Bayesian Intercompany Credibility in Loss Reserving

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Motivation

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We want

- to use similar lines of business to inform our forecasting.
- a structured and flexible way to incorporate that information.
- a simple model structure, easily interpreted by stakeholders.
- visible assumptions for easy audit.

Data

Data

We will be comparing our models using the schedule P data from the NAIC database. We will focus on:

- Paid losses (though incurred are also available)
- End-of-year 1997 valuation date
- Accident years 1988-1997
- 10 development lags
- Commercial auto insurance
- 15 insurers

Data

Table: Run-off triangle from Schedule P of NAIC

Acc. Year	0	1	2	3	4	5	6	7	8	9		
1988	×	×	×	×	×	×	×	×	×	×		
1989	×	×	×	×	×	×	×	×	×		←	1998
1990	×	×	×	×	×	×	×	×			←	1999
1991	×	×	×	×	×	×	×				←	2000
1992	×	×	×	×	×	×					←	2001
1993	×	×	×	×	×						←	2002
1994	×	×	×	×							←	2003
1995	×	×	×								←	2004
1996	×	×									←	2005
1997	×										←	2006

Cumulative Paid Losses

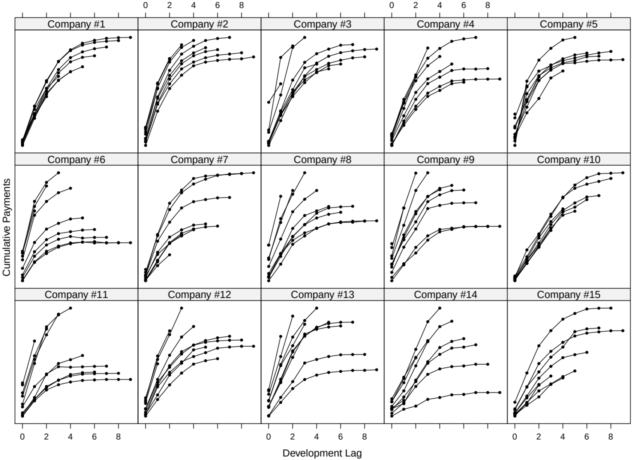


Figure: Multiple time series plot of cumulative paid loss

Model

Basic Model Structure

We start with the model

$$C_{ij}^{(n)} \sim N\left(C_{ij-1}^{(n)}\beta_j^{(n)}, \left(\sigma_j^{(n)}C_{ij-1}^{(n)}\right)^2\right)$$
$$\beta_j^{(n)} \sim N(\mu_j, \theta^2)$$

For

- Accident year i
- Development year j
- Insurer n

As $\theta^2 \to \infty$, the data is given full credibility.
As $\theta^2 \to 0$, the development factors approach the overall mean.

Prior Specification

This model is very flexible. You can incorporate basically any mean structure you like through the prior distribution of μ_j. Here are two options for the prior specification of μ_j.

$$\mu_j \sim N(a, b^2) \qquad \text{[Common prior]}$$

$$\mu_j \sim \begin{cases} N(a, b^2) & \text{if } j < k \\ N(1, 0.0001^2) & \text{if } j \geq k \end{cases} \qquad \text{[Changepoint prior]}$$

$$k \sim DU(1, 10)$$

Results

Normality Assumption

Our model depends on normality. Checking the normal qq plot and the residuals, there appear to be no real concerns.

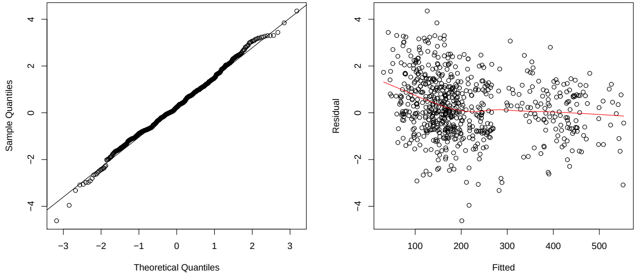
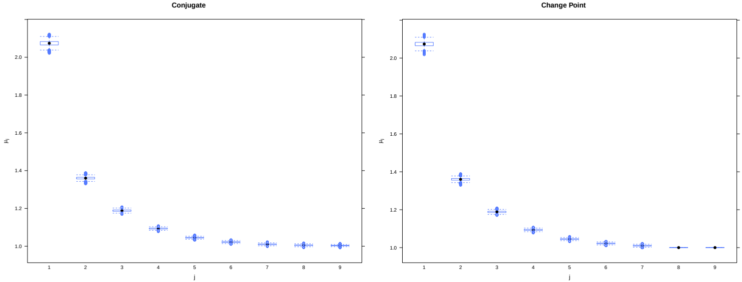


Figure: Normal qq plot and residual plot

Posterior Distribution of μ_j

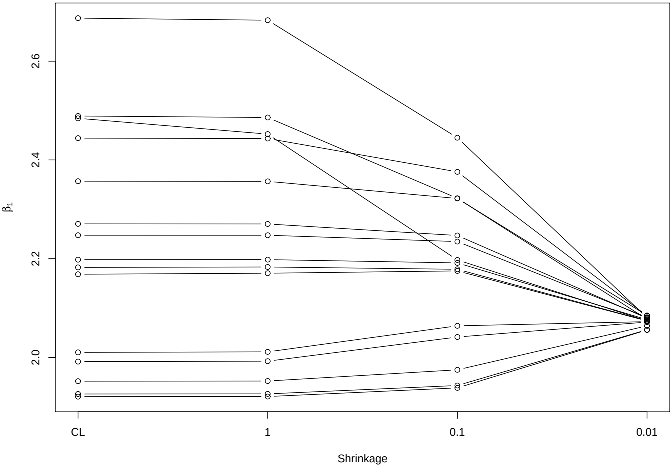
The posterior distribution of μ_j depends on our choice of prior structure.



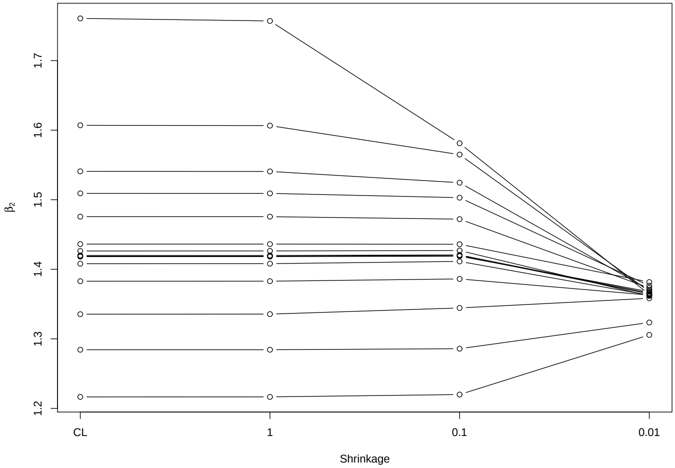
Development Factor Shrinkage

- As the value of θ decreases, the posterior means of the development factors (β_j) shrink to the overall mean.
- Note that θ is an absolute (not relative) value, so how small it is will depend on the size of the β_j.

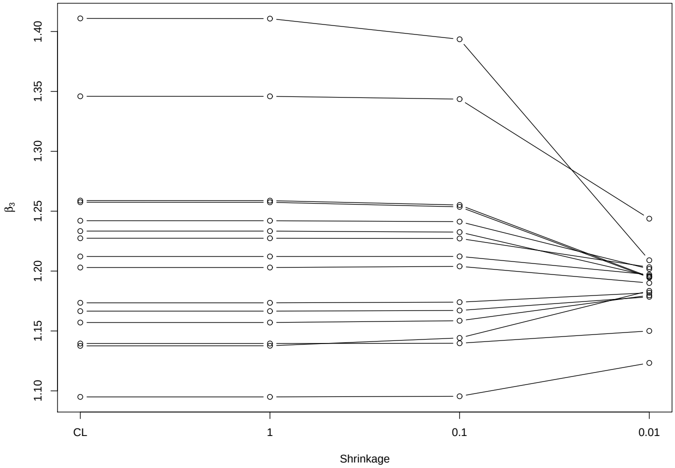
Shrinkage of β_1



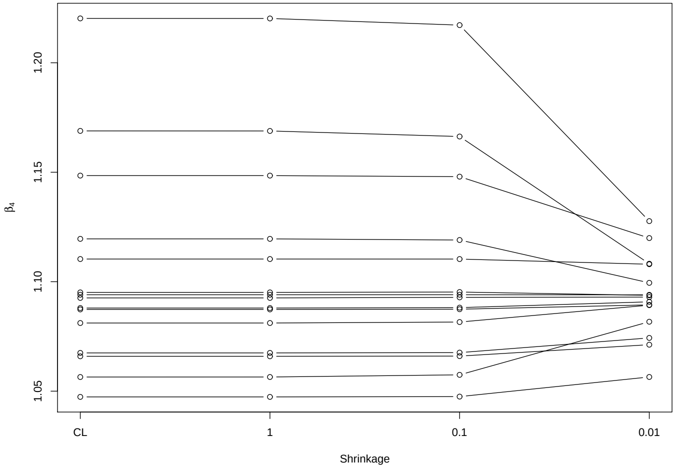
Shrinkage of β_2



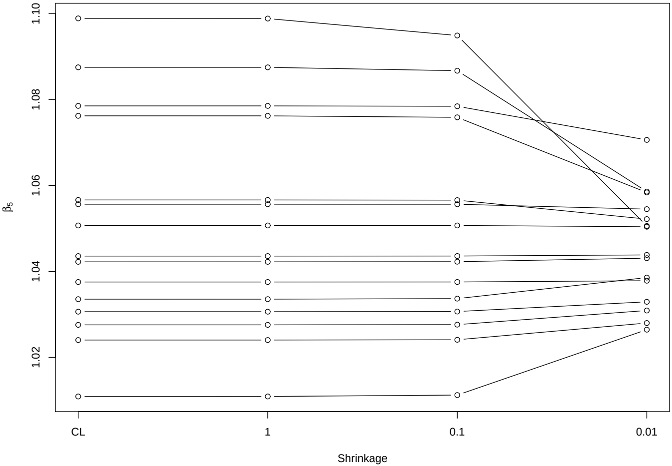
Shrinkage of β_3



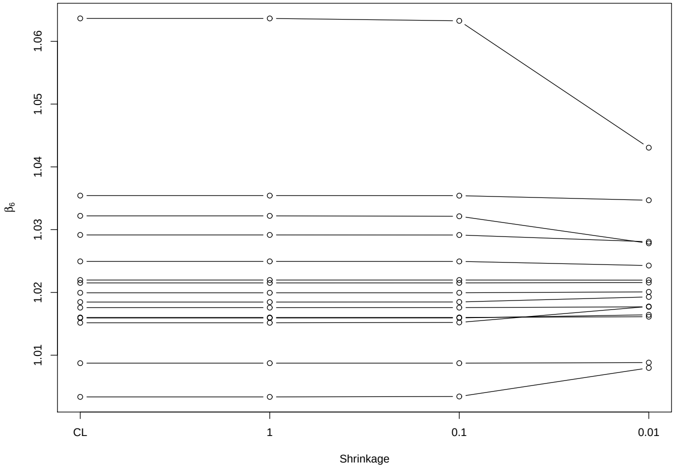
Shrinkage of β_4



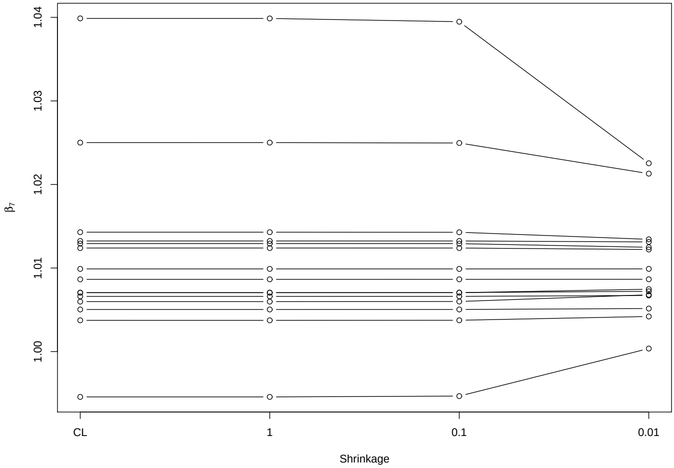
Shrinkage of β_5



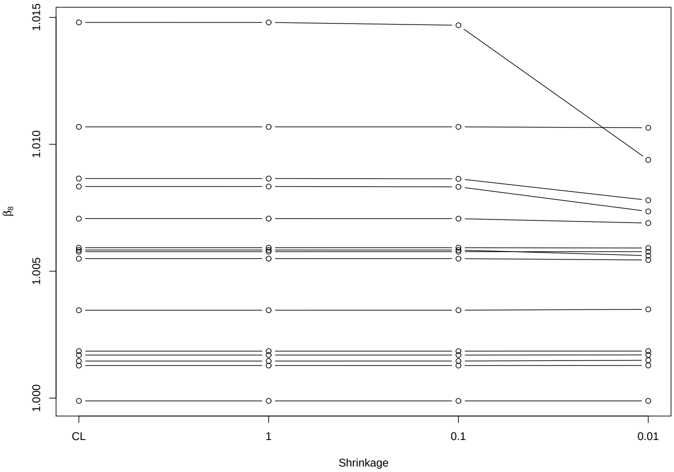
Shrinkage of β_6



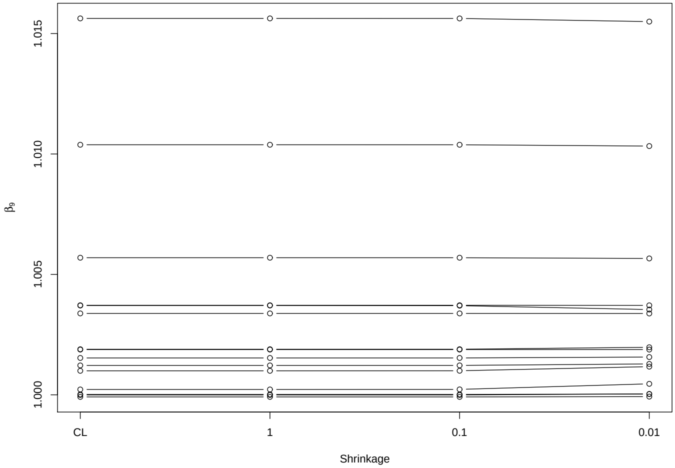
Shrinkage of β_7



Shrinkage of β_8



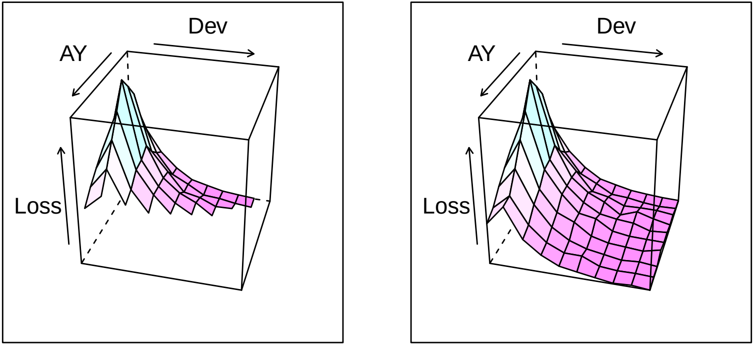
Shrinkage of β_9



Preview of Current Work

- I am currently working with my former student (Nathan Lally) on another way to think about the loss reserving triangle.
- When trying to incorporate the accident year, development year, and calendar year effects, you can run into issues of non-identifiability.
- Alternatively, we can think of the triangle as two-dimensional space.
- Then we can use all of the tools from spatial statistics to solve loss reserving problems.

Spatial Reserving Interpretation



State Farm Workers' Comp Schedule P Incremental Paid Claims

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