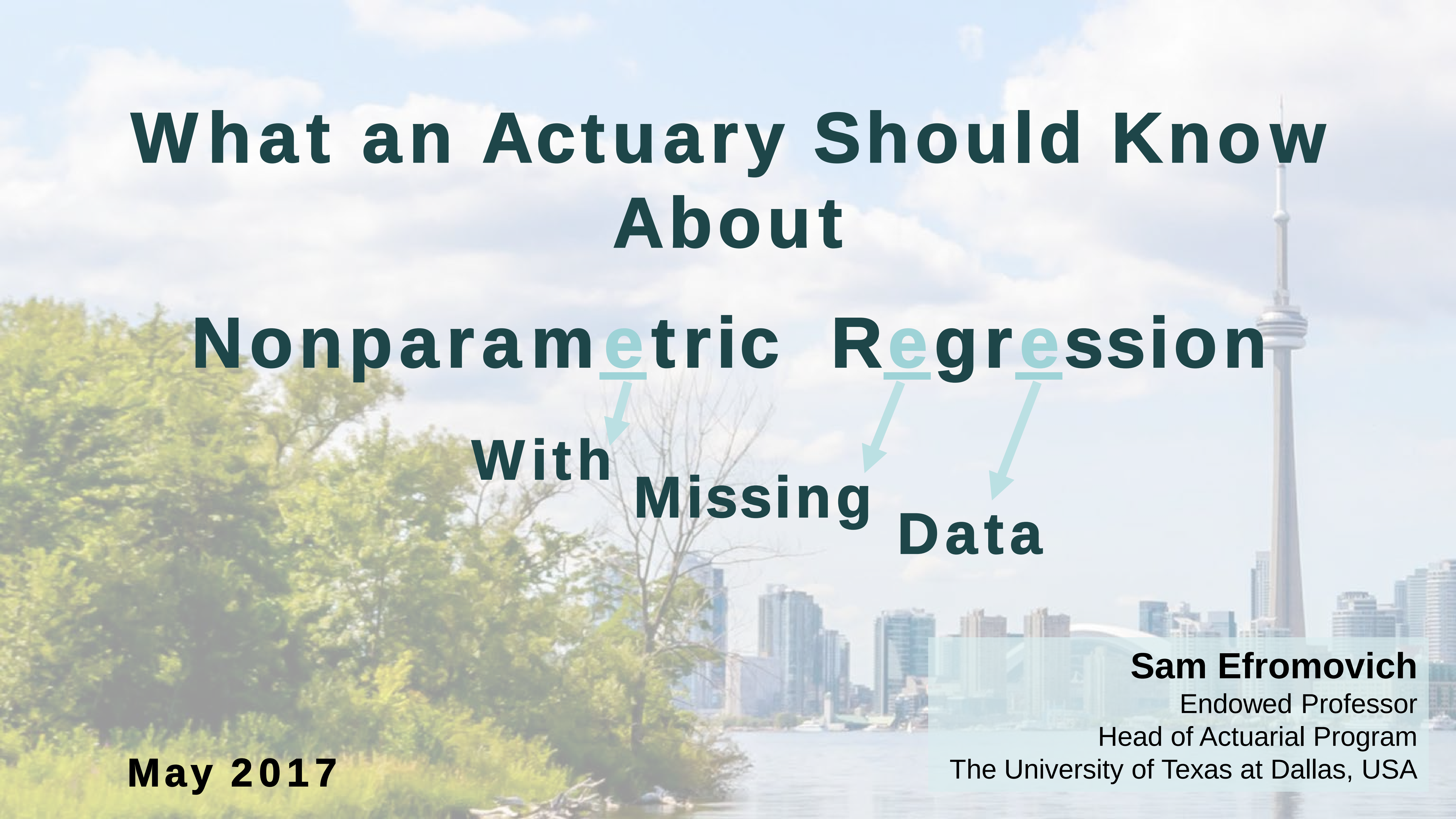

What an Actuary Should Know About

Nonparametric Regression

With Missing Data

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The “Whats” of Missing Data

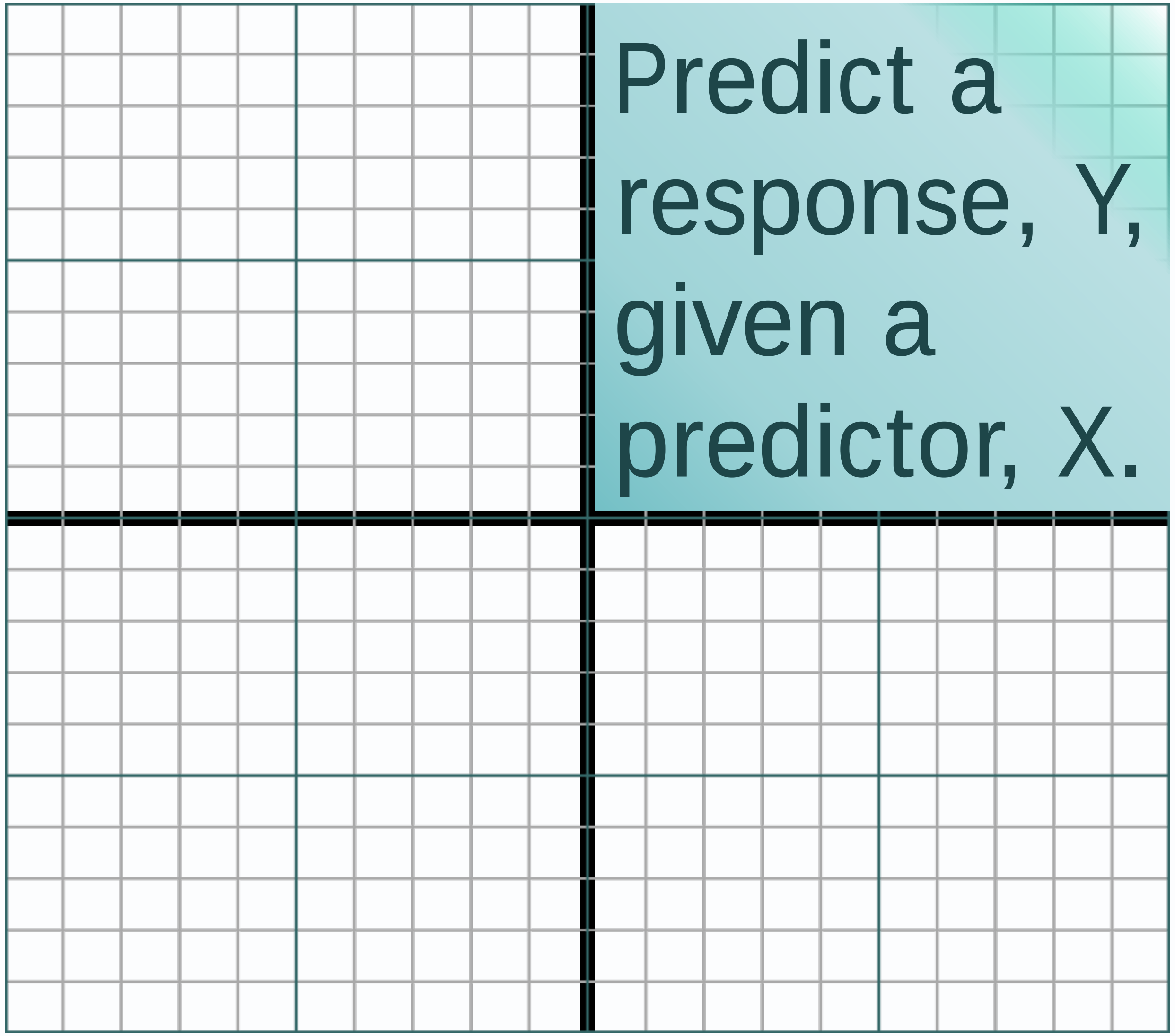
What?
So what?
Now what?

What is nonparametric regression?

What if I’m missing some data?

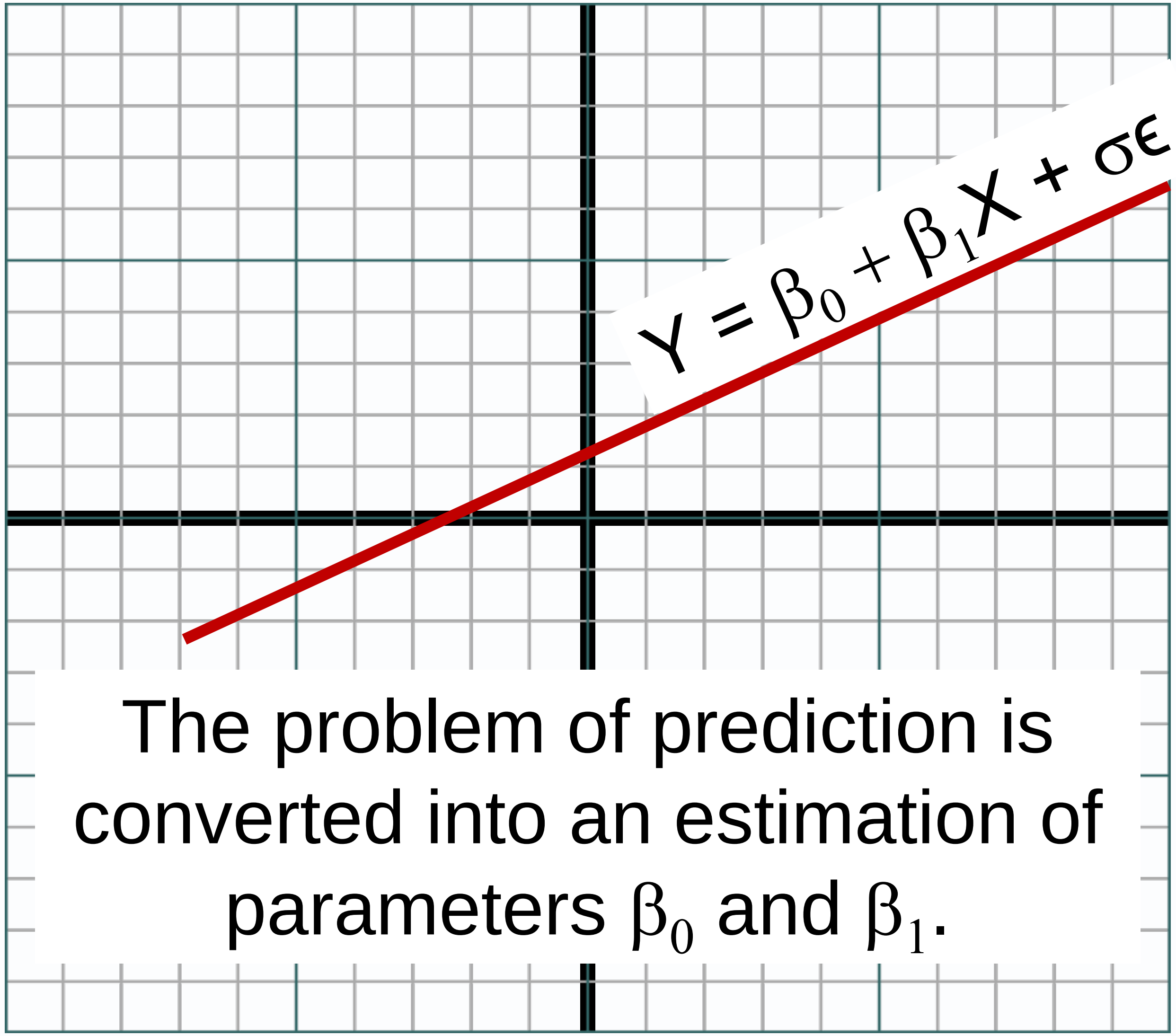
How do I deal with missing data?

Regression



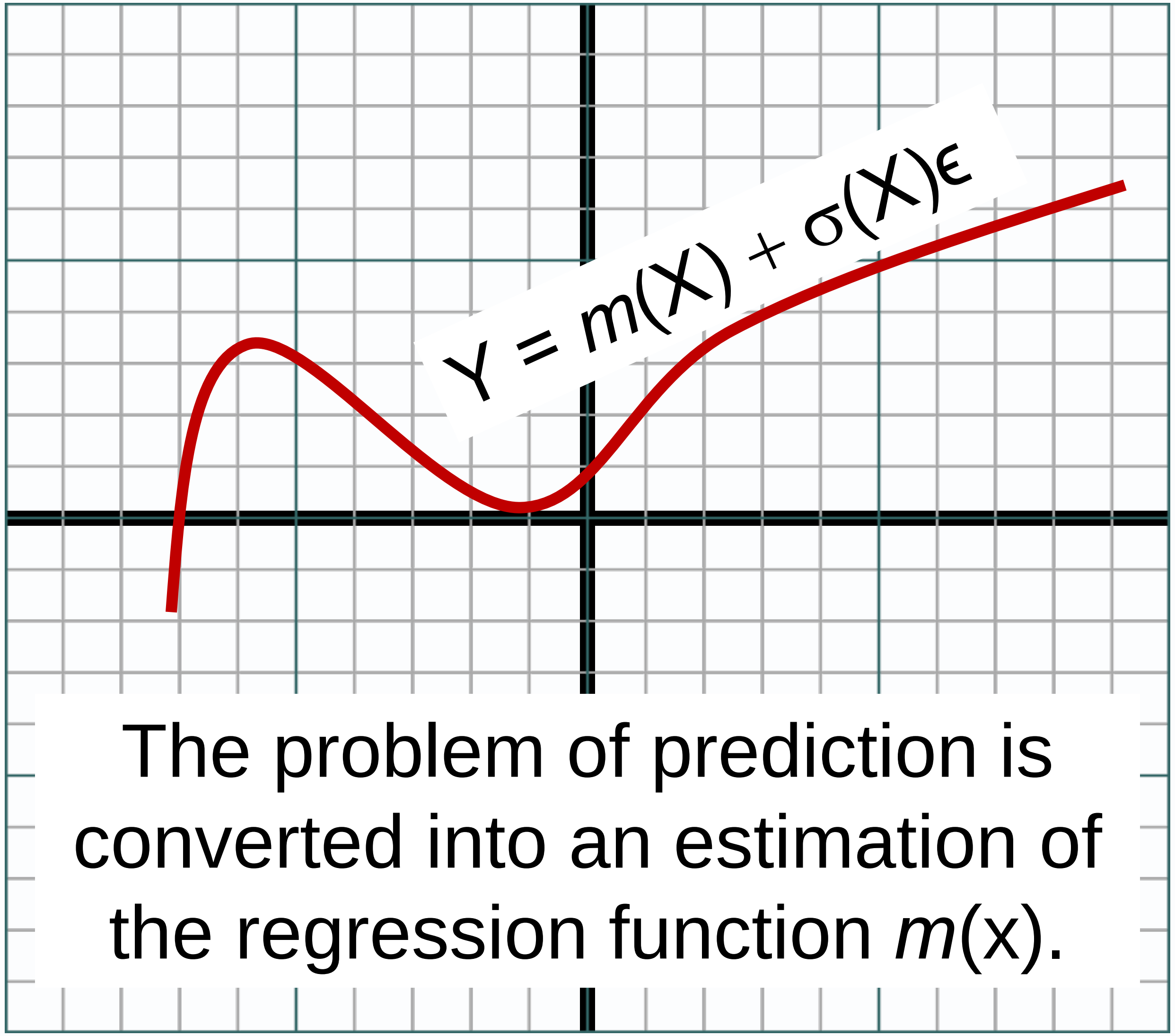
Predict a response, Y, given a predictor, X.

Parametric (Linear) Regression



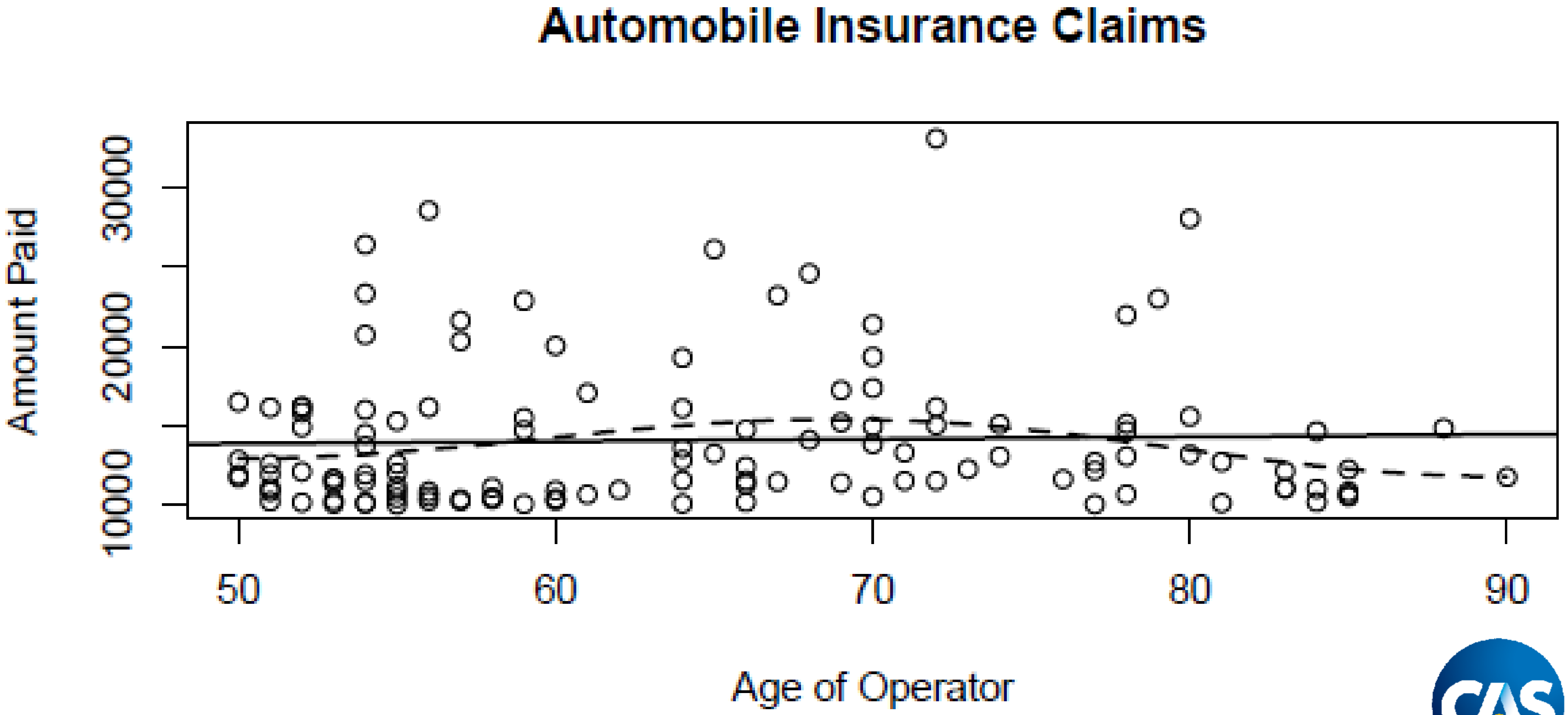
The problem of prediction is converted into an estimation of parameters β_0 and β_1.

Nonparametric Regression

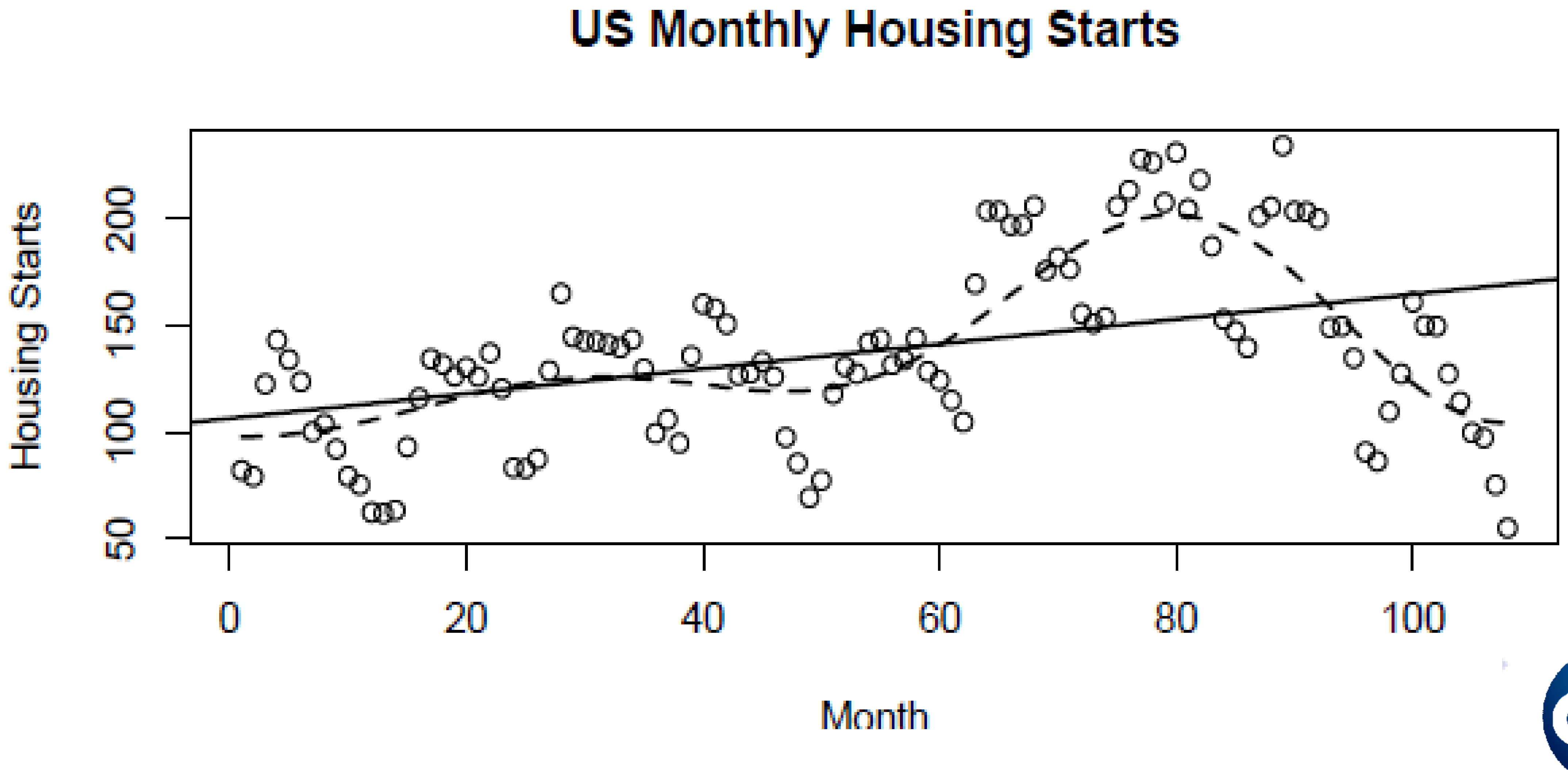


The problem of prediction is converted into an estimation of the regression function *m*(x).

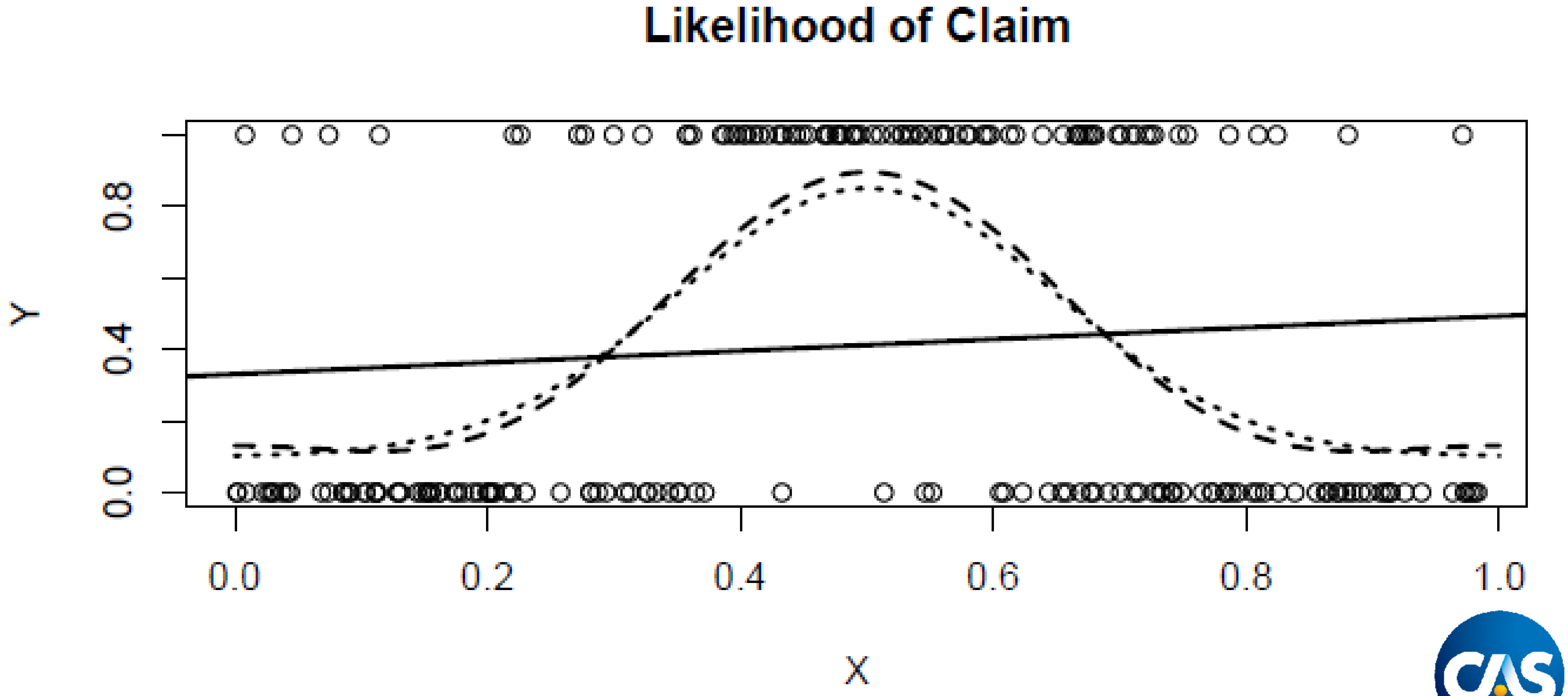
Linear vs. Nonparametric Regression



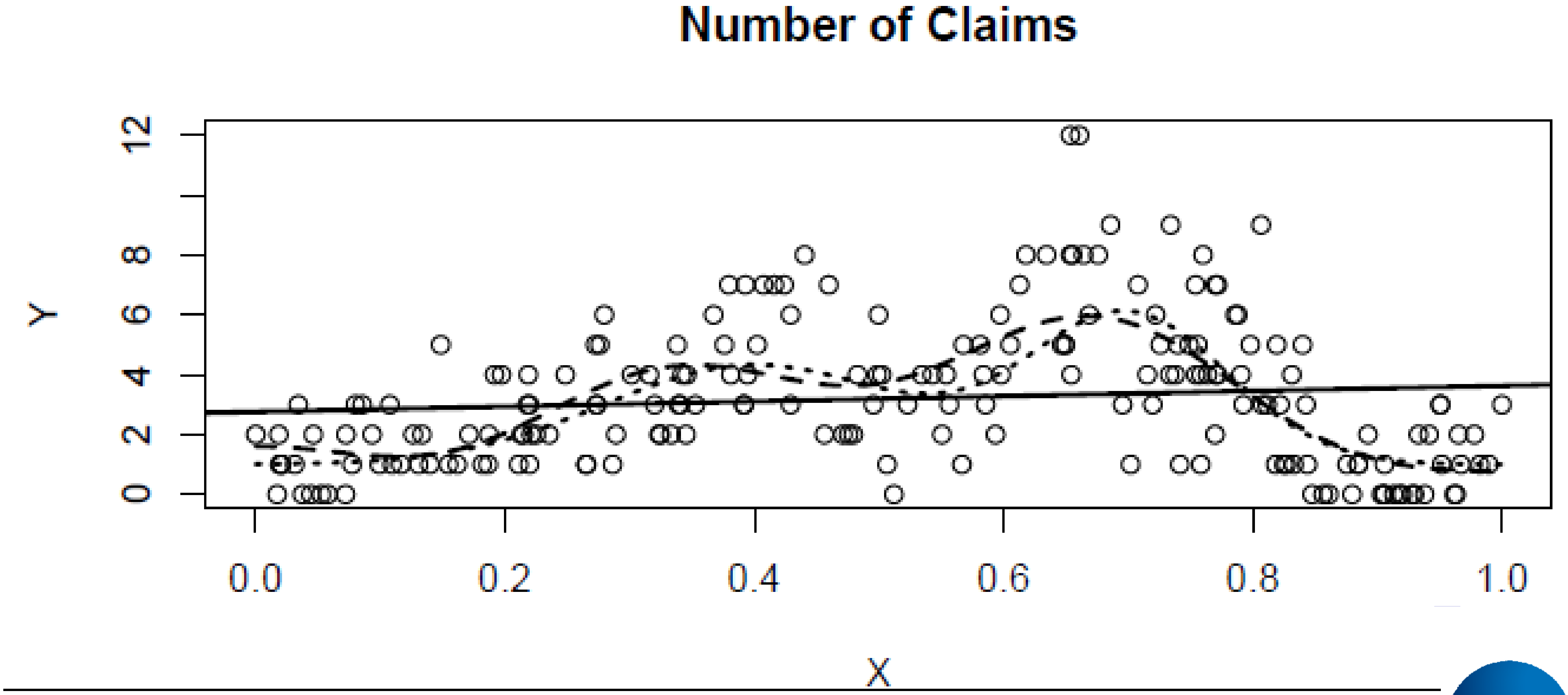
Linear vs. Nonparametric Regression



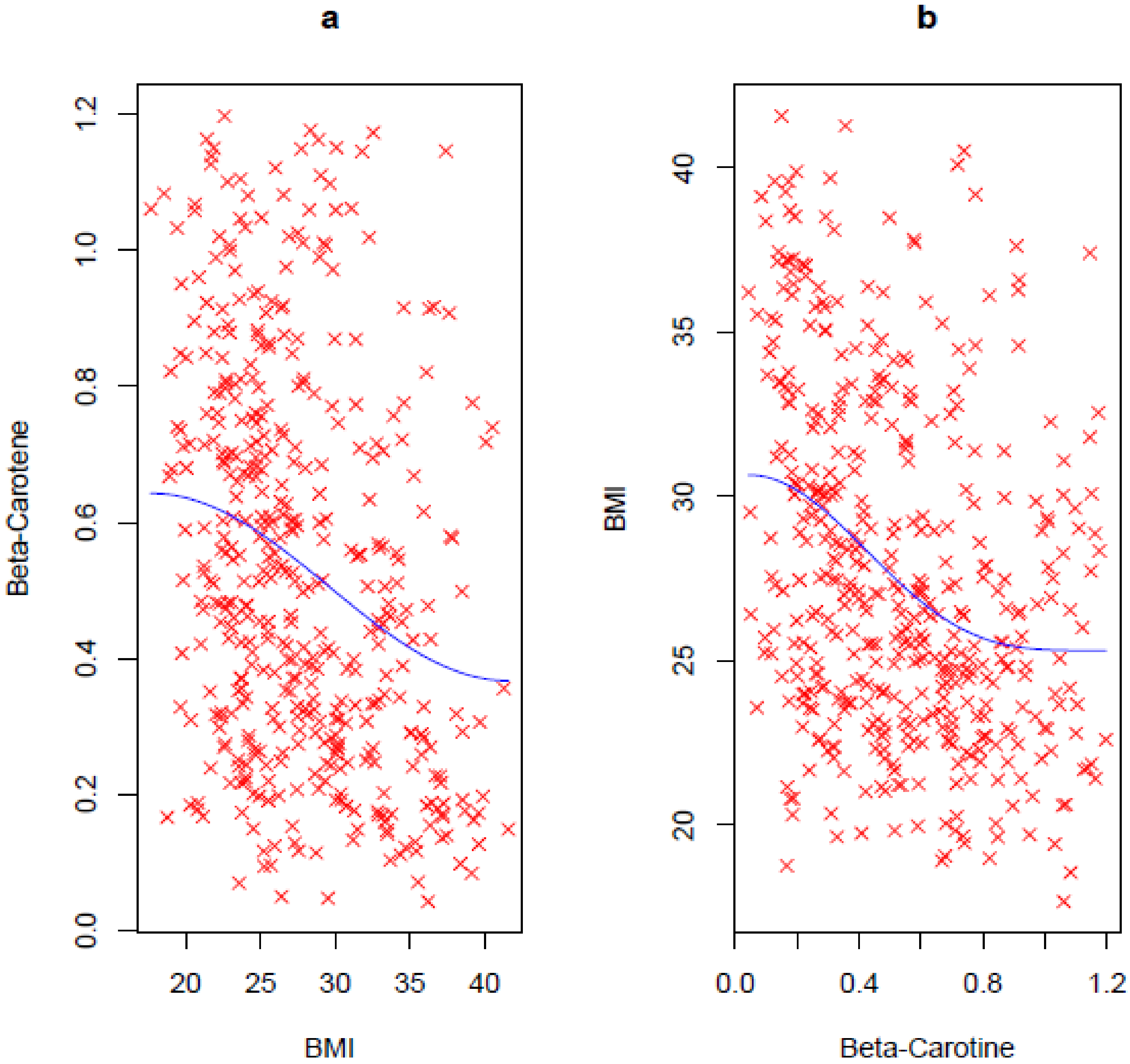
Simulated Bernoulli and Poisson Regression



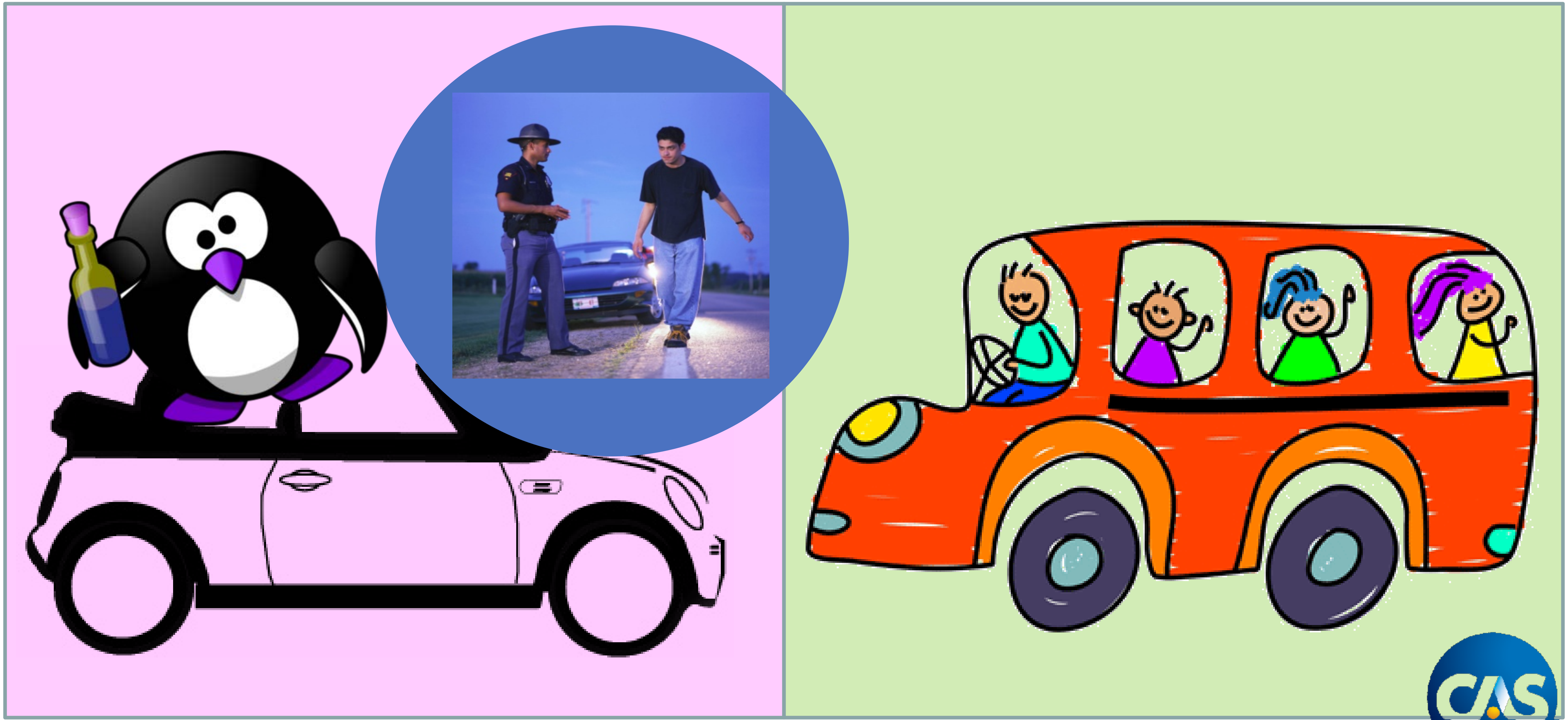
Simulated Bernoulli and Poisson Regression



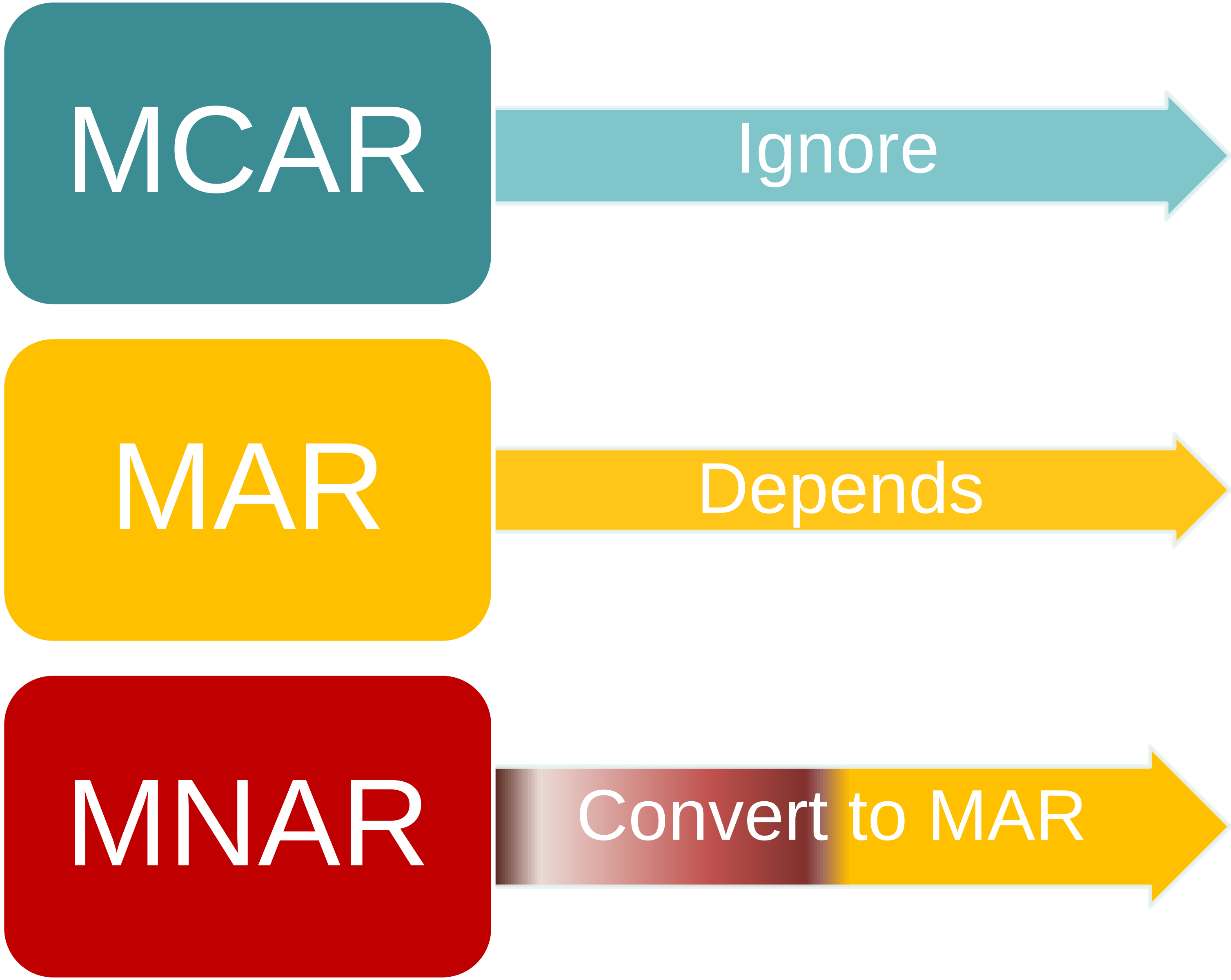
Nonparametric Regression: Body Mass Index vs. Beta Carotene



Example of Missing that Creates Biased Data



Main Types of Missing Data



MCAR → Ignore

MAR → Depends

MNAR → Convert to MAR

Regression with Missing Responses - MNAR

$$\Upsilon = m(X) + \sigma(X)\varepsilon$$

The underlying regression model

Available sample is from: $(A\Upsilon, A, X)$

A is the availability variable (Bernoulli)

Availability likelihood: □$(A=1|X,\Upsilon) = h(\Upsilon)$

Regression with Missing Responses – MNAR

$\Upsilon = \mathrm{m}(\mathrm{X}) + \sigma(\mathrm{X})\varepsilon$ ← The underlying regression model

Available sample is from: $(\mathrm{A}\Upsilon, \mathrm{A}, \mathrm{X})$

A is the availability variable (Bernoulli)

Availability likelihood: $\square(\mathrm{A}=1|\mathrm{X},\Upsilon) = h(\Upsilon)$

The joint density is (set $\psi(y,x) := h(y)f^{\Upsilon|\mathrm{X}}(y|x)$)

Regression with Missing Responses - MNAR

- The underlying regression model is

$$Y = m(X) + \sigma(X)\varepsilon.$$

- Available sample is from (AY, A, X) where: (i) The availability variable A is Bernoulli; (ii) The availability likelihood is

$$\mathbb{P}(A = 1|X, Y) = h(Y).$$

- The joint density is (set $\psi(y,x) := h(y)f^{Y|X}(y|x)$)

$$f^{X,AY,A}(x, ay, a) = [\psi(y,x)f^{X}(x)]^{a}[(1 - \int_{-\infty}^{\infty} \psi(y,x)dy)f^{X}(x)]^{1-a}$$

- We can estimate only the product $\psi(y,x) = h(y)f^{Y|X}(y|x)$, and this implies the MNAR (destructive missing) unless $h(y)$ is known.

Regression with Missing Responses - MAR

- Assume that the availability likelihood is

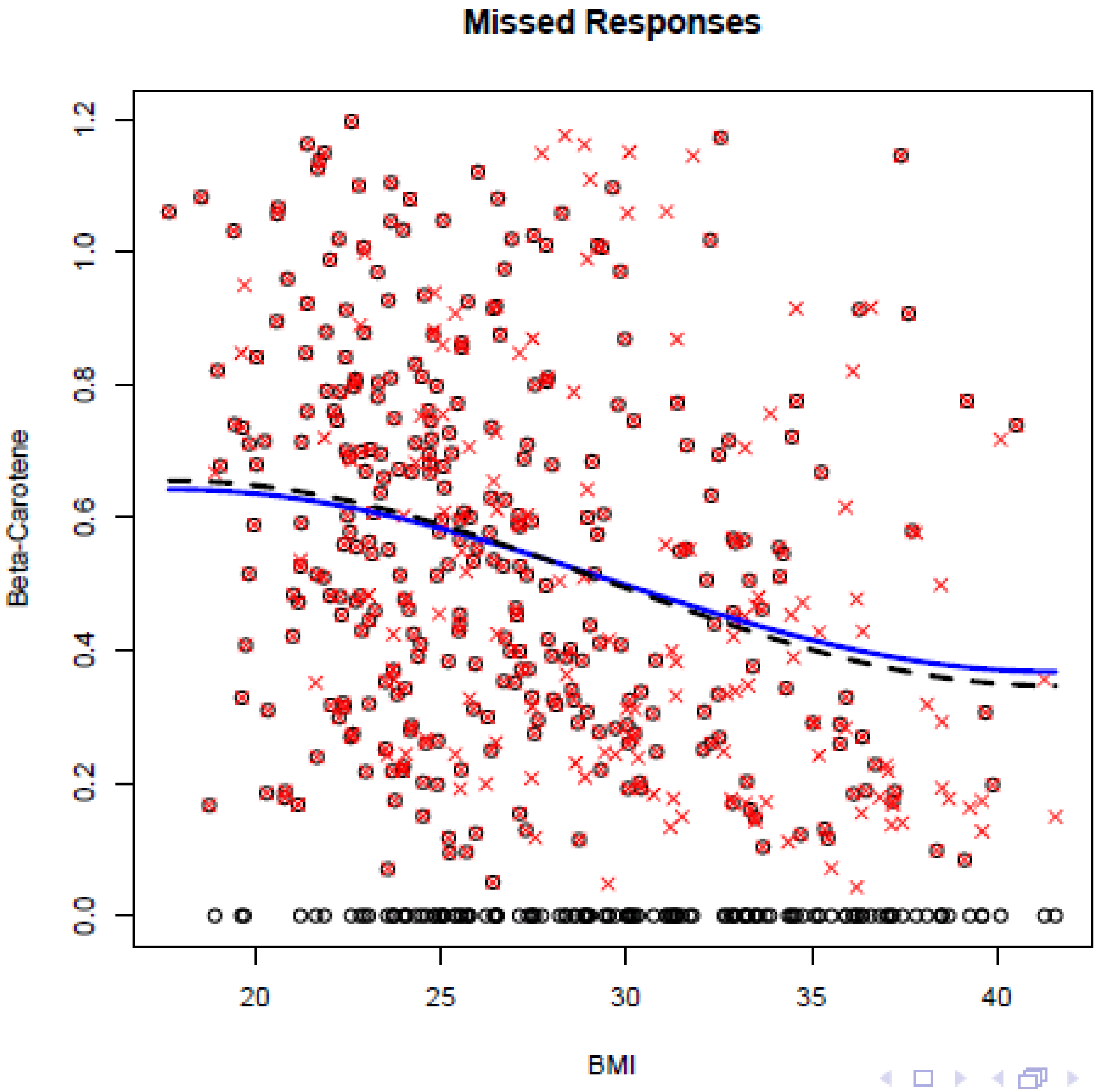
$$\mathbb{P}(A = 1|X, Y) = h(X).$$

- The joint (mixed) density of the triplet is

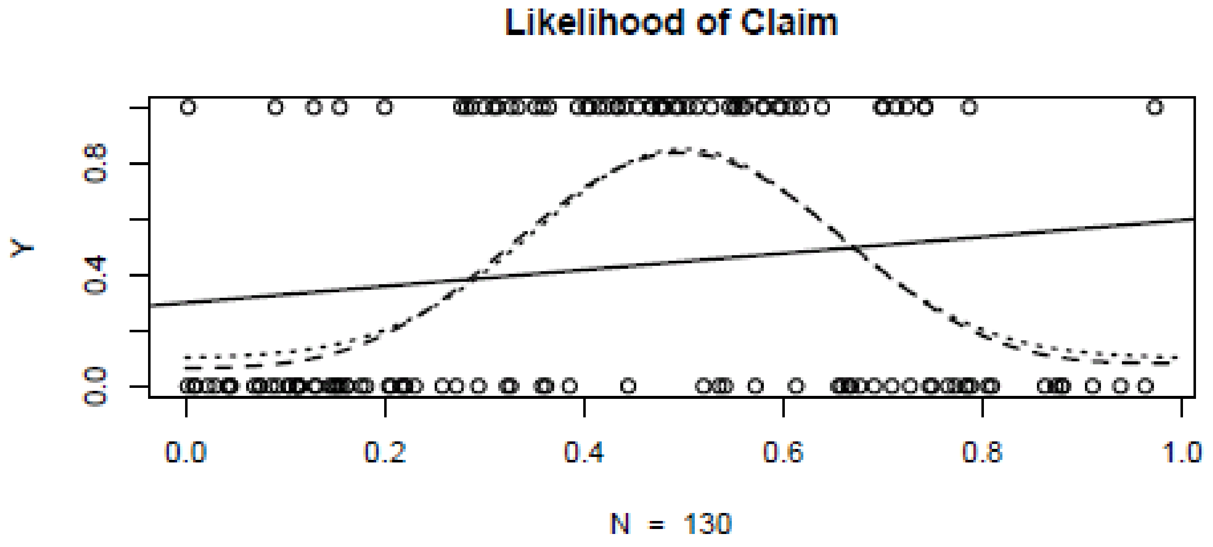
$$f^{X,AY,A}(x, ay, a) = [f^{Y|X}(y|x)h(x)f^X(x)]^a[(1 - h(x))f^X(x)]^{1-a}.$$

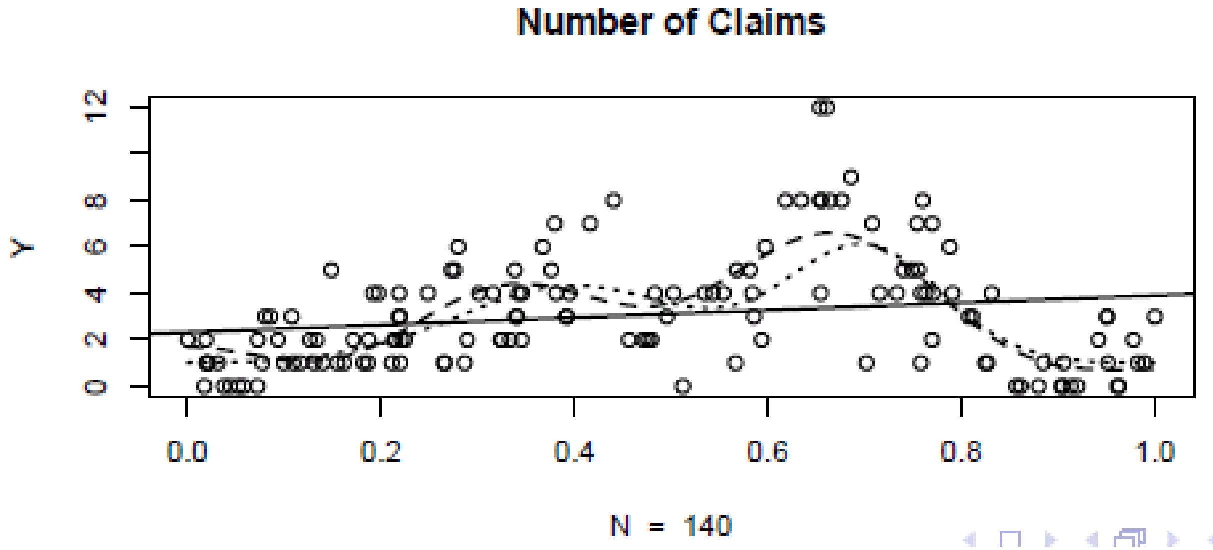
- In a subsample of complete cases the "new" design density is $g^X(x) = h(x)f^X(x)/q$, where $q := \int_0^1 h(x)f^X(x)dx = \mathbb{P}(A = 1)$. This is what allows us to use only complete cases.
- Binomial number $N := \sum_{l=1}^{n} A_l$ of complete cases; sequential estimation looks attractive.
- Traditional Methods: Imputation, Maximum Likelihood, EM, etc.; Vast Literature; Controversy.
- MAR typically does not affect rate of convergence, and the rate is the only issue that the mainstream literature is concerned about.

Regression with Missed Responses, $n = 441$, $N = 312$



Bernoulli and Poisson Regressions with Missed Responses, $n = 200$





Regression with MAR Predictors

- A sample is observed from (Y, AX, A) and the aim is to estimate $m(x) = \mathbb{E}\{Y|X = x\}$.

- It is assumed that the availability likelihood is (MAR)

$$\mathbb{P}(A = 1|X, Y) = \mathbb{P}(A = 1|Y) = h(Y).$$

- The joint density is

$$f^{AX,Y,A}(ax, y, a) = [f^{Y|X}(y|x)h(y)f^{X}(x)]^{a}[(1-h(y))f^{Y}(y)]^{1-a}, a \in \{0, 1\}.$$

- We could use only complete cases if $h(y)$ and $f^{X}(x)$ were known.

Regression Estimation for MAR Predictors

For the case of a complete case when $A = 1$,

$$f^{AX,Y,A}(x, y, 1) = f^{Y|X}(y|x)h(y)f^{X}(x).$$

Steps in regression estimation:

1. Estimate the density of response $f^{Y}(y)$ for $y = Y_l$ where $A_l = 1$.

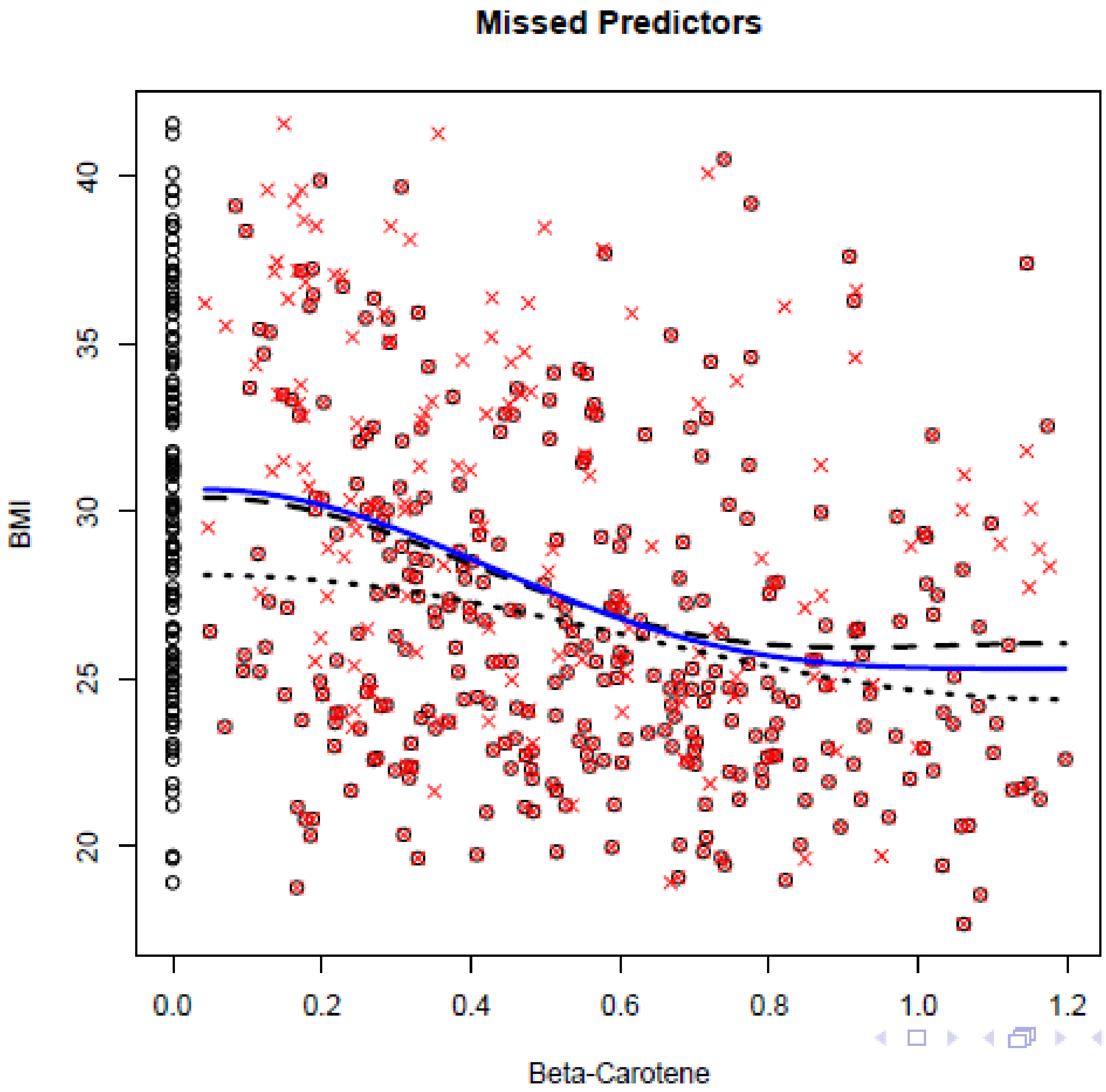
   Note: This is the only place where we need all n observations! (May use a smaller extra sample from Y.)

2. Estimate the availability likelihood $h(y)$ for $y = Y_l$ where $A_l = 1$.

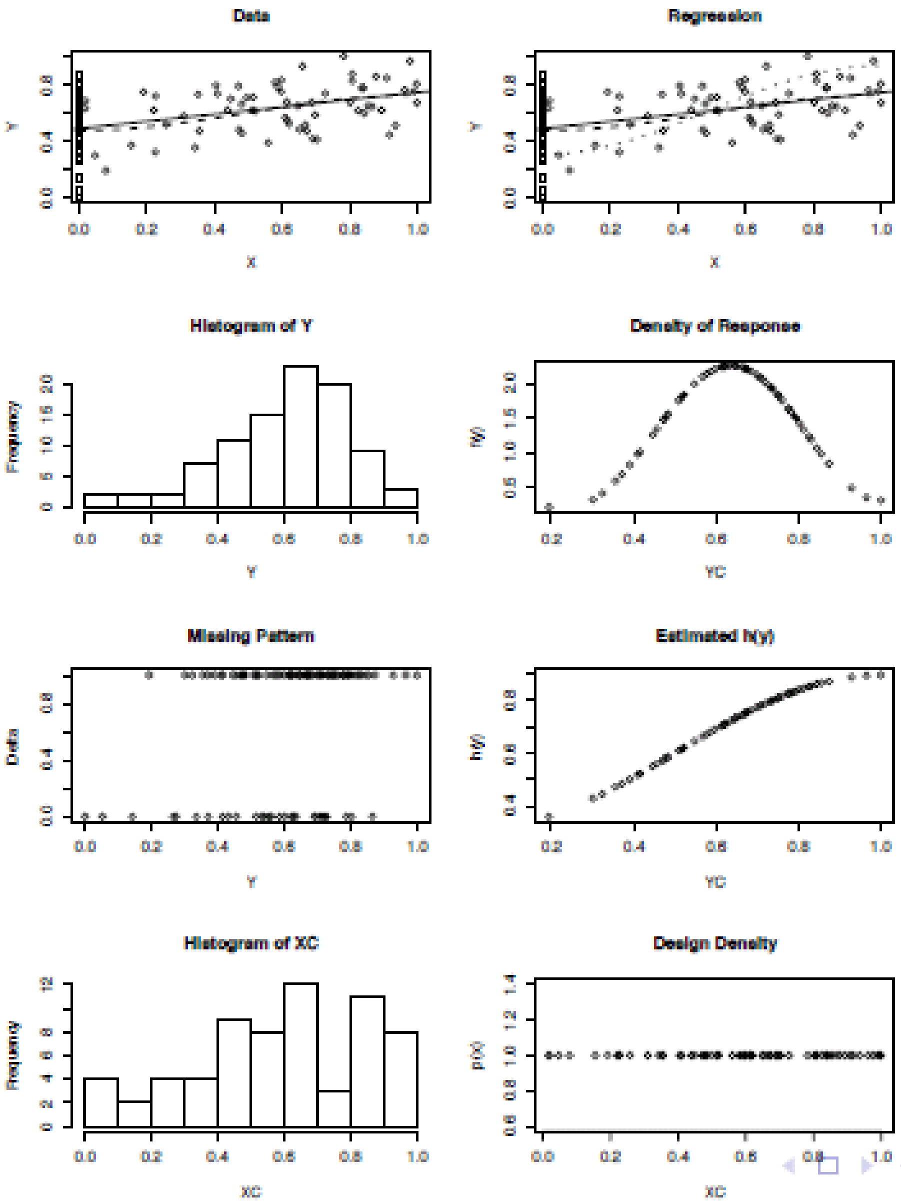
3. Estimate the design density $f^{X}(x)$ for $x = X_l$ where $A_l = 1$.

4. Estimate the regression function based on complete cases.

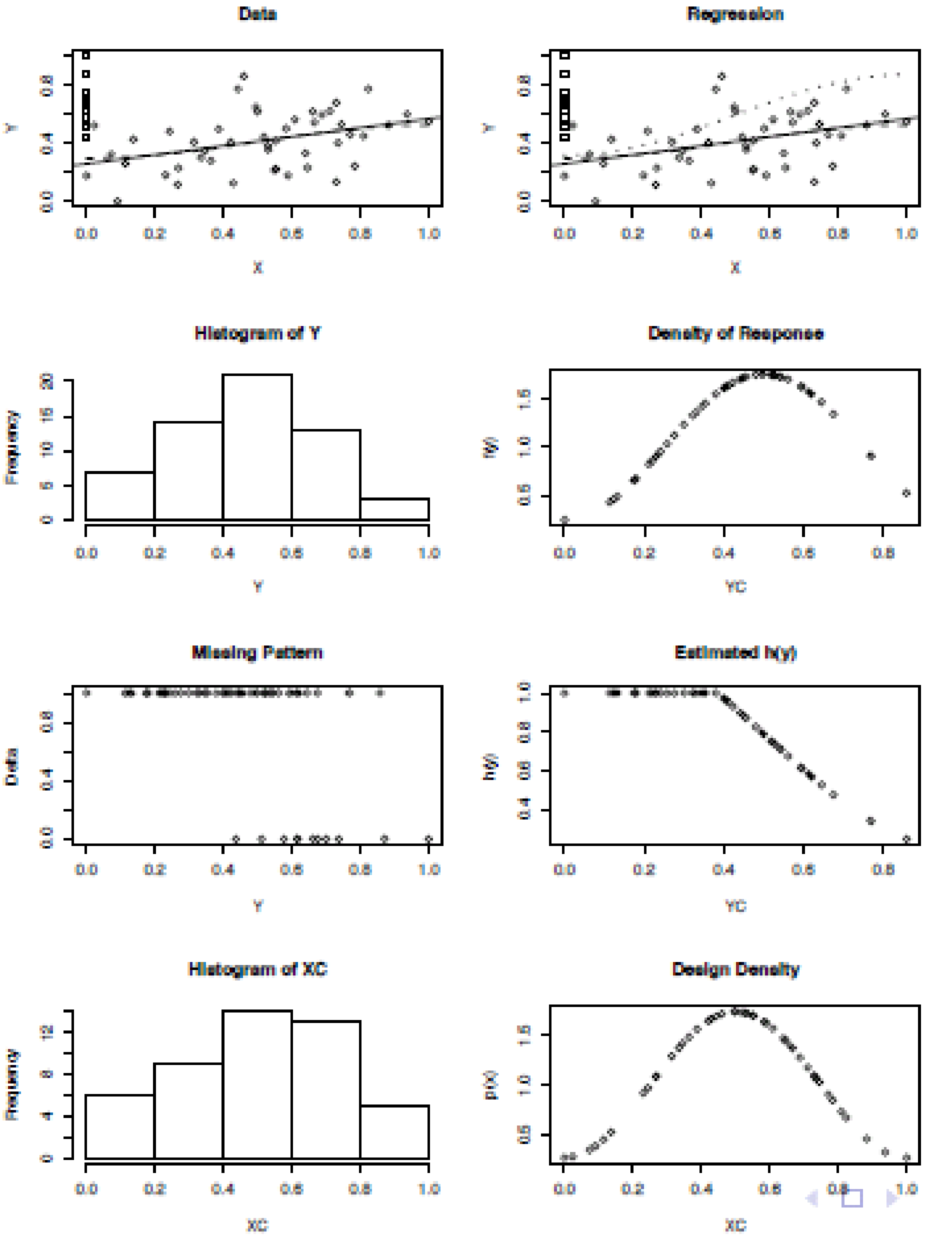
Regression with Missed Predictors, $n = 441, N = 312$



GPA (Y) versus Credit Score (X), Class A: $n = 94$, $N = 65$.



GPA (Y) versus Credit Score (X), Class B: $n = 58$, $N = 47$.



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