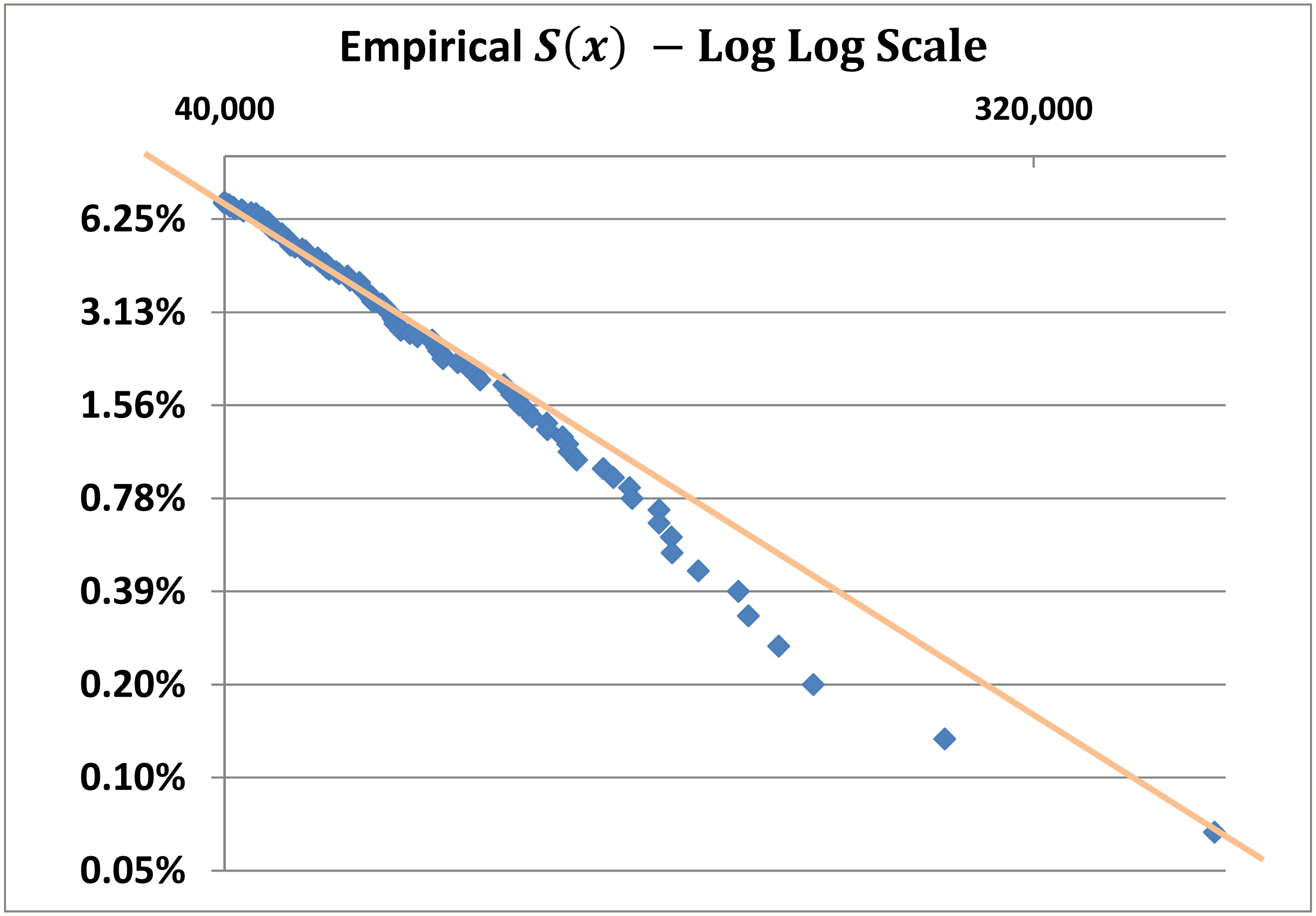
Survival Function – Log Log Scale

The Tail of the Pareto Distribution is a **Line** on a log log graph



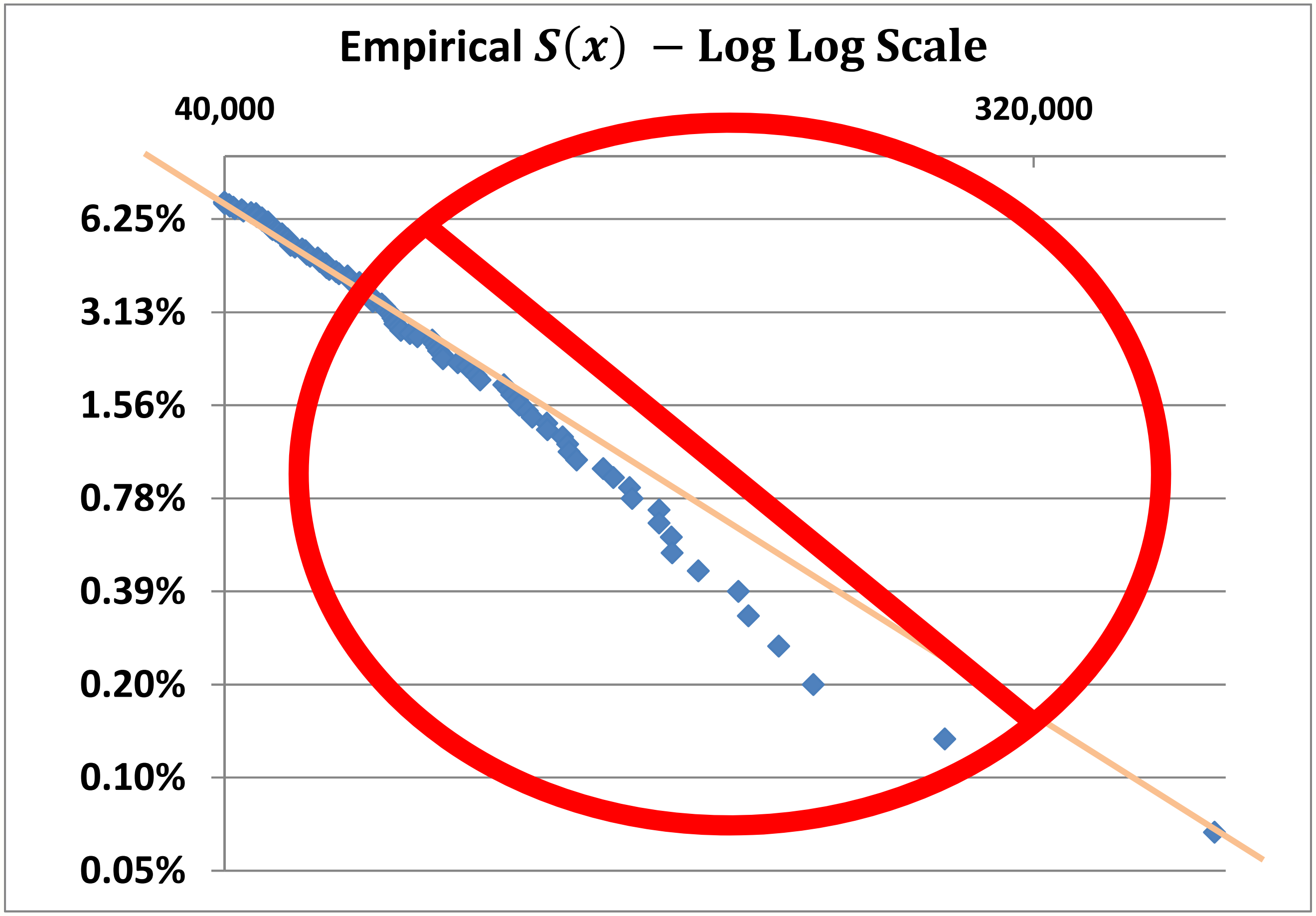
$S(x) = 1 - F(x)$

Survival Function – Log Log Scale



Don't pick a line based on just the end points

Survival Function – Log Log Scale



Don't pick a line based on just the end points

Select Parameters - Severity

- $u = 50{,}000$
- $F(u) = 95.26\%$
- $S(u) = 4.74\%$
- You can try different u's to see if the results are similar (or different)
- u should be deep enough in the tail, that the graph is a line
- Want enough points past u, so we have confidence in $F(u)$

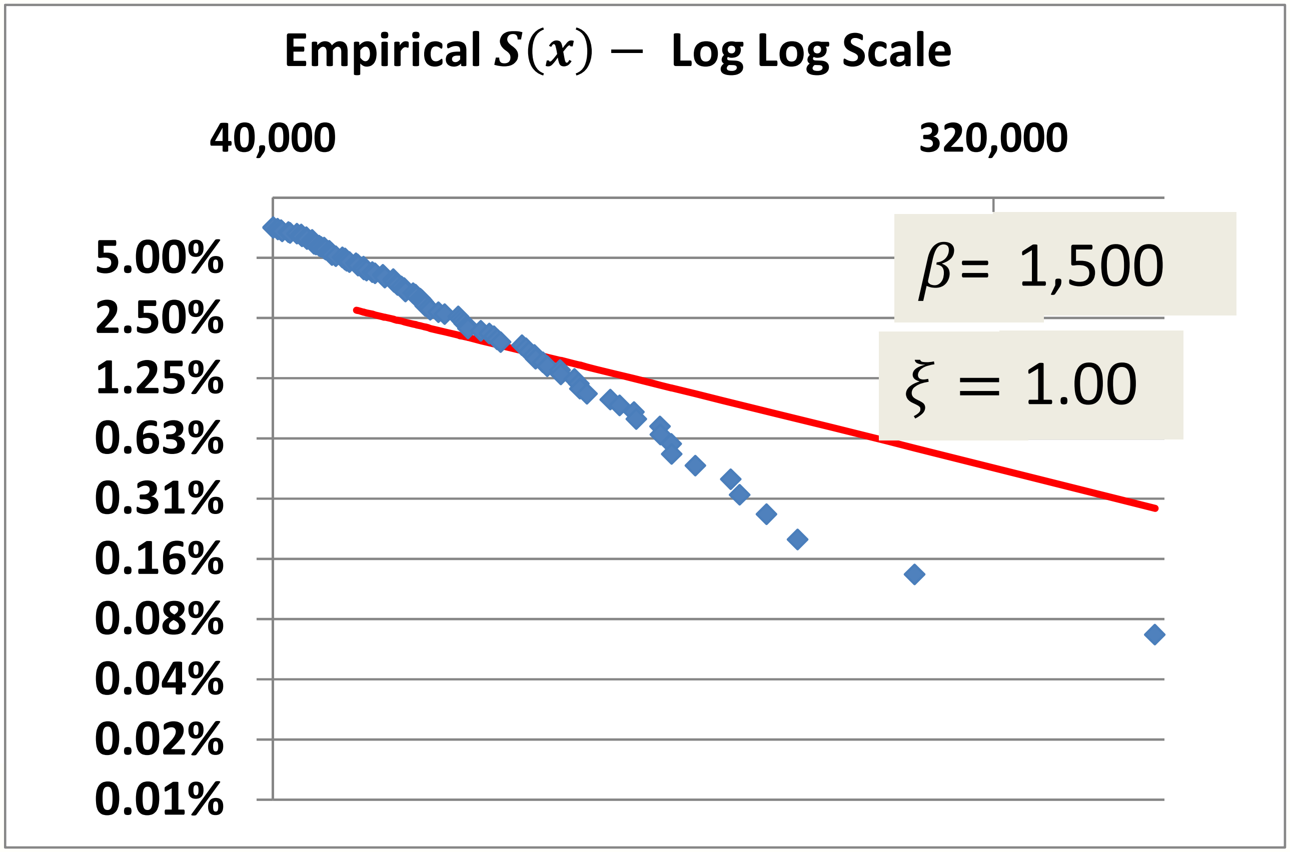
Select u, determine $F(u)$

Claim	Quarter	Amount	Empirical $F(x)$
1348	2014Q3	49,302	95.12%
592	2011Q4	49,479	95.19%
105	2010Q2	49,885	95.25%
		50,000	**95.26%**
686	2012Q1	50,862	95.32%
682	2012Q1	51,085	95.38%
639	2011Q4	51,160	95.45%

$$F(x) = \frac{ranking}{\boldsymbol{n+1}}$$

n =Number of Claims

First Parameter Selection

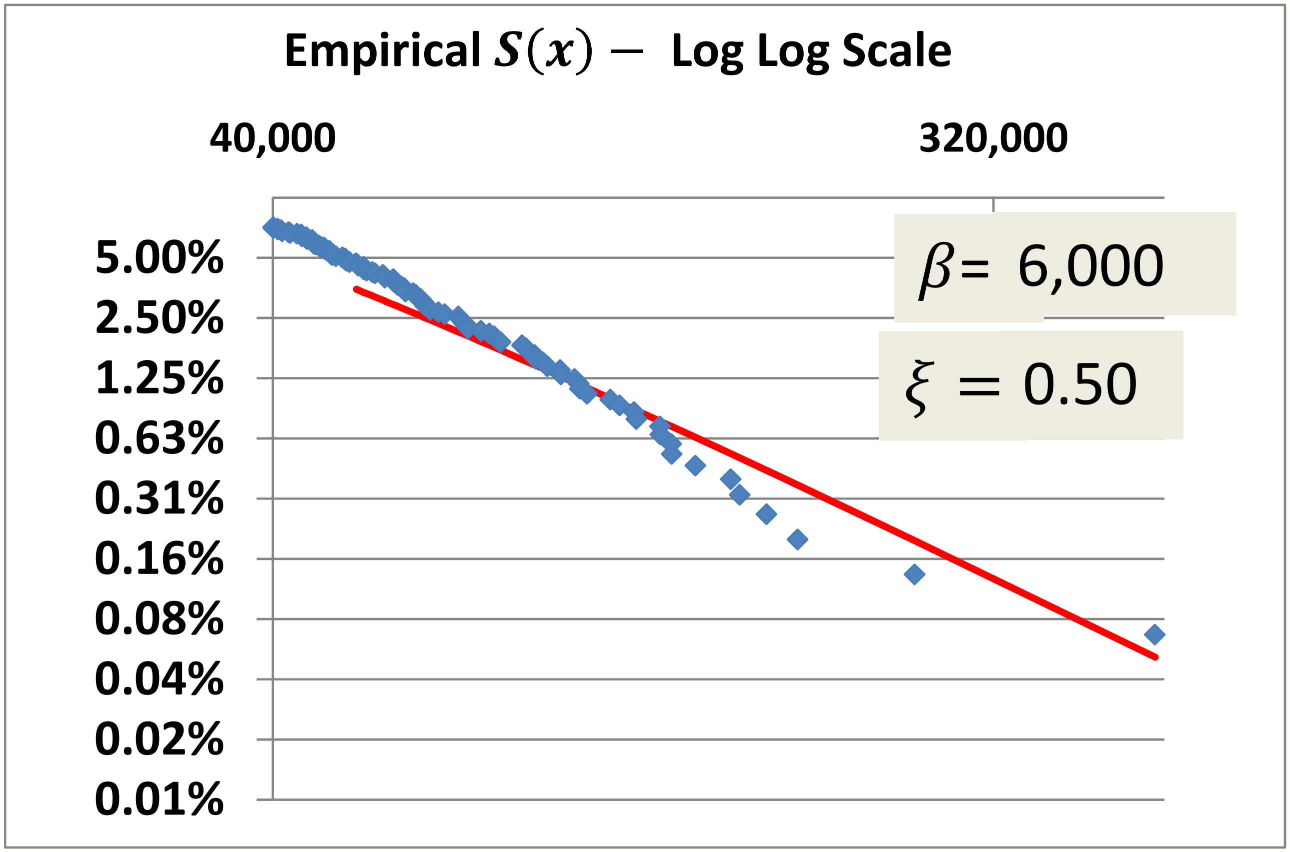


$u = 50{,}000$

$S(u) = 4.74\%$

- Select ξ first, then β
- The selected ξ is too high

Second Parameter Selection

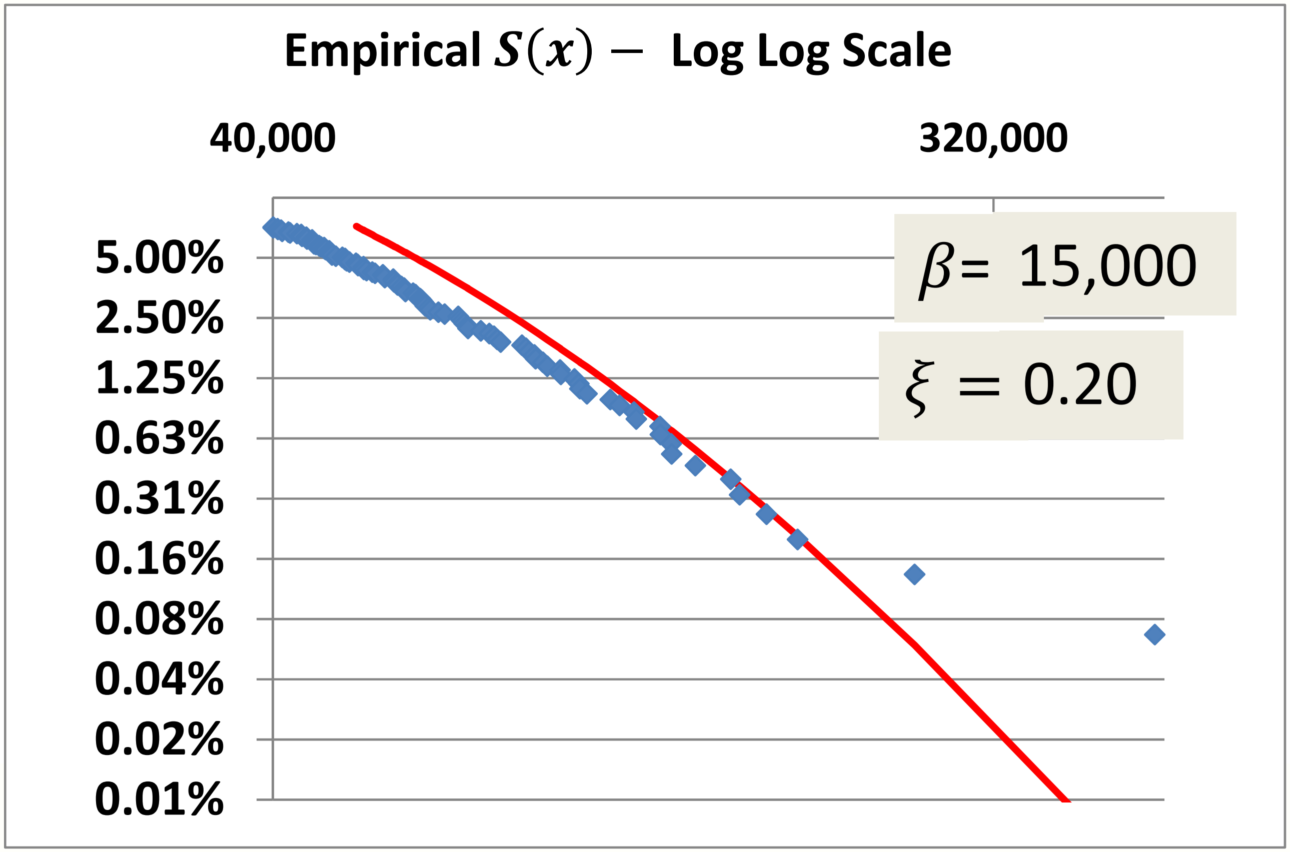


$u = 50{,}000$

$S(u) = 4.74\%$

- Better. The slope is steeper, but it's still too shallow

Third Parameter Selection



$u = 50{,}000$

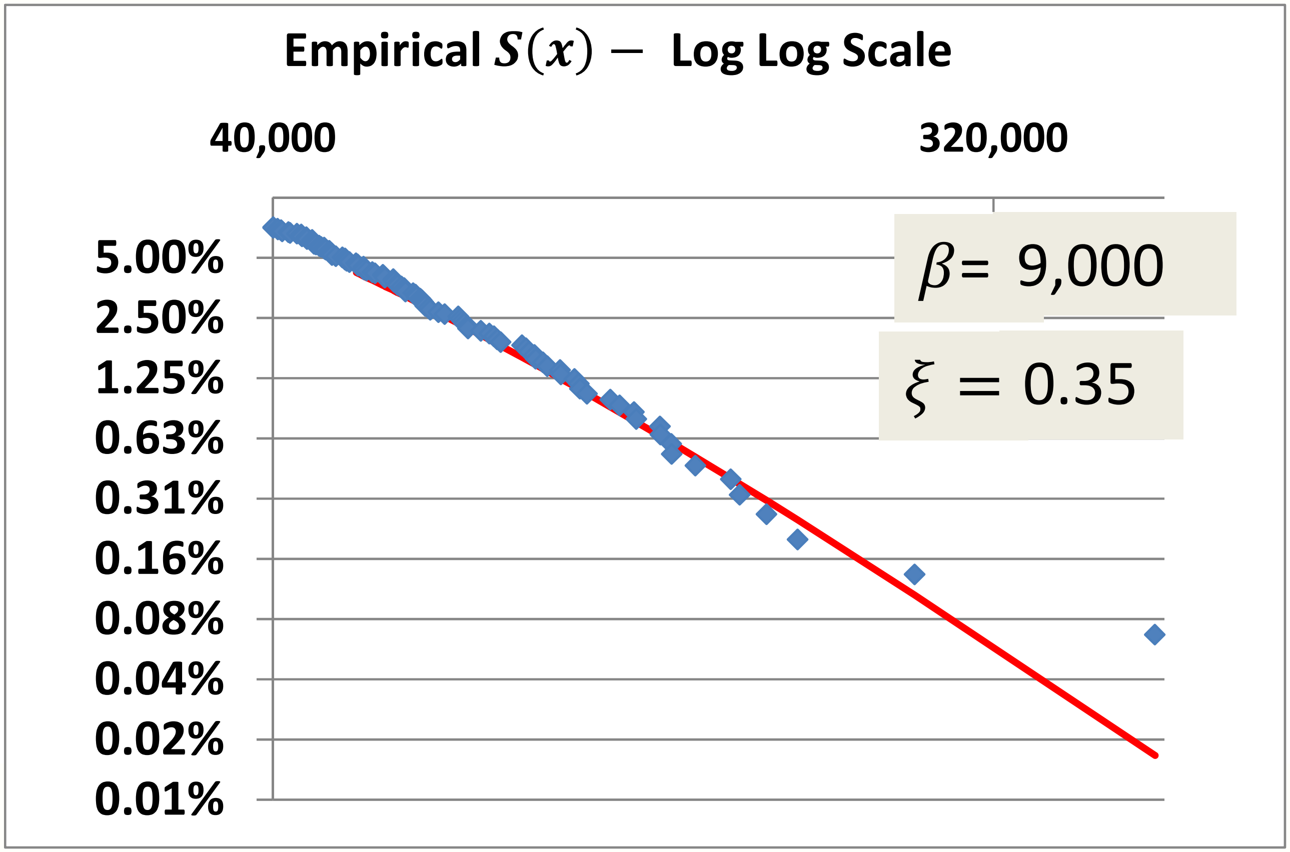
$S(u) = 4.74\%$

Empirical F(x)

Pareto S(x)

- Now, it drops too quickly
- $\xi \in [0.2, 0.5]$

Fourth Selection of Parameters



$u = 50{,}000$

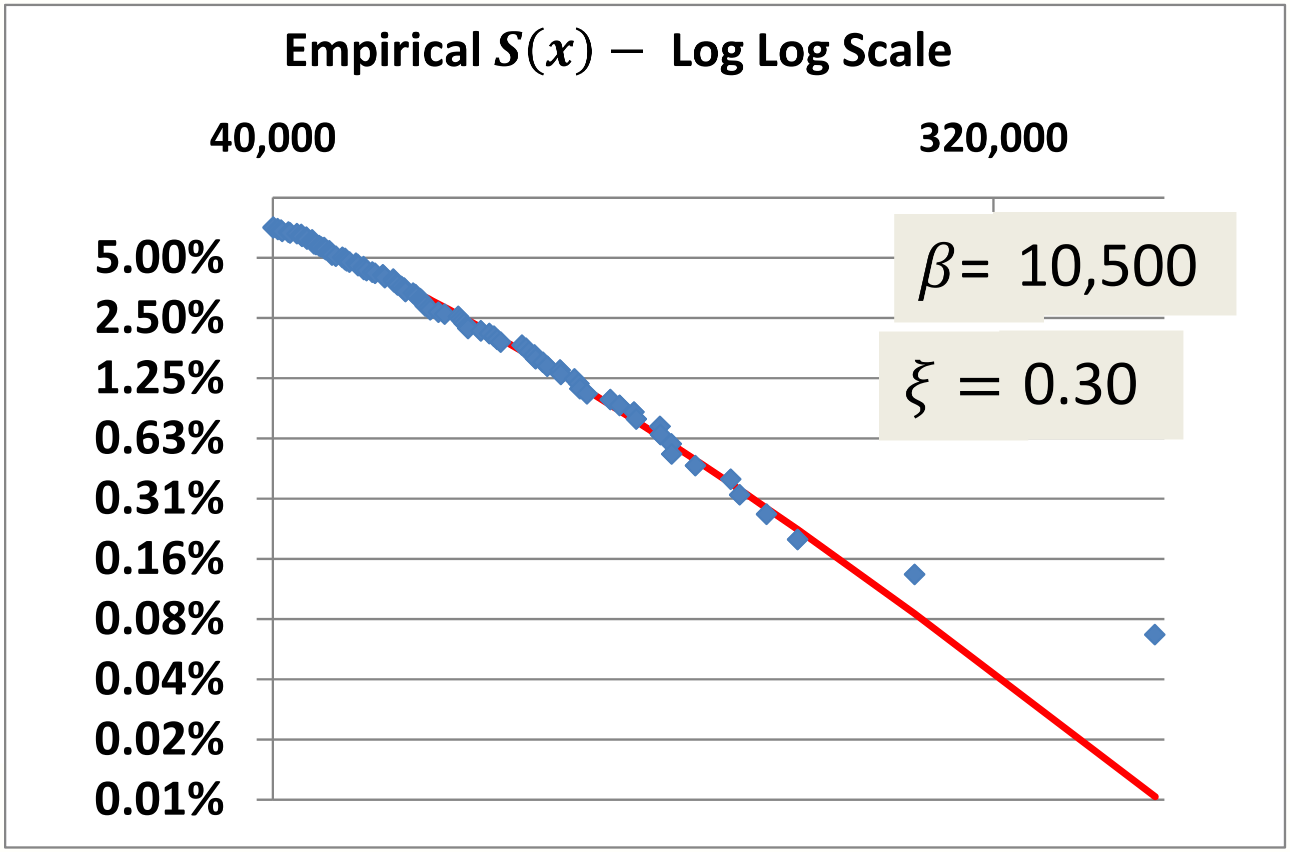
$S(u) = 4.74\%$

Empirical F(x)

Pareto S(x)

- This is much better
- Let's look a little lower and higher

Fifth Selection of Parameters

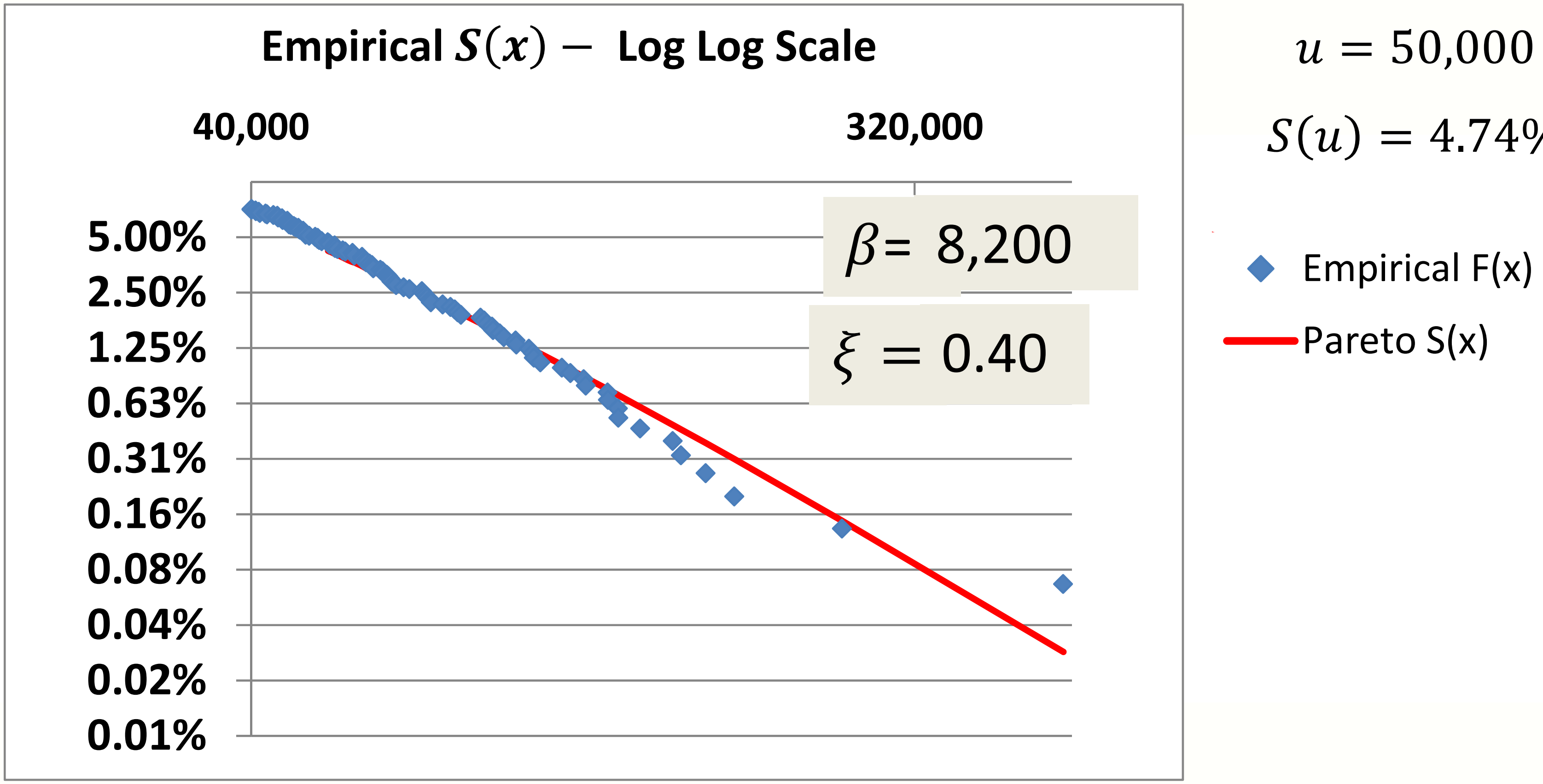


$u = 50{,}000$

$S(u) = 4.74\%$

- This is also quite good

Sixth Selection of Parameters

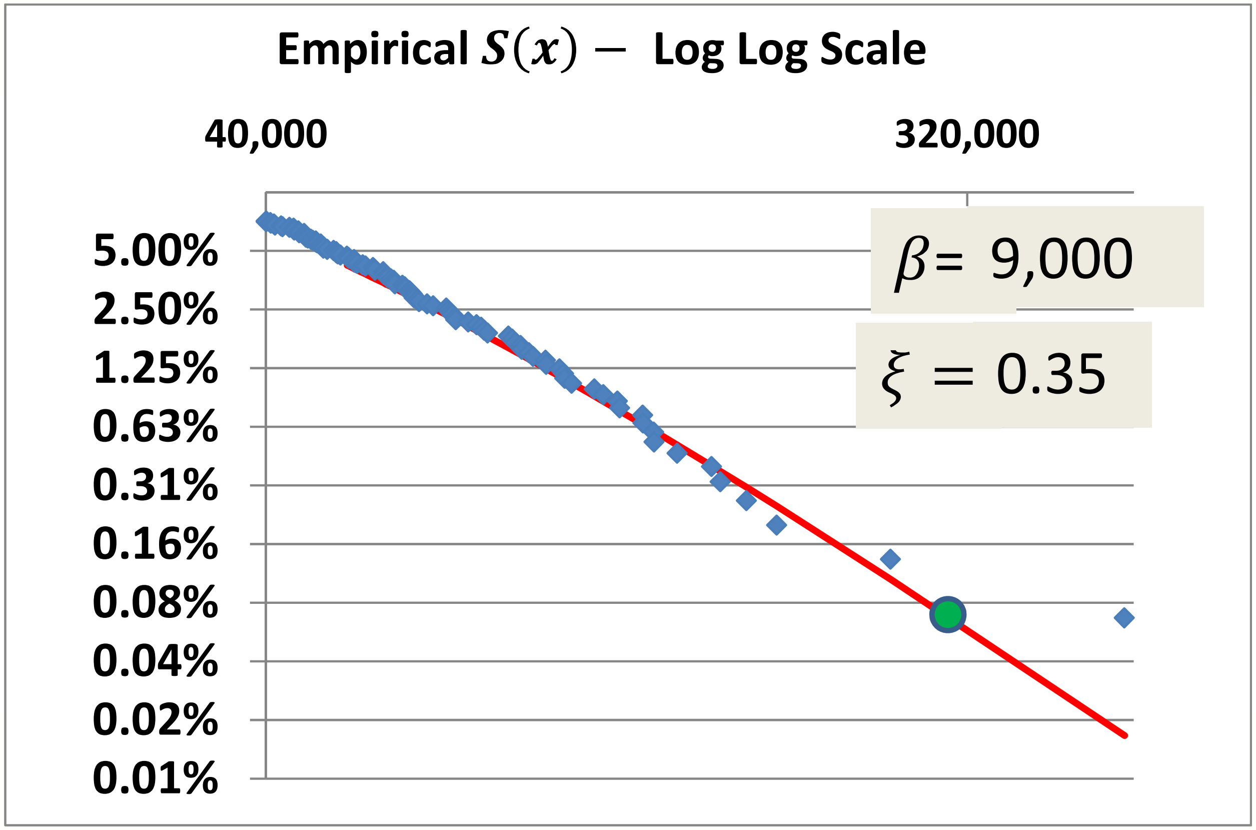


$u = 50{,}000$

$S(u) = 4.74\%$

- Excellent Fit from 40k to about 120k, not as good for the last 7 points or so

Final Selection of Parameters



$u = 50{,}000$

$S(u) = 4.74\%$

S(x) empirico

S(x) Pareto

- 300k was the size of the expected largest claim

Largest Expected Loss

ξ	β	**Largest Loss** 99.93%ile
0.30	10,500	277,000
0.35	9,000	304,000
0.40	8,200	358,000

Resumen Severidad

- We estimate a distribution for the bely
  - Up to 95.26%
- For the Tail, we use Pareto with the following parameters:
  - $u = 50{,}000$
  - $F(u) = 95.26\%$
  - $\xi = 0.35$
  - $\beta = 9{,}000$

Steps

- Estimate Frequency Distribution
  - Generally, estimate frequency and apply to projected exposures
- Estimate Severity Distribution
  - First the Meat (belly) of the distribution
  - Then the Tail

Assumptions

- Book is Homogenous
  - Homes, Apartments
- Useful Exposure Base
  - Cars, Homes, Revenue
- Make Loss Trend Adjustments
  - Market Inflation, or more detailed, if available
- Model Risk
  - Exposures, Inflation

Thank You